L.B. OKUN

WEAK INTERACTION OF ELEMENTARY PARTICLES

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FOREWORD

THIS book is based on lectures delivered by the author in 1960 and 1961 at the Institute of Theoretical and Experimental Physics of the Academy of Sciences of the USSR and at the Joint Institute of Nuclear Research. The book is meant for experimental physicists working in the field of elementary particles and high energies, and for young theoretical physicists specializing in this field.

The author has set himself two tasks: first, to make the reader familiar with the basic ideas and problems of the theory of the weak interaction of elementary particles; second, to make the reader familiar with the methods of calculation within the theory and to show him how the methods are to be applied.

The overall content of the book is concentrated about two pivotal hypotheses: the universality of the weak interaction, and the composite model of strongly interacting particles. These hypotheses allow the contents to be expounded in a more concise way and to retrace the connection between various problems in the theory of weak and strong interactions. Like every extrapolation, the hypotheses of the universality and of the composite model will undoubtedly be improved in the future and in part modified in the light of new experimental facts. In the form in which they are presented in the book, these hypotheses are to be considered as a "zero approximation".

The author is deeply grateful to A. I. Alikhanov and I. Y. Pomeranchuk, with whose initiative the lectures were delivered and the book was published, to I. Y. Kobzarev for his valuable advice, to V. B. Berestetskii who read the manuscript and made a number of useful remarks, and also to V. A. Kolkunov, E. P. Shabalin, V. V. Solovyev and N. S. Libova for their help in preparing the book for publication.

FOREWORD TO THE ENGLISH EDITION

In preparing this edition only minor amendments were made in the basic text of the book. To keep up, if only in part, with recent developments in this field, the chapter "Weak Interaction and Unitary Symmetry" was added. I take this opportunity to express my deep gratitude to V. B. Berestetskii, V. B. Mandeltsveig, I. Y. Pomerachnuk, J. Prentki, I. S. Shapiro, V. V. Sudakov, V. V. Vladimirskii and V. I. Zakharov for their discussions on various problems of unitary symmetry, which were very useful to me in writing the chapter quoted. I am in particular grateful to I. Y. Kobzarev, who read the manuscript and made a number of valuable remarks.

L. B. Okun'

CHAPTER 1

PARTICLES. INTERACTIONS. MODELS

CLASSIFICATION OF ELEMENTARY PARTICLES

All matter around us is made of elementary particles. All known processes and interactions in nature are due to the interaction between elementary particles.

The present known elementary particles can be divided into four classes. The first class contains only one particle—the photon. The second consists of leptons: electron, muon, neutrino, and their antiparticles. The third one comprises the mesons: three π-mesons and four K-mesons. The fourth one contains baryons (nucleons, Λ-, Σ- and Ξ-hyperons) and antibaryons. All these particles are enumerated in Table 1.

Beside the mesons and baryons enumerated in Table 1 other particles are known which are not included in the table because of their extremely short lifetimes. These "particles" live for such a short time that they manifest themselves only in the form of resonances in reactions at high energies. Such resonances are often considered as excited states of the π-meson, K-meson, nucleon, Λ-hyperon and so on.†

TYPES OF INTERACTIONS

There are four types of elementary-particle interactions, sharply differing from one another: the gravitational, the electromagnetic, the strong and the weak.

The gravitational interaction has a very small coupling constant (it is very weak), and, if its character does not change sharply at small distances, its role is insignificant for the phenomena that we are going to consider. Indeed, the energy of gravitational interaction of two protons set apart at a distance r is equal to

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† See tables of baryon and meson resonances on pp. 234 and 235 (note added in 1964).

$$\varkappa \frac{m^2}{r},$$

where $\varkappa$ is the Newtonian constant, $\varkappa = 6 \times 10^{-39}/m^2$, while m is the proton mass.† When $r \sim 1/m$ this energy amounts to $\sim 10^{-38}m$

TABLE 1. MASSES AND MEAN LIVES OF STABLE ELEMENTARY PARTICLES

Class	Particle	Spin	Mass (MeV)	Mean life (sec)
Photon	γ	1	0	∞
Leptons	ν_e	$\frac{1}{2}$	$<2 \times 10^{-4}$	∞
	ν_μ	$\frac{1}{2}$	<4	∞
	e	$\frac{1}{2}$	$0·511006 \pm 0·000002$	∞
	μ	$\frac{1}{2}$	$105·659 \pm 0·002$	$(2·2001 \pm 0·0008) \times 10^{-6}$
Mesons	$\pi^\pm$	0	$139·60 \pm 0·05$	$(2·551 \pm 0·026) \times 10^{-8}$
	π^0	0	$135·01 \pm 0·05$	$(1·80 \pm 0·29) \times 10^{-16}$
	$K^\pm$	0	$493·8 \pm 0·2$	$(1·229 \pm 0·008) \times 10^{-8}$
	K_1^0	0	$498·0 \pm 0·5$	$(0·92 \pm 0·02) \times 10^{-10}$
	K_2^0	0	$498·0 \pm 0·5$	$(5·62 \pm 0·68) \times 10^{-8}$
	η^0	0	$548·7 \pm 0·5$	$\Gamma < 10$ MeV
Baryons	p	$\frac{1}{2}$	$938·256 \pm 0·005$	∞
	n	$\frac{1}{2}$	$939·550 \pm 0·005$	$(1·01 \pm 0·03) \times 10^3$
	Λ^0	$\frac{1}{2}$	$1115·40 \pm 0·11$	$(2·62 + 0·02) \times 10^{-10}$
	Σ^+	$\frac{1}{2}$	$1189·41 \pm 0·14$	$(0·788 \pm 0·027) \times 10^{-10}$
	Σ^0	$\frac{1}{2}$	$1192·3 \pm 0·3$	$< 1·0 \times 10^{-14}$
	Σ^-	$\frac{1}{2}$	$1197·08 \pm 0·19$	$(1·58 \pm 0·05) \times 10^{-10}$
	Ξ^0	$\frac{1}{2}$	$1314·3 \pm 1·0$	$(3·06 \pm 0·40) \times 10^{-10}$
	Ξ^-	$\frac{1}{2}$	$1320·8 \pm 0·2$	$(1·74 \pm 0·05) \times 10^{-10}$
	Ω^-	?	1675 ± 3	$\sim 0·7 \times 10^{-10}$

The table is taken from the article: A. ROSENFELD *et al.*, *Rev. Mod. Phys.* **36**, 977 (1964).

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† Here and in what follows we shall use the system of units in which $\hbar = c = 1$. In this system the action and velocity are dimensionless quantities. Hence energy, momentum and mass have the same dimensions:

$$[E] = [p] = [m], \quad \text{since} \quad E = mc^2, \quad p = mv.$$

Dimensions of the time and length are also expressed in terms of those of mass:

$$[l] = [t] = [m^{-1}], \quad \text{since} \quad l = vt, \quad Et \sim \hbar, \quad pl \sim \hbar.$$

The reaction cross section has the dimensions $[\sigma] = [m^2]$, while the decay probability has the dimensions $[w] = [m]$.

It is easy to pass to ordinary units by taking into account the fact that $\hbar/mc \approx 2 \times 10^{-14}$ cm, $\hbar/mc^2 \approx 7 \times 10^{-25}$ sec, $1/m \approx 6 \times 10^{23}$ g^{-1}, m being the proton mass.

and, consequently, is negligibly small in comparison with the proton mass m.

The electromagnetic interaction, i.e. interaction of charged particles with photons and (in consequence of photon exchange) with each other, is characterized by the value of electric charge e. The energy of the Coulomb interaction of two protons set apart at a distance r is equal to α/r, where $\alpha = e^2 = \frac{1}{137}$, while e is the proton charge. For distances $r \sim m^{-1}$ between particles the energy of electromagnetic interaction amounts to αm and is small in comparison with the proper energy m of the particles. The relative weakness of electromagnetic interaction was used in constructing quantum electrodynamics, i.e. theory of interaction of electrons with photons. The smallness of the constant α allows one to consider electromagnetic interaction as a small perturbation and to develop in quantum electrodynamics the methods of perturbation theory. The mathematical methods of quantum electrodynamics enable calculations to be carried out with an accuracy exceeding that of contemporary experiments.

The strong interaction, i.e. the interaction between mesons and baryons, in contrast to the gravitational or electromagnetic, is short-range. The energy of the strong interaction between two particles at a distance larger than 10^{-13} to 10^{-12} cm is negligible, but at smaller distances ($r \sim 10^{-14}$ cm) the energy of strong interaction becomes of the same order of magnitude as the mass of the strongly interacting particles.†

If the photon emission by the electron is characterized by the dimensionless constant α, the pion emission by the nucleon can be characterized by the constant $g^2 \approx 14$. It is evident that such a strong interaction cannot be considered in terms of perturbation theory. Development of a theory of strong interaction has been, for about 30 years, one of the main problems in the physics of elementary particles. Recently there has been an intensive investigation of a dispersion approach to the theory of strong interaction, based on such general principles as causality and unitarity.

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† The term "strongly interacting particles" is cumbersome and inconvenient. Lack of a brief term for strongly interacting particles has led, for example, to the fact that decays into strongly interacting particles are called "non-leptonic". Such a term is inaccurate, since "non-leptonic" may also mean "photonic".

It is reasonable to call strongly interacting particles hadrons, and the corresponding decays—hadronic. In Greek the word hadros means "large", "massive", in contrast to the word leptos, which means "small", "light". The term hadron refers to long-lived mesons and baryons, as well as to resonances.

The weak interaction, responsible, in main, for elementary-particle decays, is very short-range: its effective radius is apparently substantially smaller than that of the strong interaction. It amounts to about 10^{-17} cm. The radius of the weak interaction and, consequently, its dependence on the momentum of the interacting particles, are only beginning to be investigated. We shall return to this later on. At low momenta the weak interaction can be characterized by the weak interaction constant $G = 10^{-5}/m^2$. The corresponding energy of the weak interaction of two protons set apart at a distance $1/m$ amounts to about $10^{-5}\ m$. This allows one to consider the weak interaction, in a number of cases, as a small perturbation, and to calculate slow processes due to it in the first order of a perturbation theory in the constant G. We shall discuss the validity of this procedure when we come back to the problem of the radius of the weak interaction (see Chapter 18).

ARE ELEMENTARY PARTICLES ACTUALLY ELEMENTARY?

It is now time to ask the question: are elementary particles actually elementary? This is quite natural when one takes into account the number of elementary particles that seem to exist at present. If, in addition, the number of extremely unstable excited states (resonances) is added in, the sum is not lower than that of elements Mendeleyev knew. It is therefore understandable why numerous attempts to "lower" the number of elementary particles have been made. The most radical of these attempts proceed from the assumption that all observed particles and interactions are manifestations of a unique non-linear spinor field whose non-linearity is characterized by a constant. Unfortunately, in the concrete development of this attractive idea (for example, in studies by the Heisenberg group) there are very serious difficulties. Such quantum numbers as electric charge, baryonic charge, strangeness, leptonic charge, leptonic strangeness (which distinguishes the muon from the electron) cannot as yet be obtained as characteristic numbers of some unique fundamental Lagrangian.

However, the question need not necessarily be formulated in the form that either all particles (fields) are elementary, or that there is only one fundamental field. One may adopt the standpoint that all elementary particles are characteristic states of a system of several fundamental fields interacting with one another. In this case individual fields act like carriers of a certain set of quantum numbers. This idea underlies numerous composite models of elementary particles. In such an approach it is natural to take as a principle of the theory the requirement that the number of fundamental fields and constants of interaction between them should be minimum.

COMPOSITE MODELS

Because the largest family of particles are strongly interacting, it is these that it is most important to reduce to a minimum number of fundamental ones. Besides, the large intensity of strong interactions leads one to expect, at least in principle, that the energies of interaction and, consequently, the mass differences between the particles, are of an order of magnitude comparable to the masses themselves. In 1949 Fermi and Yang pointed out that the π-meson may represent a bound state of the nucleon and antinucleon. In this case the binding energy must reach the enormous value of $940 \times 2 - 140 = 1740$ MeV. Data on the interaction of antinucleons with nucleons, obtained subsequently, do not contradict the assumption of such a strong attraction between these particles. With the discovery of strange particles a number of models appeared, in which some particles were chosen as fundamental ones, whereas the remaining particles were obtained as secondary or composite ones. The most economical model of such a type is the one proposed in 1956 by the Japanese physicist Sakata. In the Sakata model, fundamental particles are the three baryons: proton p, neutron n, and lambda hyperon Λ, and the corresponding antibaryons: antiproton $\tilde{p}$, antineutron $\tilde{n}$ and antilambda $\tilde{\Lambda}$. Thus, instead of more than twenty metastable mesons and baryons and about the same (or, maybe, a substantially larger) number of "excited" particles—resonances—the model contains three fundamental particles.

ISOTOPIC MULTIPLETS

One of the first successes of this model was that it allowed one to explain simply the classification of strongly interacting particles—the so-called scheme of isotopic multiplets, established by Gell-Mann and Nishijima few years earlier.

We have already mentioned that mesons are divided into the two groups π and K, while baryons are divided into the four groups: nucleons, Λ-hyperons, Σ-hyperons and Ξ-hyperons. These groups are called *isotopic multiplets*. The term is somewhat inaccurate, because the particles pertaining to a given multiplet are, according to nuclear physics, not isotopes but isobars, since they are of the same mass and different charge. It would be more correct to call these multiplets isobaric, as has been done, for example, in the journal *Nuclear Physics*. But, following tradition and custom, we shall, in what follows, use the term *isotopic*.

The particles pertaining to a given multiplet have not only almost the same masses but also analogous strong interactions. (The small mass differences within one multiplet are apparently of electromagnetic origin.) Hence different particles belonging to a given multiplet can be considered to be different charge states of the same particle. Thus, proton and neutron represent two charge states of the nucleon; Σ^+, Σ^- and Σ^0 represent three charge states of the Σ-hyperon, and so on. For describing isotopic multiplets it is convenient to make use of the isotopic-spin formalism (for more details see p. 83).

The number N of particles in a given multiplet is expressed directly in terms of the isotopic spin T characterizing this multiplet:

$$N = 2T + 1.$$

From this formula and from Table 1 it follows that the isotopic spin of Λ-hyperon is equal to zero (isosinglet); the isotopic spin of K-mesons, nucleons and Ξ-hyperons is equal to $\frac{1}{2}$ (isodublets), and the isotopic spin of π-mesons and Σ-hyperons is equal to 1 (isotriplets).

Not only metastable particles but also resonances possess a definite isotopic spin. The strong interaction conserves isotopic spin. This conservation law is due to the so-called isotopic invariance of strong interaction.

To the particles belonging to a given multiplet there correspond different values of the projection of isotopic spin onto the z-axis of a so-called isotopic space. Thus, to the proton and neutron there correspond, respectively, $T_3 = +\frac{1}{2}$ and $T_3 = -\frac{1}{2}$, while to the π^+, π^0 and π^- meson there correspond, respectively, $T_3 = +1$, $T_3 = 0$ and $T_3 = -1$.

The electric charge Q of a particle, its baryon number n and the third component of isotopic spin T_3 determine its strangeness S:

$$Q = T_3 + \frac{n}{2} + \frac{S}{2}.$$

It can easily be seen that $S = 0$ for π-mesons and nucleons, $S = +1$ for K-mesons, $S = -1$ for Λ- and Σ-hyperons and $\tilde{K}$-mesons, and $S = -2$ for Ξ-hyperons. Particles for which $S \neq 0$ are called in the literature strange particles. Strangeness conservation (equivalent to T_3 conservation) accounted for numerous specific properties of the reactions involving the creation and mutual conversion of strange particles. On the basis of the scheme of isotopic multiplets the existence of Σ^0- and Ξ^0-particles, unknown at the time, and dual properties of neutral K-mesons were predicted (see Chapter 15).

SCHEME OF ISOMULTIPLETS IN THE SAKATA MODEL

Assume that at small distances ($\sim 10^{-14}$ cm) there is attraction between an arbitrary fundamental baryon (p, n, Λ) and an arbitrary fundamental antibaryon ($\tilde{p}$, $\tilde{n}$, $\tilde{\Lambda}$), whereas there is repulsion between two baryons (or two antibaryons). In attracting each other the baryon and antibaryon form a bound state—a meson. Thus, a proton and an antineutron form a π^+-meson, a neutron and an antiproton form a π^--meson, p and $\tilde{\Lambda}$ form a K^+-meson, n and $\tilde{\Lambda}$ a K^0-meson, Λ and $\tilde{p}$ a K^--meson, Λ and $\tilde{n}$ a $\tilde{K}^0$-meson. π^0-meson is a combination of $p\tilde{p}$ and $n\tilde{n}$ with isotopic spin unity:

$$\pi^0 = \frac{1}{\sqrt{2}}(p\tilde{p} - n\tilde{n}).\dagger$$

In analogy with mesons, "composite" hyperons are easy to obtain. They contain at least three particles: two baryons and an

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† In essence, we operate here and in what follows with symbols of the type p and $\tilde{p}$ as with wave functions of corresponding particles and antiparticles. The states $p\tilde{p}$ and $n\tilde{n}$ may go over into each other. Their linear combinations $\frac{1}{\sqrt{2}}(p\tilde{p} - n\tilde{n})$ and $\frac{1}{\sqrt{2}}(p\tilde{p} + n\tilde{n})$ do not go over into each other. The first of these represents a component of an isotopic vector, while the second one represents an isotopic scalar (for more details see p. 94).

antibaryon. Thus, $\Sigma^+ = p\tilde{n}\Lambda, \Sigma^- = n\tilde{p}\Lambda, \Sigma^0 = \frac{1}{\sqrt{2}}(p\tilde{p} - n\tilde{n})\Lambda$. Analogously $\Xi^- = \tilde{p}\Lambda\Lambda, \Xi^0 = \tilde{n}\Lambda\Lambda$. We could write the structure of these hyperons even more briefly: $\Sigma = \Lambda\pi$, $\Xi^- = \Lambda\tilde{K}$, where π and $\tilde{K}$ are themselves composite particles. Table 2 represents the scheme of composite particles in the Sakata model.

TABLE 2. SAKATA MODEL

Mesons	S	T_3	Hyperons	S	T_3
$\pi^+ = p\tilde{n}$	0	$+1$	$\Sigma^+ = p\tilde{n}\Lambda$	-1	$+1$
$\pi^- = \tilde{p}n$	0	-1	$\Sigma^- = \tilde{p}n\Lambda$	-1	-1
$\pi^0 = \frac{p\tilde{p} - n\tilde{n}}{\sqrt{2}}$	0	0	$\Sigma^0 = \frac{p\tilde{p} - n\tilde{n}}{\sqrt{2}}\Lambda$	-1	0
$K^- = \tilde{p}\Lambda$	-1	$-\frac{1}{2}$	$\Xi^- = \tilde{p}\Lambda\Lambda$	-2	$-\frac{1}{2}$
$\tilde{K}^0 = \tilde{n}\Lambda$	-1	$+\frac{1}{2}$	$\Xi^0 = \tilde{n}\Lambda\Lambda$	-2	$+\frac{1}{2}$
$K^+ = p\tilde{\Lambda}$	1	$+\frac{1}{2}$			
$K^0 = n\tilde{\Lambda}$	1	$-\frac{1}{2}$			

S = strangeness, T_3 = third component of the isotopic spin, ~ denotes antiparticles.

In order that composite mesons and hyperons may be produced, a strong attraction between fundamental particles is sufficient, whereas for grouping these composite particles into multiplets it is necessary that the strong interaction should be isotopically invariant. As will be seen in what follows, this means that the strong interactions of the proton and neutron, as well as their masses, must be equal.

QUASIPARTICLES IN THE SAKATA MODEL

A remarkable property of the Sakata model is the fact that, beside a simple interpretation of known "long-lived" particles, the model provides a natural explanation of the existence of numerous *quasiparticles—resonances.* In order that a composite particle may exist two conditions are necessary: first, between the constituent fundamental particles there must exist a sufficiently strong attraction in order that they may form a bound state; second, the mass of this state must be smaller than the sum of the masses of particles which go over into this state as a result of the strong interaction. If the second condition is not fulfilled, then there arises a *quasiparticle* the lifetime of which is of the order of 10^{-21} to 10^{-23} sec. Therefore the mass spectrum of long-lived particles is in a sense "accidental". For example, Σ^+-hyperon represents a *compound* state of p, $\tilde{n}$ and Λ with a very high binding energy (of the order of 2 BeV). But if this energy were lower by only hundred MeV, Σ^+-hyperon would change from a universally recognized particle into an extremely unstable state, decaying in nuclear time according to the scheme $\Sigma^+ \to \Lambda^0 + \pi^+$. The presence of a strong interaction between baryons and antibaryons provides a basis for assuming that in nature there must be many such "unrealized" particles. Thus, for instance, in addition to the π-meson two other neutral pseudoscalar states of a system baryon + antibaryon with isotopic spin zero are possible. These states are a linear superposition of the systems lying on the diagonal of Table 3:

TABLE 3. PSEUDOSCALAR MESONS IN THE SAKATA MODEL

	p	n	Λ
$\tilde{p}$	$p\tilde{p}$	π^-	K^-
$\tilde{n}$	π^+	$n\tilde{n}$	$\tilde{K}^0$
$\tilde{\Lambda}$	K^+	K^0	$\Lambda\tilde{\Lambda}$

$$\frac{\alpha}{\sqrt{2}}(p\tilde{p} + n\tilde{n}) + \beta\Lambda\tilde{\Lambda},$$

$$\frac{\beta}{\sqrt{2}}(p\tilde{p} + n\tilde{n}) - \alpha\Lambda\tilde{\Lambda},$$

$$\alpha^2 + \beta^2 = 1.$$

It is likely that the recently discovered η^0-meson represents one of these states.

If the attraction between a baryon and an antibaryon in a state with spins parallel is not much weaker than in a state with spins antiparallel, then, beside pseudoscalar pions and kaons (which represent the ${}^1S_0$ states of the system baryon + antibaryon), there must exist the following vector mesons: three ϱ-particles and four K^*-particles (representing the ${}^3S_1$ states of the system baryon + antibaryon). In nuclear times ($\sim 10^{-22}$ sec) they will decay according to the schemes

$$\varrho \to 2\pi,$$
$$K^* \to K + \pi.$$

In addition to these seven vector mesons, there may exist two more neutral vector mesons with an isotopic spin equal to zero. It is possible that ω^0-meson—discovered experimentally in the form of a resonant state of three π-mesons—represents one of these vector mesons.

The establishment of the mass spectrum of elementary particles is one of the main tasks of the physics of elementary particles. In the literature the hope has often been expressed that knowledge of the mass spectrum will allow hidden regularities of strong interactions to be revealed, just as investigation of optical atomic spectra led to the quantum model of the atom and, subsequently, to quantum mechanics. From what we have said it is evident that the establishment of the spectrum of resonances (of their energies, spins, parities) is an integral part of the project of establishing the mass spectrum of elementary particles and, consequently, is a task of paramount importance.

UNIVERSAL, UNITARY-SYMMETRIC STRONG INTERACTION

In the Sakata model all strongly interacting particles consist of three baryonic fields: the proton field, the neutron field and the Λ-hyperon field. The strong interaction between these fields underlies all "nuclear" processes. What is the form of this strong interaction of fundamental fields?

The simplest form of interaction of two fermion fields, for example, of the proton field and neutron field, is the four-fermion interaction. It is shown graphically in Fig. 1. By making use of the

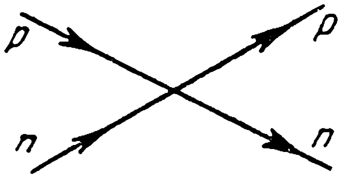

FIG. 1

operators of particle creation and annihilation the Lagrangian of this interaction can be written in the form

$$g(\bar{p}p)(\bar{n}n)$$

where g is the interaction constant, the symbol p denotes the annihilation of the proton (p) or creation of the antiproton ($\tilde{p}$), while the symbol $\bar{p}$ denotes the creation of (p) or annihilation of ($\tilde{p}$); the same holds for the symbols n and $\bar{n}$. The interaction $g(\bar{p}p)(\bar{n}n)$ can be considered as an interaction between two currents: the proton current ($\bar{p}p$) and neutron current ($\bar{n}n$).†

Dynamical properties of the strong interaction will be discussed at length later on; here only its isotopic properties will be considered. The now well-established universality of gravitational and electromagnetic interactions along with the successes of the theory of a universal weak interaction (which will be considered below) make the idea of the universality of strong interaction very attractive. Following this idea we could try to write the strong interaction in the form gj_sj_s, where $j_s = \bar{p}p + \bar{n}n + \bar{\Lambda}\Lambda$. All three baryonic fields are treated completely symmetrically in this interaction. If the masses of p, n and Λ were equal, the strong interaction would possess a symmetry which, in the literature, is called unitary symmetry. In the unitary-symmetric theory not only p, n and Λ would have the same properties but also composite particles would satisfy very strict symmetry requirements. In particular, the masses of π-mesons, K-mesons and the η^0-meson would be equal to one another. Since, in reality, the masses of these mesons are different, this means that unitary symmetry is violated in nature.

One can imagine several possible mechanisms of unitary-symmetry violation. Perhaps the "initial" mass of the Λ-hyperon is not equal to the "initial" masses of the nucleons, and this is responsible for unitary symmetry violation. Maybe there is an as yet unknown interaction which is manifested by the Λ-hyperon but not by the nucleons. One way or another, the symmetry between the nucleons and Λ-hyperon is certainly violated.

ISOTOPIC INVARIANCE IN THE SAKATA MODEL

The symmetry of the current j_s with respect to the proton and neutron is one of the most remarkable properties of the model. The point is that just this symmetry, along with the equality of the

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† We use here the terms *interaction*, *Lagrangian* and *current* without defining more precisely their meaning. Some explanation will be given later on. For more details about these concepts see the monographs on the quantum theory of fields quoted in the bibliography at the end of the book.

"initial" masses of the proton and neutron, underlies the isotopic invariance of strong interactions. The symmetry between the proton and neutron leads to the fact that composite particles are grouped into isotopic multiplets, that the particles of one multiplet have equal masses and analogous strong interactions, and that strong interactions conserve isotopic spin and, consequently, obey numerous selection rules. All this is a consequence of the current j_s being an isotopic scalar.

Thus, in the Sakata model, the isotopic invariance of the strong interaction, in a sense, reduces to the equality of the "strong charges" of the proton and neutron, and in this respect is analogous to such phenomena as the equality of the electric charges of different charged particles. Such an interpretation of isotopic invariance is particularly natural if it is assumed that the masses of the particles are determined by their interactions and that two particles (n and p) having the same interactions must automatically have the same masses.

CONSERVATION OF FUNDAMENTAL PARTICLES

In the Sakata model, fundamental particles are conserved in strong interactions, undergoing no conversion into one another. In particular three quantum numbers are conserved: (1) the number of protons minus the number of antiprotons, (2) the number of neutrons minus that of antineutrons, and (3) the number of Λ-hyperons minus that of anti-Λ-hyperons. This is expressed by graphs in which the lines representing these particles have neither a beginning nor an end. If, for example, a proton scattering takes place, then the diagram has the form shown in Fig. 2. (The proton comes from the infinite past, undergoes an interaction at the point A, and goes off into the infinite future; the dotted arrow shows the trend of time.)

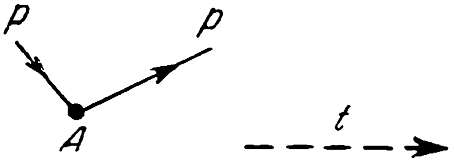

FIG. 2

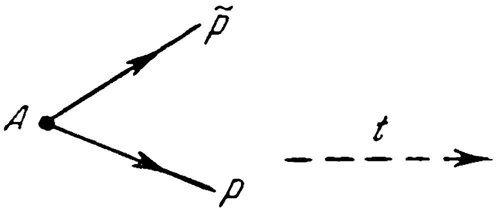

FIG. 3

If a proton + antiproton pair is created, the diagram has the form shown in Fig. 3.

The motion of antiproton can be described mathematically as the motion of proton backwards in time, and the preceding diagram can be presented by Fig. 4. (The proton comes backwards from the future and goes away into the future.)

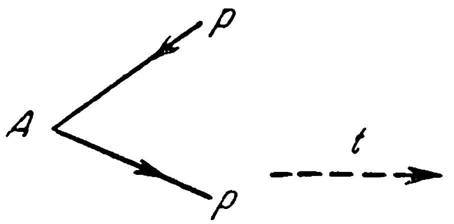

FIG. 4

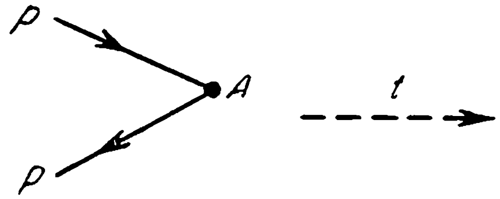

FIG. 5

The annihilation of a proton and antiproton is described in an analogous way (Fig. 5). (The proton comes from the past and goes away towards the past.)

All that we have said above about the proton also refers to the neutron and Λ-hyperon. In particular, conservation of Λ-hyperons accounts in the Sakata model for strangeness conservation: Λ-hyperons either go from the initial to the final state, as is the case in reactions of the type

$$K^- + n \rightarrow \Lambda^0 + \pi^-,$$

or, what is the same,

$$(\tilde{p}\Lambda) + n \rightarrow \Lambda^0 + (\tilde{p}n),$$

or are created in pairs, as in the reaction

$$\pi^- + p \rightarrow \Sigma^- + K^+,$$

or, what is the same,

$$(\tilde{p}n) + p \rightarrow (\Lambda\tilde{p}n) + (\tilde{\Lambda}p).$$

Fundamental particles are also conserved in electromagnetic interactions, which are due, in the Sakata model, to the coupling of the photon with the electric charge (and the Dirac magnetic moment) of the proton. Only the weak interaction is able to annihilate and create the fundamental Sakata particles.

UNIVERSAL WEAK INTERACTION

The weak interaction is responsible for elementary-particle decays. In a universe in which the weak interaction is "switched off" all the particles enumerated in Table 1 would be stable. Only the

π^0-meson and Σ^0-hyperon, which decay on account of the electromagnetic interaction, would be exceptions. The weak interaction causes not only particle decays but also various processes occurring in the collisions of particles. But the cross sections for these processes are very small in comparison with those for strong interactions. For thc latter a cross section of the order of 10^{-27} cm^2 is characteristic, whereas the former have cross sections of the order of 10^{-40} cm^2.

The time intervals which characterize known decays due to the weak interaction are not less than 10^{-10} sec, whereas strong interactions have time intervals of the order of 10^{-23} sec. Therefore processes resulting from weak interactions are frequently called slow processes.

Slow processes are very numerous and diverse, ranging from the β-decay of nuclei to the oscillations of a neutral K-meson beam. None the less, it is still possible that the interaction which causes these processes is universal, i.e. that it is of the same form and magnitude for different particles. The hypothesis of a universal weak interaction does not contradict any experimental fact, although, as yet, it cannot account for all of them.

We postulate that the weak interaction has the form

$$\frac{G}{\sqrt{2}} j_w j_w^+ ,$$

where G is the weak interaction constant ($G = 10^{-5}/m^2$), the current j_w is of the form

$$j_w = \bar{e}\nu + \bar{\mu}\nu + \bar{n}p + \bar{\Lambda}p,$$

while the Hermitian conjugate current j_w^+ is

$$j_w^+ = \bar{\nu}e + \bar{\nu}\mu + \bar{p}n + \bar{p}\Lambda .$$

Here, as before, the symbol $\bar{p}$ denotes the operator of proton creation and antiproton annihilation, while the symbol p is the operator of the proton annihilation and antiproton creation. The same holds for the symbols of the other particles, p, n, Λ, e^-, μ^- and ν being regarded as particles, and $\tilde{p}$, $\tilde{n}$, $\tilde{\Lambda}$, e^+, μ^+ and $\tilde{\nu}$ being regarded as antiparticles.

It is easy to see that the current j_w lowers the electric charge Q by unity, $\Delta Q = -1$, while the current j_w^+ increases the charge, $\Delta Q = 1$. The product $j_w j_w^+$ is Hermitian and, of course, conserves the electric charge. The dynamical properties of weak interactions and their behaviour under mirror reflection and charge conjugation will be considered in detail in what follows. For the present we shall only mention that the weak interaction does not conserve parity.

The current j_w represents the sum of four currents: the electronic current $j_e = \bar{e}\nu$, muonic current $j_\mu = \bar{\mu}\nu$, nucleonic current $j_n = \bar{n}p$ and strange current $j_\Lambda = \bar{\Lambda}p$. All four currents are treated completely symmetrically in the weak interaction, in accordance with the requirement of universality.

TABLE OF WEAK INTERACTIONS

We enumerate certain processes resulting from the interaction of weak currents (see Table 4).

TABLE 4. SCHEME OF WEAK INTERACTIONS

	$\bar{e}\nu$	$\bar{\mu}\nu$	$\bar{n}p$	$\bar{\Lambda}p$
$\bar{\nu}e$	(1) $e + \nu \to e + \nu$			
$\bar{\nu}\mu$	(2) $\mu \to e + \nu + \tilde{\nu}$	(3) $\nu + \mu \to \nu + \mu$ $\nu \to \nu + \mu^+ + \mu^-$		
$\bar{p}n$	(4) $n \to p + e + \tilde{\nu}$ $\pi \to e + \nu$ $\pi^+ \to \pi^0 + e + \nu$	(5) $\mu + p \to n + \nu$ $\pi \to \mu + \nu$	(6) $n + p \to n + p$ $p + p \to p + p$ $n + n \to n + n$	
$\bar{p}\Lambda$	(7) $\Lambda \to p + e + \tilde{\nu}$ $\Sigma \to n + e + \tilde{\nu}$ $K \to \pi + e + \nu$	(8) $\Lambda \to p + \mu + \nu$ $\Sigma \to n + \mu + \nu$ $K \to \mu + \nu$ $K \to \pi + \mu + \nu$	(9) $\Lambda + p \to n + p$ $\Lambda \to p + \pi$ $K \to 2\pi, 3\pi$	(10) $\Lambda + p \to$ $\to \Lambda + p$

The squares of the table which are reflected with respect to the diagonal squares contain, respectively, the direct and inverse processes. Therefore we shall not fill in the six squares above the diagonal.

In the first square, we encounter at once a process which has not as yet been observed experimentally. This is the neutrino–electron scattering. The theoretically predicted cross section for this process is so small that it cannot be detected without an essential improvement in experimental techniques.

The product $j_e j_\mu^+$ of the electronic and muonic current gives the muon decay (the second square). In the third square we see the neutrino–muon scattering resulting from the interaction of the muonic current with itself. A direct experimental observation of this scattering is apparently impossible. However, data on this interaction can be obtained if muon-pair creation by neutrinos in a nuclear Coulomb field is investigated:

$$\nu + Z \rightarrow \nu + Z + \mu^+ + \mu^-.$$

The next row describes the interaction of the nucleonic current with the electronic current, muonic current, and itself. The fourth square contains the neutron β-decay: $n \rightarrow p + e^- + \sim\tilde{\nu}$. Since proton creation is equivalent to antiproton annihilation, the same interaction must also give another process: $\tilde{p} + n \rightarrow e^- + \tilde{\nu}$. According to our model a neutron and an antiproton form a π^--meson. Thus, the fourth square contains not only the neutron β-decay but also the decay of the π-meson into the electron and neutrino. In the same square there is one more process: $\pi^+ \rightarrow \pi^0 + e^+ + \nu$, observed experimentally in 1962. In general, the fourth square contains all possible β-processes in which the strangeness of strongly interacting particles is not changed; in particular, it contains strangeness conserving decays of strange particles, for instance $\Sigma^- \rightarrow \Lambda^0 + e^- + \tilde{\nu}$.

The fifth square shows the interaction of strongly interacting particles with the pair $\tilde{\mu}\nu$. This square contains the muon capture by the proton $\mu^- + p \rightarrow n + \nu$, and the pion decay $\pi \rightarrow \mu + \nu$.

The sixth square contains neutron–proton scattering. As distinct from ordinary neutron–proton scattering the weak scattering should exhibit parity non-conservation, which should give a small admixture of parity non-conservation (of the order 10^{-7} in the amplitude) in all nuclear reactions (even those at low energies). It should be noted that, owing to virtual strong interactions, parity non-conservation will take place not only in np-scattering but also in pp- and nn-scattering as well as in reactions at high energies (for example, in π-meson production). The accuracy of existing experiments is about two orders lower than that which is necessary for observing this phenomenon.†

The lower row describes interactions of the strange current. The seventh square contains the β-decay of the Λ-hyperon:

$$\Lambda \rightarrow p + e^- + \tilde{\nu},$$

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† The phenomenon of parity non-conservation in nuclear forces has recently been observed experimentaly (note added in 1964).

and that of the sigma hyperon:

$$\Sigma^- \to n + e^- + \tilde{\nu},$$

as well as K-meson decays: $K \to e + \nu$, $K \to \pi + e + \nu$, and so on.

In all these processes the decay is through the Λ-hyperon which is contained in one or the other particles.

The eighth square contains analogous decay processes with the muon emission: $\Lambda \to p + \mu + \nu$, $K \to \mu + \nu$, and so on.

The ninth square contains processes in which leptons do not take part. These processes have, in the Sakata model, as a basis, the reaction $n + p \to \Lambda + p$. One cannot at present observe this reaction in its pure form as its cross section is too small.

However, proton annihilation is equivalent to antiproton creation and, consequently, there must exist the transition

$$\Lambda \to p + \tilde{p} + n.$$

If an antiproton combines with a proton, then the following decay takes place:

$$\Lambda \to n + \pi^0.$$

If an antiproton combines with a neutron, then there occurs the decay

$$\Lambda \to p + \pi^-.$$

Thus, the interaction of the strange and the nucleonic current gives the Λ-hyperon decay. Non-leptonic decays of other particles ($K \to 2\pi$, $K \to 3\pi$, $\Sigma \to N + \pi$) are obtained as a result of the decay of the Λ-hyperon contained in these particles. So, the ninth square includes all non-leptonic decays of strange particles.

Finally, the last square describes the rare parity non-conserving process of Λ-hyperon–nucleon scattering. Observation of this process is a matter for the far future.

Summing up, it can be said that the scheme of weak interactions we are considering accounts, in principle, for all experimentally observed slow processes, and predicts no "extra" (experimentally forbidden) process.

NEUTRAL CURRENTS?

It is natural to ask why the weak interaction involves only the charged (or, more precisely, the charge changing) current

$$i_w = \bar{e}\nu + \bar{\mu}\nu + \bar{n}p + \bar{\Lambda}p$$

and does not involve any of the charge conserving neutral currents

$$\bar{\nu}\nu, \quad \bar{e}e, \quad \bar{\mu}\mu, \quad \bar{\mu}e, \quad \bar{p}p, \quad \bar{n}n, \quad \bar{\Lambda}\Lambda, \quad \bar{\Lambda}n.$$

Such a choice of currents is not based on any general principles, but on agreement with experiment. Interaction of the currents $\bar{\mu}e$ and $\bar{e}e$ would lead to the $\mu^+ \to 2e^+ + e^-$ decay which has not been observed experimentally. Interaction of the currents $\bar{\nu}\nu$, $\bar{e}e$, $\bar{\mu}\mu$, $\bar{\mu}e$ with the current $\bar{\Lambda}n$ would lead to the decays

$$\begin{aligned}
K^+ &\to \pi^+ + e^+ + e^-,\\
K^+ &\to \pi^+ + \mu^+ + \mu^-,\\
K^+ &\to \pi^+ + \nu + \bar{\nu},\\
K^+ &\to \pi^+ + \mu^- + e^-,\\
\Sigma^+ &\to p + e^+ + e^-,\\
\Lambda^0 &\to n + e^+ + e^- \quad \text{etc.},
\end{aligned}$$

which have not as yet been observed. Therefore it can with certainty be said that weak interaction does not involve the total neutral current. However, the presence of individual products of the type $(\bar{p}p + \bar{n}n)$ $(\bar{\Lambda}n)$, $(\bar{\nu}\nu)$ $(\bar{p}p)$ or $(\bar{\mu}\mu)$ $(\bar{p}p)$ in the weak interaction Lagrangian cannot, so far, be excluded. On the contrary, as will be seen in what follows, a large amount of data on the non-leptonic decays of strange particles could find a natural explanation on the basis of an interaction including $(\bar{p}p + \bar{n}n)$ $(\bar{\Lambda}n)$, whereas the ordinary non-leptonic interaction $(\bar{p}n)$ $(\bar{\Lambda}p)$ cannot account for them (we refer here to the so-called $\Delta T = \frac{1}{2}$ rule). The problem of neutral currents needs further investigation.

TWO NEUTRINOS

Another important problem is whether there are neutrinos of several kinds. We have written the electronic current in the form $j_e = \bar{e}\nu$, and muonic current in the form $j_\mu = \bar{\mu}\nu$, assuming tacitly that the neutrinos included in these currents are the same. However, there are no experimental grounds for such an assumption.

The assumption that ν_e and ν_μ are the same ($\nu_e = \nu_\mu$) might be suggested by the symmetry which exists between the three leptons (μ, e and ν) and the three fundamental baryons (Λ, n and p) in the Sakata model. This symmetry is sometimes called the Kiev symmetry (it was first submitted for discussion at the Kiev Conference on High-Energy Physics in 1959). Indeed, the weak interaction that we wrote down possesses symmetry with respect to the exchange

$$\Lambda \leftrightarrow \mu, \quad n \leftrightarrow e, \quad p \leftrightarrow \nu.$$

Introduction of two kinds of neutrinos would violate this symmetry. Unfortunately, the problem of the relation between baryons and leptons is for the present completely unclear. Hence Kiev symmetry cannot be taken as a reliable guide.

The assumption that ν_e and ν_μ are different particles ($\nu_e \neq \nu_\mu$) is favoured by the absence of the decays $\mu \to e + \gamma$ and $\mu \to 3e$ (see p. 211). An experiment that would allow one to solve the problem of whether ν_e and ν_μ are the same was proposed by Pontecorvo and performed by a group from Columbia University using the Brookhaven accelerator. In this experiment (the results of which were published in the summer of 1962) it was established that the neutrino produced in the $\pi^\pm \to \mu^\pm \pm \nu$ decays does not give rise, in colliding with nucleons, to reactions of the type $\tilde{\nu} + p \to n + e^+$ or $\nu + n \to p + e^-$ which should take place if ν_e and ν_μ were identical particles.

Quite a number of schemes which could account for the results of this experiment are discussed in the literature. One of these is a scheme in which e^-, μ^+, ν are leptons, while e^+, μ^-, $\tilde{\nu}$ are anti-leptons, and in which the transitions $\mu - e$ are forbidden by leptonic charge conservation. The ordinary muon decay proceeds in this scheme with the emission of two ν's (and not a ν and a $\tilde{\nu}$). According to this proposal the neutrino, like other fermions, is a four-component particle, its two left-handed components being contained in the electron part, while its two right-handed components belong to the muon part.† This scheme is attractive for it is economic: no special muon charge is introduced in it, and use is made of all four components in the neutrino wave function. If the neutrino mass differs from zero, then, in this scheme, the transition of the electronic components of the neutrino to the muonic ones should exist, so that neutrinos produced in the $\pi^- \to \mu^- + \nu$ decay might give rise to the reaction $\nu + n \to e^- + p$. But because of the small neutrino mass this effect would be extremely small. If the neutrino mass equals zero, one cannot devise any experiment which would

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† In this case the lepton current is unlike the form that is at present universally adopted (see pp. 33–34), and has the form

$$\bar{\nu}\gamma_\alpha(1 + \gamma_5)\, e + \bar{\mu}\gamma_\alpha(1 - \gamma_5)\, \nu.$$

be able to distinguish this scheme from that in which the muon and muon neutrino have a conserved "muonic charge", and where e^-, ν_e, μ^-, ν_μ have a leptonic charge of $+1$, both ν_μ and ν_e being left-polarized. According to the latter scheme there exists a conserved quantity, the muonic charge, which equals $+1$ for μ and ν_μ, and zero for the other particles. In this case it is natural to expect the muon neutrino to have properties completely unlike those of the electron neutrino. For example, it may have a mass. (The present upper limit for the muon neutrino mass is ~ 3 MeV.) Moreover, the muon neutrino may have anomalous interactions. This possibility is particularly likely if the muon itself proves to have an anomalous interaction which is absent for the electron.

MINIMAL MODEL

Thus, the Sakata model, in principle, allows a large number of phenomena to be described in a unified manner. However, for a quantitative description of phenomena mathematical methods are needed, and not just a qualitative model. But a real theory of strong interactions is, so far, lacking. We wrote the strong interaction Lagrangian in the form of the coupling of the currents of three fundamental fields. This Lagrangian possesses all the necessary symmetry properties. It is hoped that all the observed mesons and baryons will arise as the eigenstates of this Lagrangian, like levels in the Schroedinger equation, while resonances will arise like virtual levels. We have within the framework of the Sakata model a consistent scheme of weak interactions. It is undoubtedly of heuristic value. However, without a theory of strong interactions, any statement as to the agreement of the model with experiment becomes, to a considerable degree, an "act of belief".

It may even turn out that without a theory of strong interactions the very problem of the existence of fundamental fields makes no sense. And indeed, even in the Sakata model, all elementary particles are equally non-elementary. Virtual strong interactions "dress" in a cloud of strongly interacting particles not only the "composite" Σ-hyperon but also the "fundamental" Λ-hyperon. The physical proton, neutron and Λ-hyperon are by no means identical with initial fundamental "bare" particles, for they contain an infinite number of baryon-antibaryon pairs.

None the less the Sakata model, even in the absence of a theory of strong interactions, gives a number of quantitative predictions.

These predictions are determined by the fact that this model, possessing a minimum number of fundamental fields and interactions, has a number of characteristic symmetry properties. These symmetry properties distinguish the Sakata model from a theory in which all mesons and hyperons are elementary and are contained in the initial Lagrangian.

The symmetry properties due to the minimality of initial fields and interactions are in no way associated with the obvious concepts of composite particles, considered above. They are quite strict consequences of the hypothesis that the Lagrangian includes only three strongly interacting (baryonic) fields, two of which represent an isotopic doublet, and the third represents an isotopic singlet. These fields may be not only p, n, Λ but also Ξ^-, Ξ^0, Λ or, in general, any three fundamental fields A, B, C which might not even correspond to any of the known particles. We shall call all such models, based on the hypothesis of minimality, minimal models. The Sakata model is a particular example of a minimal model.

Of course, individual symmetry properties inherent to a minimal model can be introduced purely phenomenologically and, indeed, have been introduced within the framework of the ordinary theory which takes all particles as elementary. However, in such a phenomenological approach these symmetry properties turn out to have no interrelation and are quite arbitrary. In the minimal model these properties follow from the very basis of the model.

In what follows we shall analyse in detail the symmetry properties of the minimal model, and consider possible ways of checking them experimentally.

CHAPTER 2

SPINORS. AMPLITUDES. CURRENTS

IN CHAPTER 1 we presented a qualitative picture of the interactions between elementary particles. In order to proceed to a quantitative description of the phenomena we shall need some results from relativistic field theory.

SCALARS AND VECTORS

It is well known that the laws of physics, in general, and those of elementary particle physics, in particular, are invariant with respect to Lorentz transformations: in different inertial systems the laws have the same form.

All quantities involved in physical equations are divided into classes according to their behaviour under Lorentz transformations. The simplest class are scalar quantities, which do not change under these transformations; for example, a particle's mass m.

Another group are the vectors. A particle's 4-coordinate is a vector, which we shall denote by x_μ ($\mu = 1, 2, 3, 4$):

$$x_1 = x, \quad x_2 = y, \quad x_3 = z, \quad x_4 = t.$$

A particle's 4-momentum is also a four-dimensional vector formed by its energy E and momentum p: $p_\mu = (E, p)$. Scalar products of vectors are invariants. In particular, the square of its 4-momentum is an invariant:

$$p^2 = p_4 p_4 - p_1 p_1 - p_2 p_2 - p_3 p_3 = E^2 - p^2 = p_\mu p_\mu = m^2.$$

In what follows, if in a product there are two identical subscripts (for example, $x_\mu p_\mu$) this will denote a special sum of products, the product of the space terms subtracted from the products of the time terms. This corresponds to the fact that four-dimensional space is not Euclidean but pseudo-Euclidean.

The product of two vectors can be written also in a somewhat different way, if the second-rank symmetric tensor $\delta_{\mu\nu}$ is introduced:

$$px = p_\mu x_\mu = p_\mu x_\nu \delta_{\mu\nu},$$

where

$$\delta_{\mu\nu} = \begin{cases} 1, & \text{if } \mu = \nu = 4 \\ -1, & \text{if } \mu = \nu = 1, 2, 3 \\ 0, & \text{if } \mu \neq \nu. \end{cases}$$

WAVE FUNCTION

The wave function of a particle with spin zero (for example, that of the π-meson) behaves under Lorentz transformations like a scalar. For a free particle the wave function φ satisfies the Klein–Gordon equation, which in momentum representation has the form

$$(p^2 - m^2)\,\varphi = 0,$$

where p is the 4-momentum of the particle, while m is its mass.

It is evident that one function is insufficient for describing a fermion, because such a particle has at least two states (depending on the direction of the spin). Hence it is obvious that at least two functions are necessary for describing a particle with a spin of $\frac{1}{2}$.

SPINORS

Indeed, a particle with spin $\frac{1}{2}$ can be described by two functions satisfying a system of two second-order equations similar to the Klein–Gordon equation. The set of two such functions is called the spinor and is denoted as

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

In the literature a particle with spin $\frac{1}{2}$ is usually described not by two but by four functions satisfying a system of four first-order equations. These wave functions are called bispinors or four-component spinors and, to within a normalization factor, are of the form

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}.$$

In the rest system of the particle

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

if its spin is directed along the z-axis, and

$$u = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

if the spin is directed in the opposite direction.

DIRAC EQUATION

The system of four equations satisfied by bispinors (in what follows, for the sake of brevity, we shall call these simply spinors) can be written in the form of a matrix equation, if use is made of γ-matrices. This equation is called the Dirac equation. From the fact that the spinor u is a four-component spinor it follows that γ-matrices must be 4×4-matrices. The Dirac equation, written in terms of γ-matrices, in the momentum representation is of the form

$$(\hat{p} - m)\, u = 0,$$

where it is understood that m is multiplied by the unit 4×4-matrix I, while $\hat{p} = p_\mu \gamma_\mu = p_4 \gamma_4 - \mathbf{p} \,.\, \boldsymbol{\gamma}$. An explicit form of the γ-matrices will be considered below; meanwhile we shall find commutation relations for these matrices.

We multiply the Dirac equation from the left by $(\hat{p} + m)$. Then we obtain

$$(\hat{p} + m)(\hat{p} - m)\, u = (p_\mu p_\nu \gamma_\mu \gamma_\nu - m^2)\, u = 0.$$

If we now require that the condition $p^2 = m^2$ be fulfilled, then this equation must have the form

$$(p^2 - m^2)\, u = 0,$$

whence, taking into account the symmetry of the tensor $p_\mu p_\nu$, we obtain

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\delta_{\mu\nu}.$$

σ-MATRICES

4×4-matrices γ_μ satisfying this relation are conveniently constructed from the Pauli 2×2-matrices σ. We shall deal with the following 2×2-matrices:

$$\tilde{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The product of two matrices is a new matrix. Its element (i, k), where i is the number of a row, while k is the number of a column, is obtained by multiplying an ith row of the first matrix by a kth column of the second matrix. Carrying out such a multiplication, one can easily see, for example, that

$$\sigma_1 \sigma_2 = i\sigma_3.$$

The result of the multiplication of two matrices depends on their order in the product. In particular,

$$\sigma_1\sigma_2 = -\sigma_2\sigma_1 = i\sigma_3.$$

It can also easily be shown that

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \tilde{I}.$$

The properties of the σ-matrices can be expressed by the formula

$$\sigma_\alpha\sigma_\beta = \tilde{\delta}_{\alpha\beta}\tilde{I} + i\tilde{\varepsilon}_{\alpha\beta\gamma}\sigma_\gamma;$$

where $(\alpha, \beta, \gamma) = 1, 2, 3$, $\tilde{\delta}_{\alpha\beta}$ is a second-rank symmetric tensor:† $\tilde{\delta}_{\alpha\beta} = 1$ if $\alpha = \beta$, or $\tilde{\delta}_{\alpha\beta} = 0$ if $\alpha \neq \beta$, while $\tilde{\varepsilon}_{\alpha\beta\gamma}$ is a completely antisymmetric third-rank tensor:

$$\tilde{\varepsilon}_{\alpha\beta\gamma} = \begin{cases} 1, & \text{if } \alpha, \beta, \gamma \text{ are obtained from 123 by even permutation,} \\ -1, & \text{if } \alpha, \beta, \gamma \text{ are obtained from 123 by odd permutation,} \\ 0, & \text{if among } \alpha,\ \beta,\ \gamma \text{ there are at least two identical indices.} \end{cases}$$

In what follows we shall encounter the product $\tilde{\varepsilon}_{\alpha\beta\gamma}\tilde{\varepsilon}_{\sigma\varrho\nu}$, in which a summation is to be carried out over certain indices. It can easily be seen that the product of two antisymmetric tensors $\tilde{\varepsilon}$ is expressed by the determinant

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† We introduced here the symbol ~ in order to distinguish between the three-dimensional δ-symbol ($\tilde{\delta}$) and the four-dimensional one (δ), as well as between the unit 2×2-matrix ($\tilde{I}$) and the 4×4-matrix (I). Henceforth, in the cases when this does not lead to any misunderstandings, we shall not make this distinction, and shall write 1 instead of I or $\tilde{I}$.

$$\tilde{\varepsilon}_{\alpha\beta\gamma}\tilde{\varepsilon}_{\mu\nu\varrho} = \begin{Vmatrix} \tilde{\delta}_{\alpha\mu} & \tilde{\delta}_{\alpha\nu} & \tilde{\delta}_{\alpha\varrho} \\ \tilde{\delta}_{\beta\mu} & \tilde{\delta}_{\beta\nu} & \tilde{\delta}_{\beta\varrho} \\ \tilde{\delta}_{\gamma\mu} & \tilde{\delta}_{\gamma\nu} & \tilde{\delta}_{\gamma\varrho} \end{Vmatrix}.$$

The sum of such products over one pair of indices is equal to

$$\tilde{\varepsilon}_{\alpha\beta\gamma}\tilde{\varepsilon}_{\alpha\nu\varrho} = \begin{Vmatrix} \tilde{\delta}_{\beta\nu} & \tilde{\delta}_{\beta\varrho} \\ \tilde{\delta}_{\gamma\nu} & \tilde{\delta}_{\gamma\varrho} \end{Vmatrix} = \tilde{\delta}_{\beta\nu}\tilde{\delta}_{\gamma\varrho} - \tilde{\delta}_{\beta\varrho}\tilde{\delta}_{\gamma\nu}.$$

Multiplying this relation by $\tilde{\delta}_{\beta\nu}$ and taking into account that $\tilde{\delta}_{\alpha\beta}\tilde{\delta}_{\alpha\beta} = 3$, we obtain

$$\tilde{\varepsilon}_{\alpha\beta\gamma}\tilde{\varepsilon}_{\alpha\beta\varrho} = 2\tilde{\delta}_{\gamma\varrho}.$$

In an analogous way one obtains

$$\tilde{\varepsilon}_{\alpha\beta\gamma}\tilde{\varepsilon}_{\alpha\beta\gamma} = 6.$$

γ-MATRICES

4×4-matrices γ are expressed in terms of 2×2-matrices σ as follows:

$$I = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \qquad \gamma_1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}, \qquad \gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix},$$

$$\gamma_3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}, \qquad \gamma_4 = \begin{pmatrix} \tilde{I} & 0 \\ 0 & -\tilde{I} \end{pmatrix},$$

$$\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4 = -\begin{pmatrix} 0 & \tilde{I} \\ \tilde{I} & 0 \end{pmatrix}.$$

Notation in a detailed form is more cumbersome, for example,

$$\gamma_4 = \begin{pmatrix} \tilde{I} & 0 \\ 0 & -\tilde{I} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

It is easy to see that the γ_μ-matrices possess the necessary commutation properties:

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}I \qquad (\mu,\ \nu = 1,\ 2,\ 3,\ 4).$$

Analogous relations hold for the γ_5-matrix:

$$\gamma_\mu\gamma_5 + \gamma_5\gamma_\mu = 0, \qquad \gamma_5^2 = I.$$

Digressing somewhat from the theme of this section, we shall present some relations satisfied by γ-matrices which we shall use.

SOME RELATIONS

We shall denote the scalar product of the 4-vector A_α and γ_α-matrices by $\hat{A}$:

$$\hat{A} = A_\alpha \gamma_\alpha = A_4\gamma_4 - A_1\gamma_1 - A_2\gamma_2 - A_3\gamma_3 = A_4\gamma_4 - \boldsymbol{A}\boldsymbol{\gamma}.$$

From the commutation relation for γ-matrices it follows that

$$\gamma_\mu \gamma_\mu = 4,$$

$$\hat{A}\hat{B} + \hat{B}\hat{A} = 2AB, \quad \text{where} \quad AB = A_\mu B_\mu \quad \text{is a number;}$$

$$\gamma_\mu \hat{A} + \hat{A}\gamma_\mu = 2A_\mu.$$

From this last equality, multiplying it from the right by $R\gamma_\mu$, where R is an arbitrary expression, it is easy to see that

$$\gamma_\mu \hat{A} R \gamma_\mu + \hat{A}\gamma_\mu R \gamma_\mu = 2R\hat{A}.$$

Taking into account that $\gamma_\mu\gamma_\mu = 4$, we obtain

for $R = 1$: $\quad \gamma_\mu \hat{A} \gamma_\mu = -2\hat{A}$,

for $R = \hat{B}$: $\quad \gamma_\mu \hat{A}\hat{B}\gamma_\mu = 2\hat{B}\hat{A} + 2\hat{A}\hat{B} = 4AB$,

for $R = \hat{B}\hat{C}$: $\quad \gamma_\mu \hat{A}\hat{B}\hat{C}\gamma_\mu = -2\hat{C}\hat{B}\hat{A}$.

In what follows we shall make considerable use of these relations.

CALCULATION OF TRACES

A fundamental quantity characterizing a matrix is its trace (in German *Spur*, in Russian *sled*), i.e. the sum of the diagonal matrix elements. It can easily be seen that for all γ-matrices the trace equals zero, while for the unit matrix I the trace is equal to 4:

$$\mathrm{Sp}\,\gamma_\mu = 0, \qquad \mathrm{Sp}\,\gamma_5 = 0, \qquad \mathrm{Sp}\,I = 4.$$

From the very definition of the trace it follows that the trace of the product of n matrices does not change under cyclic permutation of the matrices:

$$\mathrm{Sp}\,\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n} = \mathrm{Sp}\,\gamma_{\alpha_2}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n}\gamma_{\alpha_1}.$$

Using this fact and the relation $\gamma_\mu\gamma_5 = -\gamma_5\gamma_\mu$, where $\mu = 1, 2, 3, 4$, it can easily be seen that the trace equals zero if n is odd. Indeed,

$$\begin{aligned}\mathrm{Sp}\,\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n} &= \mathrm{Sp}\,\gamma_5\gamma_5\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n}\\ &= \mathrm{Sp}\,\gamma_5\,\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n}\gamma_5 = -\,\mathrm{Sp}\,\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n} = 0.\end{aligned}$$

If the relation $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$ is used, then one can derive in an analogous way the following reduction formula for the case when n is even:

$$\begin{aligned}\mathrm{Sp}\,\gamma_{\alpha_1}\gamma_{\alpha_2}\gamma_{\alpha_3}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n} &= \delta_{\alpha_1\alpha_2}\,\mathrm{Sp}\,\gamma_{\alpha_3}\gamma_{\alpha_4}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n}\\ &-\delta_{\alpha_1\alpha_3}\,\mathrm{Sp}\,\gamma_{\alpha_2}\gamma_{\alpha_4}\cdots\gamma_{\alpha_{n-1}}\gamma_{\alpha_n} + \dots + (-1)^n\delta_{\alpha_1\alpha_n}\,\mathrm{Sp}\,\gamma_{\alpha_2}\gamma_{\alpha_3}\cdots\gamma_{\alpha_{n-1}}.\end{aligned}$$

For $n = 2$ and $n = 4$ we have

$$\begin{aligned}\tfrac{1}{4}\,\mathrm{Sp}\,\gamma_\mu\gamma_\nu &= \delta_{\mu\nu},\\ \tfrac{1}{4}\,\mathrm{Sp}\,\gamma_\mu\gamma_\nu\gamma_\varrho\gamma_\sigma &= \delta_{\mu\nu}\delta_{\varrho\sigma} + \delta_{\nu\varrho}\delta_{\mu\sigma} - \delta_{\mu\sigma}\delta_{\nu\sigma}.\end{aligned}$$

From the definition of the matrix $\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4$ it follows that

$$\tfrac{1}{4}\,\mathrm{Sp}\,\gamma_5\gamma_\mu\gamma_\nu\gamma_\varrho\gamma_\sigma = -i\varepsilon_{\mu\nu\varrho\sigma},$$

where $\varepsilon_{\mu\nu\varrho\sigma}$ is a completely antisymmetric four-rank tensor:

$$\varepsilon_{\mu\nu\varrho\sigma} = \begin{cases} 1, \text{ if } \mu, \nu, \varrho, \sigma \text{ are obtained from 1234 by even permutation,}\\ -1, \text{ if } \mu, \nu, \varrho, \sigma \text{ are obtained from 1234 by odd permutation,}\\ 0, \text{ if among } \mu, \nu, \varrho, \sigma \text{ there are at least two identical indices.}\end{cases}$$

In what follows we shall encounter the product $\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\mu\nu\varrho\sigma}$, in which the sum is to be carried out over some of the indices.

It can easily be seen that the product of two antisymmetric tensors ε is expressed by the determinant

$$\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\mu\nu\varrho\sigma} = -\begin{Vmatrix}\delta_{\alpha\mu} & \delta_{\alpha\nu} & \delta_{\alpha\varrho} & \delta_{\alpha\sigma}\\ \delta_{\beta\mu} & \delta_{\beta\nu} & \delta_{\beta\varrho} & \delta_{\beta\sigma}\\ \delta_{\gamma\mu} & \delta_{\gamma\nu} & \delta_{\gamma\varrho} & \delta_{\gamma\sigma}\\ \delta_{\delta\mu} & \delta_{\delta\nu} & \delta_{\delta\varrho} & \delta_{\delta\sigma}\end{Vmatrix}.$$

The fact that the determinant must actually have a negative sign is simple to verify by taking into account that $\varepsilon_{1234}\,\varepsilon_{1234} = +1$, while $\delta_{11}\,\delta_{22}\,\delta_{33}\,\delta_{44} = -1$, because $\delta_{11} = \delta_{22} = \delta_{33} = -1$ while $\delta_{44} = +1$. If this product is summed over the first index (which is equivalent to its being multiplied by $\delta_{\alpha\mu}$), then we obtain

$$\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\alpha\nu\varrho\sigma} = -\begin{Vmatrix} \delta_{\beta\nu} & \delta_{\beta\varrho} & \delta_{\beta\sigma} \\ \delta_{\gamma\nu} & \delta_{\gamma\varrho} & \delta_{\gamma\sigma} \\ \delta_{\delta\nu} & \delta_{\delta\varrho} & \delta_{\delta\sigma} \end{Vmatrix}$$

$$= -\delta_{\beta\nu}\delta_{\gamma\varrho}\delta_{\delta\sigma} - \delta_{\beta\varrho}\delta_{\gamma\sigma}\delta_{\delta\nu} - \delta_{\beta\sigma}\delta_{\gamma\varrho}\delta_{\delta\varrho}$$

$$+ \delta_{\beta\varrho}\delta_{\gamma\nu}\delta_{\delta\sigma} + \delta_{\beta\nu}\delta_{\gamma\sigma}\delta_{\delta\varrho} + \delta_{\beta\sigma}\delta_{\gamma\varrho}\delta_{\delta\nu}.$$

(Here we took advantage of the fact that $\delta_{\alpha\beta}\,\delta_{\alpha\beta} = 4$, $\delta_{\alpha\beta}\,\delta_{\alpha\varrho} = \delta_{\beta\varrho}$.) In an analogous way one obtains the relations

$$\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\alpha\beta\varrho\sigma} = -2\begin{Vmatrix} \delta_{\gamma\varrho} & \delta_{\gamma\sigma} \\ \delta_{\delta\varrho} & \delta_{\delta\sigma} \end{Vmatrix} = -2(\delta_{\gamma\varrho}\delta_{\delta\sigma} - \delta_{\gamma\sigma}\delta_{\delta\varrho}),$$

$$\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\alpha\beta\gamma\sigma} = -6\delta_{\delta\sigma},$$

$$\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\alpha\beta\gamma\delta} = -24.$$

OPERATIONS *, ~, +

We now define the operations of complex conjugation *, transposition ~, and Hermitian conjugation (+). Complex conjugation of a matrix is simply that of all its elements. It can easily be seen that

$$\gamma_1^* = \gamma_1, \quad \gamma_2^* = -\gamma_2, \quad \gamma_3^* = \gamma_3, \quad \gamma_4^* = \gamma_4, \quad \gamma_5^* = \gamma_5.$$

In transposing the matrix its rows are replaced by columns: the matrix is reflected about the diagonal. In this case

$$\tilde{\gamma}_1 = -\gamma_1, \quad \tilde{\gamma}_2 = \gamma_2, \quad \tilde{\gamma}_3 = -\gamma_3, \quad \tilde{\gamma}_4 = \gamma_4, \quad \tilde{\gamma}_5 = \gamma_5.$$

Hermitian conjugation is the result of complex conjugation and transposition; hence

$$\gamma_1^+ = -\gamma_1, \quad \gamma_2^+ = -\gamma_2, \quad \gamma_3^+ = -\gamma_3, \quad \gamma_4^+ = \gamma_4, \quad \gamma_5^+ = \gamma_5.$$

The matrix that is the Hermitian conjugate of the product $\gamma_\mu\gamma_\nu$ is

$$(\gamma_\mu\gamma_\nu)^+ = \gamma_\nu^+\gamma_\mu^+.$$

CONJUGATED SPINOR

Define the conjugated spinor $\bar{u}$ as follows:

$$\bar{u} = u^+\gamma_4.$$

It is easy to see that, if u represents the column

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix},$$

then $\bar{u}$ represents the row

$$\bar{u} = (u_1^*, \quad u_2^*, \quad -u_3^*, \quad -u_4^*).$$

Let us find the equation that is satisfied by $\bar{u}$. For this let us write an equation which is the Hermitian conjugate of the Dirac equation:

$$u^+(\hat{p}^+ - m) = 0.$$

Multiplying this equation from the right by γ_4 and taking advantage of the fact that $\gamma_4^2 = 1$, $\gamma_4\hat{p}^+\gamma_4 = \hat{p}$, we obtain

$$\bar{u}(\hat{p} - m) = 0.$$

FIVE BILINEAR COVARIANTS

Both u and $\bar{u}$ are four-component spinors, hence, 16 bilinear combinations can be made of them. The bilinear combinations can be grouped into five different covariant quantities:

$\bar{u}u$ — scalar (S),
$\bar{u}\gamma_\alpha u$ — polar 4-vector (V),
$\bar{u}\sigma_{\alpha\beta}u$ — antisymmetric tensor (T) containing six different components;

$$\sigma_{\alpha\beta} = \tfrac{1}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha),$$

$\bar{u}\gamma_\alpha\gamma_5 u$ — axial 4-vector (A), and
$\bar{u}\gamma_5 u$ — pseudoscalar (P).

The proof that these quantities *are* actually Lorentz covariants is contained in any treatise on quantum field theory, and we shall not present it. Bilinear covariants play an important role in the description of processes involving fermions.

In what follows we shall consider how one can express, in terms of these covariants, the currents

$$j_{el} = \bar{p}p - \bar{e}e - \bar{\mu}\mu,$$
$$j_s = \bar{p}p + nn + \bar{\Lambda}\Lambda,$$
$$j_w = \bar{e}\nu + \bar{\mu}\nu + \bar{n}p + \bar{\Lambda}p,$$

responsable for electromagnetic, strong and weak interactions.

ELECTROMAGNETIC INTERACTION

It is well known that the electromagnetic current is a 4-vector: each of the three components of this current has the form

$$\bar{u}\gamma_\alpha u,$$

where u is the electron, muon or proton spinor. The interaction of this current with the electromagnetic field is of the form

$$ej_{el}^{\alpha}A_{\alpha},$$

where e is the electric charge, while A_{α} is the 4-potential of the electromagnetic field. All experimental data concerning the electromagnetic interaction are in excellent agreement with the assumption that the electromagnetic current has the form written above.

For a free particle the expression $\bar{u}\gamma_{\alpha}u$ contains a normal (or Dirac) magnetic moment along with the charge. The anomalous magnetic moment of a fermion is described by the term $\bar{u}\sigma_{\alpha\beta}q_{\beta}u$, where q is the momentum transferred to the electromagnetic field. The anomalous magnetic moments of the electron and muon are due to virtual electromagnetic interactions and, hence, are small. The anomalous magnetic moments of baryons are due to virtual strong interactions, and their value even exceeds that of the normal magnetic moment. The postulate according to which the fundamental electromagnetic current contains only terms of the type $\bar{u}\gamma_{\alpha}u$ and no term of the type $\bar{u}\sigma_{\alpha\beta}q_{\beta}u$ is called the principle of minimum electromagnetic interaction.

STRONG INTERACTION

For the present, little can be said about the form of the strong interaction. The hypothesis that, like the electromagnetic current, the strong current j_s is also a vector of the type $\bar{u}\gamma_{\alpha}u$ appears to be very attractive. This hypothesis is favoured by the fact that, at least in perturbation theory, such a current would lead to repulsion between the particles and to attraction between the particle and antiparticle, analogous to the Coulomb attraction between the electron and positron (see Chapter 1).

We can only conjecture as to whether the strong interaction is a contact interaction (of the type $j_s^{\alpha}j_s^{\alpha}$) or is caused by neutral vector-meson exchange, similarly to electromagnetic interaction (of the type $j_s^{\alpha}B_{\alpha}$, where B_{α} is the field of the strongly interacting vector mesons—vectons). In what follows we shall not use any special form of the strong interaction, but shall take into account phenomenologically the effects resulting from it.

WEAK INTERACTION

We have already mentioned in Chapter 1 that the initial weak interaction is of the form

$$\frac{G}{\sqrt{2}} j_w j_w^+,$$

where G is the weak interaction constant equal to $(1{\cdot}01 \pm 0{\cdot}01) \times 10^{-5}\, m_p^{-2}$; the denominator $\sqrt{2}$ is introduced for convenience and (mainly) not to break with tradition.

A weak interaction in the form of the product of two currents (nucleonic $\bar{p}n$ and electronic $\bar{e}\nu$) was first suggested by Fermi in 1934 for describing neutron β-decay. In analogy with the electromagnetic current, Fermi assumed weak currents to be vector currents: $\bar{e}\gamma_\alpha \nu$ and $\bar{p}\gamma_\alpha n$.

The experimental determination of the form of the β-decay interaction took more than twenty years, and led to a number of dramatic mistakes, searches and discoveries. Soon after Fermi's study the hypothesis was put forward, and for some time considered to be valid, that the β-decay interaction cannot at all be described by a linear combination of the five scalars which represent the scalar products of the five bilinear covariants S, V, T, A, P (see above). According to this hypothesis the β-decay interaction should also contain the derivatives of the spinor wave functions.

Subsequent experiments disproved this hypothesis and led to the erroneous conclusion that the scalar (S) variant of interaction and the tensor (T) were involved in β-decay. This idea was held up to 1958. However, several years earlier, serious changes had already taken place in the physics of weak interactions.

First, weak interaction was found (1947–1955) to be responsible not only for the neutron β-decay but also for almost all decays of elementary particles.

Second, it was discovered that the weak interaction does not conserve parity. Experiments done by the end of 1956, suggested by Lee and Yang, proved this conclusively.

The discovery of parity non-conservation stimulated a great deal of interest in β-decay. A large number of new experimental results appeared contradicting those which had led to the conclusion that the S- and T-couplings dominated the β-decay interaction.

Analysing all the experiments, Gell-Mann and Feynman and, independently, Marshak and Sudarshan, came to the conclusion in the autumn of 1957 that previous experiments were probably incorrect and that it was the *V*- and *A*-variants of the weak interaction that were responsible for the β-decay and other slow processes. Experiments done subsequently to check the *V–A*-theory of β-decay confirmed the validity of this conclusion.

The universal *V–A*-weak interaction is written in the form

$$\frac{G}{\sqrt{2}} j_w j_w^+,$$

where

$$j_w = \bar{u}_e O_\alpha u_\nu + \bar{u}_\mu O_\alpha u_\nu + \bar{u}_n O_\alpha u_p + \bar{u}_\Lambda O_\alpha u_p,$$

$$j_w^+ = \bar{u}_\nu O_\alpha u_e + \bar{u}_\nu O_\alpha u_\mu + \bar{u}_p O_\alpha u_n + \bar{u}_p O_\alpha u_\Lambda,$$

$$O_\alpha = \gamma_\alpha(1 + \gamma_5) = \gamma_\alpha - \gamma_5 \gamma_\alpha.$$

The term γ_α is a vector (*V*), while the term $\gamma_5\gamma_\alpha$ is an axial term (*A*). Such a weak interaction is universal, because for different pairs of particles it has the same form (*V–A*) and the same constant (*G*). In the next section we shall show that this interaction does not conserve the space and charge parity.

γ_5-INVARIANCE

It can easily be seen that the currents we have just considered are invariant with respect to the so-called γ_5-transformation, under which

$$u \rightarrow \gamma_5 u, \qquad \bar{u} \rightarrow -\bar{u}\gamma_5.$$

Indeed, in the electromagnetic current the terms $\bar{u}\gamma_\alpha u$ are invariant, while the terms $\bar{u}\sigma_{\alpha\beta} q_\beta u$, forbidden by the principle of minimum electromagnetic interaction, change sign under the γ_5-transformation.

If the requirement of γ_5-invariance is applied to the weak current j_w, the expression of which was given on p. 30, and if one requires that j_w not contain, in an explicit form, any momenta of the particles, then this determines uniquely the form of its terms:

$$\bar{u}_2 \gamma_\alpha (1 + \gamma_5) u_1.$$

The factor $(1 + \gamma_5)$ arises in this case because of the fact that in the current j_w, as distinct from the currents j_s and j_e, the spinors $\bar{u}$ and u refer to different particles.

It is easy to see that the matrix $(1 + \gamma_5)$ projects from the four-component spinor only two components. If $u = \begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix}$, where χ_1 and χ_2 are two-component spinors, then

$$(1 + \gamma_5)\,u = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix} = \begin{pmatrix} \chi_1 & -\chi_2 \\ -\chi_1 & +\chi_2 \end{pmatrix} = \begin{pmatrix}\varphi\\ -\varphi\end{pmatrix}.$$

Thus, $(1 + \gamma_5)\,u$ is expressed in terms of the two-component spinor $\varphi = \chi_1 + \chi_2$; the same holds for $\bar{u}(1 - \gamma_5)$. If the weak interaction is the sole interaction in which a given particle is involved—and this is apparently the case for the neutrino—and if, moreover, the mass of the particle equals zero, then the two-component spinor φ completely describes the particle. In this case the spinor $\varphi = \chi_1 - \chi_2$ is by no means connected with the spinor $\chi_1 + \chi_2$, and a relativistically invariant equation can be written for the two-component spinor φ. This equation describes a longitudinally polarized particle.

It is interesting to note that in the minimal model the gravitational interaction is also γ_5-invariant (in the linear approximation). This can easily be seen if the energy-momentum tensor is written out for a system of fermion fields, whose strong, weak and electromagnetic interactions are γ_5-invariant.

All of the aforesaid gives grounds for assuming that γ_5-invariance plays some sort of essential role in nature. A difficulty associated with the γ_5-transformation consists in the fact that the equation of motion of a free fermion possessing a mass is not γ_5-invariant. Indeed, under the γ_5-transformation in the Lagrangian the term proportional to the mass changes sign:

$$m\bar{\psi}\psi \rightarrow -m\bar{\psi}\psi.$$

What does this invariance, which is violated by the masses, mean? There is as yet no answer to this question.

Other symmetry properties of weak interaction will be considered in the next chapter. Meanwhile let us find the connection of this interaction with physically observable quantities.

LAGRANGIAN AND AMPLITUDE

The expressions we have written above for interactions correspond, in essence, to the classical picture of the interaction of a current with a field or a current with a current. This can be seen particularly clearly in the example of the electromagnetic coupling. The current structure of the weak interaction is analogous to that of the electromagnetic interaction. This analogy, already mentioned above, becomes even more complete if the assumption is made that the weak four-fermion interaction in fact results from the intermediate vector-meson exchange (see Chapter 18).

It is well known that in quantum field theory the "classical" interaction Lagrangian is replaced by a quantized Lagrangian in which the wave functions of the fields are replaced by the operators of creation and annihilation of corresponding particles and antiparticles. Thus, for example, to the "classical" interaction of the electron current with electromagnetic field

$$e\bar{u}_e\gamma_\alpha u_e A_\alpha$$

there corresponds the quantized Lagrangian

$$e\bar{\psi}\gamma_\alpha\psi_e A_\alpha,$$

where ψ_e is the operator of electron annihilation and positron creation, $\bar{\psi}_e$ is the operator of positron annihilation and electron creation, while A is the operator of photon creation and annihilation.

Knowing the quantized Lagrangian which describes the interaction of photons with electrons, one can calculate all electromagnetic processes and, in particular, the amplitude for electron scattering by an external electromagnetic field.

In quantum field theory every process is represented by an amplitude. The amplitude of the process contains all the data about it. Knowing the amplitude of the process one can calculate its probability and dependence on the spin polarization of the particles involved, on their momenta, on angles between them, and so forth.

One of the basic problems of the theory of elementary particles consists in devising methods that would allow one to calculate, on the basis of a few fundamental principles, the amplitudes of all processes involving elementary particles.

In quantum electrodynamics there is now a clear algorithm, based on perturbation theory, for finding the amplitude of any process. The classical interaction $e\bar{u}\gamma_\alpha u A_\alpha$, considered above, arises in this case as the amplitude of the electron scattering by an external field, calculated in the first order of perturbation theory. In the next order of perturbation theory this amplitude already contains corrections of order α, due to virtual photons. These corrections have been calculated, and are in excellent agreement with the relevant experimental data.

Unfortunately, the problem of finding the amplitude from a given Lagrangian has been solved only in quantum electrodynamics. For the present, we are still very far from solving this problem for strong interaction. However, quite a number of general requirements which must be satisfied by the amplitudes of one or another process are well established.

The requirement of relativistic invariance holds for all amplitudes. The amplitudes of processes involving photons must satisfy the requirement of gauge invariance. The amplitudes of all processes apparently must be invariant with respect to the transformation of combined inversion (time reversal) while the amplitudes of the strong and electromagnetic processes must be invariant with respect to the inversion of the coordinates and charge conjugation separately.

These assumptions and a number of other symmetry requirements, which will be considered later on, determine the form of amplitudes to within unknown scalar functions (of angular and energy variables) or constants.

The amplitude properties which can be established on the basis of invariance requirements are usually called *kinematical.* The scalar functions involved in the amplitude determine its *dynamical* properties.

For the weak interaction, in analogy with electrodynamics, the classical current

$$j_w = \bar{u}_e O_\alpha u_\nu + \bar{u}_\mu O_\alpha u_\nu + \bar{u}_n O_\alpha u_p + \bar{u}_\Lambda O_\alpha u_p$$

can be compared to the quantized current

$$j_w = \bar{\psi}_e O_\alpha \psi_\nu + \bar{\psi}_\mu O_\alpha \psi_\nu + \bar{\psi}_n O_\alpha \psi_p + \bar{\psi}_\Lambda O_\alpha \psi_p,$$

where ψ and $\bar{\psi}$ are operators, and the expression

$$\frac{G}{\sqrt{2}} j_w j_w^+$$

can be considered to be the quantized weak interaction Lagrangian. In cases where we have to do with the weak interaction of leptons, the amplitudes describing the corresponding processes (for example, muon decay) in the first order of perturbation theory agree with the classical Lagrangian of the interaction of currents, as in the case of the electron scattering by an external electromagnetic field.

In the cases where the weak interaction involves strongly interacting particles, the corresponding amplitudes will contain dynamical scalar functions, which we are as yet unable to calculate, and which we shall consider as phenomenological parameters.

S- AND T-MATRICES

Consider a set of N physical states which, as a result of interaction, may go over into one another. The transition from a certain state i to a certain state f will be characterized by the quantity S_{fi}. A set of N^2 quantities S_{fi} forms a scattering matrix, or S-matrix. If all interactions are switched off, then the S-matrix becomes the unit matrix I: each of the states goes over to itself. Hence physical processes take place if the T-matrix, which is determined by the relation

$$S = I + iT,$$

differs from zero. In what follows the quantity M_{fi} of

$$T_{fi} = (2\pi)^4 \delta^4 (p_f - p_i) M_{fi},$$

will be called the amplitude of the process. Here p_i and p_f are the 4-momenta of the initial and final state, while the δ-function expresses the energy-momentum conservation law in its explicit form:

$$\delta^4 (p_f - p_i) = \delta(p_f^x - p_i^x)\,\delta(p_f^y - p_i^y)\,\delta(p_f^z - p_i^z)\,\delta(E_f - E_i).$$

For the sake of brevity, the subscripts f and i will be omitted henceforth.

PROBABILITY AND CROSS SECTION

The square of the absolute value of T_{fi} determines the probability of transition from the initial state i to the final state f:

$$\bar{w}_{fi} = |T_{fi}|^2 = [(2\pi)^4 \delta^4 (p_f - p_i)]\,[(2\pi)^4 \delta^4 (0)]\,|M|^2.$$

To calculate $\bar{w}_{fi}$ we introduce the four-dimensional normalization volume VT which, of course, will not enter the final result. From the definition of δ^4 it follows that, if $V \to \infty$, $T \to \infty$, then

$$(2\pi)^4 \delta^4 (0) = VT.$$

In order to obtain the probability of a transition not to just one state f but to a group of states, we have to multiply $\bar{w}_{fi}$ by an element of phase space Φ, which has the form

$$\Phi = \prod_{l=1}^{n} \frac{dk_l V}{(2\pi)^3},$$

where n is the number of particles in the final state, while k_l is the 3-momentum of the lth particle.

We now have to find a correct normalization of the expression for the transition probability. We shall normalize the wave functions of particles in such a way that a unit volume contains $2E$ particles, where E is the energy of the particle. It is easy to see that for scalar particles such a normalization corresponds to the wave function $\varphi = e^{-ikx}$. Indeed, the density of particles is in this case equal to

$$i\left(\varphi^* \frac{\partial \varphi}{\partial t} - \varphi \frac{\partial \varphi^*}{\partial t}\right) = 2E.$$

For fermions the density of particles is given by $\bar{u}\gamma_4 u$, while the condition

$$\bar{u}\gamma_4 u = 2E$$

is equivalent to the condition

$$u_\alpha \bar{u}_\beta = (\hat{p} + m)_{\alpha\beta}.$$

(Note that in the two last relations summation over the spin states of the particle is assumed; note also that u and φ have different dimensions.) In order to obtain the normalized probability, $\bar{w}_{fi}$ is to be divided by the quantity N:

$$N = \prod_{l=1}^{n} 2E_l V \prod_{i=1}^{k} 2E_i V,$$

where k is the number of particles in the initial state. If we consider a decay, then $k = 1$, while if we consider a collision between two particles, then $k = 2$.

As a result, for the normalized transition probability per unit time we obtain

$$w_{fi} = \frac{\bar{w}_{fi}}{T} \frac{\Phi}{N} = \frac{(2\pi)^4 V}{\prod_{i=1}^{k} (2E_i V)} \cdot \delta^4(p_f - p_i) \, |M|^2 \prod_{l=1}^{n} \frac{dk_l}{(2\pi)^3 \, 2E_l}.$$

For the decay probability of a particle ($k = 1$) we find

$$w = \frac{(2\pi)^4}{2E_1}|M|^2 \prod_{l=1}^{n} \left[\frac{dk_l}{(2\pi)^3 2E_l}\right] \delta^4(p_f - p_i).$$

Collision between two particles ($k = 2$) is usually characterized by a cross section defined as follows:

$$w_{fi} = \sigma j,$$

where j is the flux density. In the laboratory coordinate system, where particle i_1 is at rest while the incident particle i_2 has the velocity v_2, the flux density is equal to

$$j = \frac{v_2}{V}.$$

As a result we obtain the cross section

$$\sigma = \frac{w_{fi}}{j} = \frac{(2\pi)^4}{2m_{i_1} 2E_{i_2} v_{i_2}}|M|^2 \prod_{l=1}^{n} \left[\frac{dk_l}{(2\pi)^3 2E_l}\right] \delta^4(p_f - p_i).$$

The quantity $I = m_1 E_2 v_2 = m_1 p_2$ can be written in the invariant form

$$I = \sqrt{[(p_1 p_2)^2 - p_1^2 p_2^2]},$$

and, finally, we have

$$\sigma = \frac{(2\pi)^4}{4\sqrt{[(p_1 p_2)^2 - m_1^2 m_2^2]}}|M|^2 \prod_{l=1}^{n} \left[\frac{dk_l}{(2\pi)^3 2E_l}\right] \delta^4(p_f - p_i).$$

CHAPTER 3

C-, *P*-, *T*- TRANSFORMATIONS

We now proceed with our consideration of the symmetry properties of elementary particles and the interaction between them. In connection with the discovery of parity non-conservation, great attention has been drawn within the past five years to the following group of discrete transformations: charge conjugation (C), inversion of coordinates (P) and time reversal (T). We shall begin our consideration of these transformations with charge conjugation.

CHARGE CONJUGATION

The thirty-year history of the investigation of charge conjugation is a brilliant illustration of the words of the well-known astrophysicist Schwarzschild:†

> If simple perfect laws uniquely rule the universe, should not pure thought be capable of uncovering this perfect set of laws without having to lean on the crutches of tediously assembled observations? True, the laws to be discovered may be perfect, but the human brain is not. Left on its own, it is prone to stray, as many past examples sadly prove. In fact, we have missed few chances to err until new data, freshly gleaned from nature, set us right again for the next steps.

The concept of particles and antiparticles, and the idea that the laws of physics contain them in a symmetric way, arose from the struggle to clarify the meaning of the relativistic equation describing elementary particles and, in particular, the electron. When Dirac obtained his equation, he tried for a long time to overcome the following difficulty: along with negatively charged particles—electrons—the equation contained also positively charged particles. At one time Dirac assumed these positively charged particles to be protons. This interpretation was soon found to be unsatisfactory because of the mass difference between the proton and electron. Then Oppenheimer suggested that these particles might be antielectrons. But this assumption met a number of objections. In 1932, Pauli wrote in connection with the Oppenheimer suggestion:

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† M. Schwarzschild, *Structure and Evolution of the Stars*, Princeton University Press, Princeton, New Jersey, 1958, p. 1.

> This explanation appears to be unsatisfactory because the laws of nature in this theory are exactly symmetric with respect to electrons and antielectrons. ... Thus, we do not believe that this explanation can be considered seriously.†

In the same year (1932) Anderson discovered the positron!

In time, the concept of "particle–antiparticle" conjugation, i.e. *C* conjugation, was extended to bosons and other fermions. The discovery of "charge-mirror" pairs μ^+, μ^-, π^+, π^-, K^+, K^- and the discovery of antiprotons and antineutrons and anti-hyperons definitely convinced physicists that the universe is symmetric with respect to charge conjugation.

Charge conjugation does not mean simply a change in the sign of the electric charge or magnetic moment of the particle; other "charge" quantum numbers—baryonic charge, leptonic charge, strangeness—also change their sign. Thus, for example, the antineutron differs from the neutron not only by the sign of its magnetic moment but, what is more important, by the sign of its baryonic charge. As a result the antineutron, as distinct from the neutron, may annihilate in collision with a nucleon.

Some elementary particles, for example γ, π^0, K_1^0, K_2^0 under charge conjugation go over into themselves: they are their own antiparticles. Such particles are called absolutely neutral.

If a particle, under charge conjugation, goes over into another particle, then one cannot ascribe to it definite parity with respect to charge conjugation. However, if under charge conjugation a particle goes into itself, then one can ask for the sign which the transformed wave function has with respect to the original function. It will be shown that the wave function of the photon, under charge conjugation, changes sign, whereas that of the π^0-meson does not. The photon is odd under charge conjugation, while the π^0-meson is even under charge conjugation. The vacuum is even under charge conjugation.

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† W. Pauli, Die Allgemeinen Prinzipien der Wellenmechanik, *Handbuch der Physik*, vol. 24, part I, p. 246.

The fact that the world around us does not contain the same number of antiprotons and protons, electrons and positrons, is not to be interpreted as saying that the laws of physics are not symmetric with respect to the exchange of particles and antiparticles. Apparently, particles and antiparticles are not distributed uniformly in the universe. In any case the vacuum, as we define it, does not prefer particles of a definite charge.

The experimentally observed symmetry between particles and antiparticles was generally accepted and expressed as a requirement of invariance with respect to charge conjugation, which can be formulated in the following way:

To any possible physical process there corresponds another one differing from the first only by the fact that in it all particles are replaced by antiparticles.

By 1956, invariance with respect to charge conjugation was considered as one of the firm bases of physics. At the end of 1956, however, experiment revealed that weak interactions are not invariant with respect to charge conjugation. It turned out that μ^+- and μ^--decays differ by the sign of the asymmetry of the electron angular distribution. Decays of positive and negative pions differ by the sign of the polarization of muons produced in the decay. The free neutrino turned out to be polarized antiparallel to its momentum, and the antineutrino parallel. Thus the theory of the free neutrino is not invariant under charge conjugation.

To understand the implications of the violation of the charge conjugation invariance and, hence, the non-conservation of charge conjugation parity, we have to recur to the mirror-reflection transformation (spatial parity).

SPACE INVERSION

We now consider the operation of the inversion of space coordinates, and the concept of space parity P associated with it. It is well known that space coordinates change the sign under P-inversion, whereas the time coordinate remains unchanged:

$$x \to -x, \qquad t \to t.$$

Let us look at the behaviour of various physical quantities under inversion. The momentum of a particle is the product of its mass and velocity:

$$p = mv = m\frac{dx}{dt}.$$

Therefore the momentum components change sign under inversion:

$$p \to -p.$$

The same reasoning gives for the energy

$$E \to E.$$

Analogously one can derive the behaviour of the angular momentum under inversion as follows: since $M = x \times p$, under inversion

$$M \to M.$$

The same holds for a particle's intrinsic moment—spin σ:

$$\sigma \to \sigma.$$

The 4-potential of electromagnetic field behaves under inversion as the 4-momentum:

$$A \to -A, \quad A_4 \to A_4.$$

Thus it follows that under inversion the electric field changes sign, whereas the magnetic field does not:

$$\mathscr{E} \to -\mathscr{E}, \quad \mathscr{H} \to \mathscr{H}.$$

Vectors which, under inversion, change sign are called polar vectors, while those which do not are called axial or pseudo-vectors.

Up to 1956, the invariance of physical equations with respect to space inversion (mirror reflection) was considered to be a fundamental law. It was assumed that to any physical process there necessarily corresponds another one which is the mirror reflection of the first. A consequence of this invariance is the space parity conservation law.

An analysis carried out in the middle of 1956 by Lee and Yang in connection with the θ–τ problem revealed that parity conservation in weak interactions was not experimentally checked, and new experiments carried out in the winter of 1956/57, following a proposal of Lee and Yang, proved irrefutably that weak interactions are not invariant with respect to mirror reflection. Thus, for instance, it turned out that μ-mesons in pion decay as well as electrons in β-decay are polarized longitudinally, and that there is an "up–down" asymmetry in the decay of polarized nuclei. All these effects can be described as spin-momentum correlations of the type $\boldsymbol{\sigma} . \boldsymbol{p}$. As is clear from what we have said, this expression changes sign under mirror reflection.

COMBINED INVERSION

In 1956, when parity conservation in weak interactions was questioned, Landau put forward the hypothesis that in physics there cannot be any interaction that is not invariant with respect to a transformation consisting of the reflection of space coordinates combined with the transition from particles to antiparticles. This combination of space inversion and charge conjugation was called by Landau combined inversion. The hypothesis of combined parity (parity with respect to combined inversion) conservation re-established, of course in a modified form, the left–right symmetry which seemed to be disproved by experiments in which space parity non-conservation in weak interactions was discovered.

According to this hypothesis, although the space parity conservation law is violated, the left–right symmetry remains, provided particles are converted into antiparticles under mirror reflection. The invariance with respect to combined inversion requires that to every physical process there should necessarily correspond another one which is obtained from the former by mirror reflection and replacement of all particles by antiparticles. Each of these operations individually is now physically meaningless, since it transforms a given physical process into one which cannot exist (for example, C-conjugation transforms the emission of a positron and a left-handed neutrino into that of an electron and a left-handed antineutrino, which does not exist in nature).

Considering combined inversion as a generalization of charge conjugation, it can be said that in a transformation from particle to antiparticle not only the sign of all the charge quantum numbers is to be changed but also that of the helicity, i.e. the projection of the spin of the particle onto its momentum.

Still, as approximate quantum numbers, the charge parity and space parity play a role: they are conserved in electromagnetic and strong interactions. The concept of combined parity is applicable only to truly neutral particles or systems of particles, i.e. to particles or systems of particles which, under combined inversion, go into themselves; for example, the π^0-meson, K_1^0-meson, K_2^0-meson, positronium, etc. The combined parity of a particle is equal to the product of its charge parity and space parity, if these have a definite value. However, as distinct from space parity and charge parity, which are *not* conserved in weak interactions, combined parity is conserved in all known physical processes.

TIME REVERSAL AND THE *CPT*-THEOREM

By analogy with space inversion, P, we consider the operation of time reversal T:

$$x \to x, \qquad t \to -t.$$

In this case we find that

$$\begin{aligned} p &\to -p, & E &\to E, \\ M &\to -M, & \sigma &\to -\sigma, \\ A &\to -A, & A_4 &\to A_4, \\ \mathcal{E} &\to \mathcal{E}, & \mathcal{H} &\to -\mathcal{H}. \end{aligned}$$

In the physics of elementary particles there is a fundamental theorem which interrelates the transformations C, P and T. This is the Lüders–Pauli theorem, which states that in any normal physical theory (what is meant by the word "normal" will be explained below) there must be invariance with respect to the product of all three transformations.

Theories which are not invariant with respect to C, P, T individually are easy to devise and, indeed, we know that the weak interaction is not invariant with respect to P and C. An example of theory which is not invariant with respect to P and T was considered by Ioffe. These theories do not contradict any basic principles of quantum mechanics and quantum field theory. But, within the framework of a normal theory, the invariance with respect to the product of all three transformations cannot be violated.

A normal theory is characterized, first, by Lorentz invariance and, second, by a correct relationship between spin and statistics. This means that particles with a half-integer spin obey Fermi–Dirac statistics, while those having integer spin obey Bose–Einstein statistics. Moreover, the theory must not involve any infinite-order derivatives. The Lüders–Pauli theorem is very convenient for finding out whether or not a certain symmetry is violated in one or another experiment.

In order to find out if charge parity is conserved we should, strictly speaking, do the same experiments with particles and their antiparticles and see if various correlations are different, as would be the case if charge symmetry is violated. In most experiments on β-decay this was not done. The experiments were carried out only with particles. For example, in the well-known Wu experiment arguing from the observed asymmetry of Co^{60}-decay electrons (without the experiments on anticobalt nuclei!) it followed that charge parity was not conserved.

The possibility of such an interpretation of the Wu experiment follows just from the *CPT* theorem. In the Wu experiment, the correlation of the nuclear spin σ and the momentum p of emitted electrons was observed. Under *P*-transformation $p \to -p$ and $\sigma \to \sigma$ and, consequently, (σp) changes sign, which means space parity non-conservation. Under *T*-transformation $p \to -p$, $\sigma \to -\sigma$ and $(\sigma p) \to (\sigma p)$. Thus the product (σp) under *PT*-transformation changes sign and, consequently, contradicts *PT*-invariance. This means that, by virtue of the Lüders–Pauli theorem, the presence of such a correlation contradicts not only *PT*-invariance but also *C*-invariance.

As distinct from β-decay, μ-decay gives not only indirect (through the *CPT*-theorem) but also direct evidence for charge parity non-conservation: electrons from $\pi^- \to \mu^- \to e^-$ decays and positrons from $\pi^+ \to \mu^+ \to e^+$ decays are polarized differently. Correlations in the muon decay point out that space parity and charge parity are not conserved, but that time reversal is, and, consequently, so is the product of the charge parity and space parity, i.e. combined parity.

It is easy to see that, if the combined parity is conserved, then the particle cannot have an electric dipole moment. Indeed, if a particle, for instance the neutron, had a dipole moment, then this moment should be directed along σ, but the product $\sigma \mathcal{E}$ changes sign under *T*-transformation. Consequently, this interaction contradicts not only parity conservation but also combined-parity conservation.

It should be noted that the above simple criteria turn out to be inadequate for treating the problem of time parity conservation in a certain process, if particles produced in this process are strongly interacting with one another. This problem is considered in detail on p. 163 and also later on.

We proceed now from a general physical consideration of C-, P- and T-transformations to the problem of their mathematical description.

INTRINSIC PARITY OF THE FERMION†

First of all let us see how the parity of fermions is determined. As is known, the fermion is described by a 4-component spinor u. Let us find out how this spinor is transformed under space inversion. Consider the Dirac equation

$$(\hat{p} - m)\,u = 0.$$

This equation can be written in the following way:

$$(E\gamma_4 - p\gamma - m)\,u = 0.$$

Under inversion of coordinates $p \to p' = -p$, $u \to u' = Pu$, and we get

$$(E\gamma_4 + p\gamma - m)\,Pu = 0.$$

We ask the question: what form should the operator P have in order that the resulting equation goes over to the initial Dirac equation? For this the operator P should be such that

$$P^{-1}(E\gamma_4 + p\gamma - m)\,P = (E\gamma_4 - p\gamma - m),$$

where P^{-1} is the inverse P-matrix:

$$P^{-1}P = 1.$$

It can easily be seen that the necessary properties are possessed by the γ_4-matrix, for which

$$\gamma_4^2 = 1, \quad \gamma_4\gamma\gamma_4 = -\gamma.$$

Thus, $P = \gamma_4$, and the spinor u under space inversion goes over to $\gamma_4 u$.

Taking into account that

$$\gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

we see that the upper and lower components of the spinor are transformed differently under inversion. Consider this result from a somewhat different standpoint.

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† For the sake of brevity the term fermion here and in what follows stands for particles with spin $\frac{1}{2}$.

In the Dirac equation let us pass from 4 × 4-matrices and four-component spinors to 2 × 2-matrices and two-component spinors, writing u in the form $u = \begin{pmatrix}\varphi\\ \chi\end{pmatrix}$:

$$(\hat{p} - m)u = \left[E\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} - p\begin{pmatrix}0 & \sigma\\ -\sigma & 0\end{pmatrix} - m\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right]\begin{pmatrix}\varphi\\ \chi\end{pmatrix}$$

$$= \begin{pmatrix}E - m, & -p\sigma\\ p\sigma, & -E - m\end{pmatrix}\begin{pmatrix}\varphi\\ \chi\end{pmatrix} = 0,$$

whence

$$\chi = \frac{\sigma p}{E + m}\varphi, \quad \varphi = \frac{\sigma p}{E - m}\chi.$$

Performing the inversion transformation $\sigma \to \sigma$, $p \to -p$, $\sigma p \to -\sigma p$, we see that φ and χ, under the reflection of coordinates, behave differently.

P-INVERSION AND V–A-INTERACTION

We have already seen that the weak interaction leads to correlations which violate invariance with respect to space inversion. We now consider how this is expressed in the weak interaction Lagrangian.

We have written the weak interaction Lagrangian in the form of a self-product of the weak interaction current consisting of terms of the type

$$\bar{u}_1\gamma_\alpha(1 + \gamma_5)u_2 = \bar{u}_1\gamma_\alpha u_2 - \bar{u}_1\gamma_5\gamma_\alpha u_2 = V_\alpha - A_\alpha.$$

How do V_α and A_α behave under P-inversion? We have seen that the spinor u under inversion goes over to $\gamma_4 u$. Let us find out what happens to $\bar{u}$. For this we perform the Hermitian conjugation

$$u^+ \to u^+\gamma_4$$

and multiply this relation from the right by γ_4. Then we obtain

$$\bar{u} \to \bar{u}\gamma_4.$$

Therefore under space inversion the expression $V_\alpha = \bar{u}_2\gamma_\alpha u_1$ transforms as follows:

$$V_\alpha = \bar{u}_2\gamma_\alpha u_1 \to (\bar{u}_2\gamma_4\gamma_\alpha\gamma_4 u_1) = \begin{cases} V_\alpha, & \alpha = 4,\\ -V_\alpha, & \alpha = 1, 2, 3.\end{cases}$$

Such a property is possessed by the 4-vector describing, say, the coordinates of a certain point in four-dimensional space: under space inversion its projections onto the spatial axes change sign, while the projection onto the time axis remains invariant. Hence the current V_α is called the vector current. It behaves as a vector not only under Lorentz transformations but also under reflection.

The expression $A_\alpha = \bar{u}_2\gamma_5\gamma_\alpha u_1$ under inversion is transformed as follows:

$$A_\alpha = \bar{u}_2\gamma_5\gamma_\alpha u_1 \to \bar{u}_2\gamma_4\gamma_5\gamma_\alpha\gamma_4 u_1 = \begin{cases} -A_\alpha, & \alpha = 4, \\ A_\alpha, & \alpha = 1, 2, 3. \end{cases}$$

Thus, under reflection A goes over to A, i.e. it is an axial vector, like the magnetic field.

So, the weak current j_w is not invariant with respect to P-inversion: some of the terms consituting it change sign under P-inversion, whereas the other ones do not. Since the weak interaction Lagrangian has the form $L_w \sim j_w^+ j_w$, L_w is also not invariant with respect to P-inversion: products of the type $V_\alpha^+ V_\alpha$ and $A_\alpha^+ A_\alpha$ involved in L_w do not change sign under P-inversion, whereas products of the type $V_\alpha^+ A_\alpha$ and $A_\alpha^+ V_\alpha$ do. It is the presence of these terms in the Lagrangian L_w that reflects parity non-conservation in the weak interaction.

CHARGE CONJUGATION FOR FERMIONS

Let us see how the spinor u is transformed under charge conjugation. Before proceeding to formal transformations, we shall explain the meaning of the operation to be carried out. The Dirac equation has a solution corresponding to positive energies, and another one corresponding to negative energies. The solution corresponding to positive energies describes the particle, whereas the other one is not a physical solution, because there are no particles with negative energies. But we can interpret this last solution as corresponding to the antiparticle.

Thus, we write the Dirac equation $(\hat{p} - m)u = 0$ and perform in it the transformation $p \to p' = -p$. Then, we look for such a transformation of the spinor u under which u' is expressed in terms of the conjugated spinor $\bar{u}$:

$$u(p) \to u'(p') = u_c(-p) = C\tilde{\bar{u}}(p).\dagger$$

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† The mark $\sim$ is usually not written above $\bar{u}$, because the transposition of $\bar{u}$ is indicated by the very order of factors in the expression $C\bar{u}$.

As a result we obtain

$$(-\hat{p} - m)\, C\tilde{\bar{u}}(p) = 0.$$

We now ask the question: what properties should the matrix C possess in order that this equation goes over into the ordinary Dirac equation for conjugated spinor $\bar{u}(\hat{p} - m) = 0$? To answer this question, we transpose the last equation:

$$(\tilde{\hat{p}} - m)\,\tilde{\bar{u}} = 0.$$

It is seen that the matrix C should satisfy the condition

$$C^{-1}(-\hat{p} - m)\, C = (\tilde{\hat{p}} - m).$$

For this it is in turn necessary that

$$C^{-1}C = 1, \quad C^{-1}\gamma_\mu C = -\tilde{\gamma}_\mu.$$

It can easily be seen that in the representation that we have chosen for the γ-matrices the necessary properties are possessed by the matrix $C = \gamma_2\gamma_4$. In this case

$$C^+ = C^{-1} = C.$$

The spinor $u_C(p) = C\tilde{\bar{u}}(-p)$ is called the charge conjugate of $u(p)$, while the matrix C is called the charge conjugation matrix.

CHARGE CONJUGATION AND *V–A*-INTERACTION

Similar to the way that we have considered the behaviour of the 4-vectors V_α and A_α under space inversion we can also consider the behaviour of these quantities under charge conjugation. The basic point is that terms of the type $(\bar{e}\nu_1)(\bar{\nu}_2\mu)$ will not go over into themselves under charge conjugation but into Hermitian conjugate terms of the type $(\bar{\mu}\nu_2)(\bar{\nu}_1 e)$. (The Lagrangian is Hermitian and hence contains these terms as well.) Carrying out the transformations we see that, although V and A are not invariant under space inversion and charge conjugation, the product of the two operations does not affect the weak interaction Lagrangian (see Chapter 5). The V–A-interaction was constructed just so that it is invariant with respect to the CP-transformation.

PARITY OF THE ANTIFERMION

It is known that the parity of the fermion differs from that of the antifermion. We may explain this in terms of C- and P-transformations. The operator C has the form $\gamma_2\gamma_4$, while P has the form of

γ_4. We may perform the charge conjugation of the spinor u, and then the inversion of space coordinates:

$$PCu = P(\gamma_2\gamma_4\tilde{\bar{u}}) = \gamma_4\gamma_2\gamma_4\tilde{\bar{u}} = -\gamma_2\tilde{\bar{u}}.$$

Now we do this transformation in the reverse order:

$$CPu = C(\gamma_4 u) = \gamma_2\gamma_4\gamma_4\tilde{\bar{u}} = \gamma_2\tilde{\bar{u}}.$$

In the first case we performed the mirror reflection of the antiparticle, and in the second case that of the particle. The results differ by a sign. This means that the parity of the fermion differs from that of the antifermion.

INTRINSIC PARITY OF THE BOSON

We can now, within the framework of the composite model, get a picture of the parities of different elementary particles. We know that the parity of the proton differs from that of the antiproton. The orbital parity of the $p + \tilde{p}$ system is equal to $(-1)^l$, where l is the orbital momentum. Consequently, the total parity of this system is equal to

$$P = (-1)^{l+1}.$$

From this relation it follows, in particular, that a proton and an antiproton form a pseudoscalar π-meson only if they are in ${}^1S_0$-state.

Because, in strong and electromagnetic interactions, Λ, p and n do not go over into one another, and in the weak interaction parity is not conserved, we can choose the parities of Λ, p and n in an arbitrary way. We shall assume these parities to be $P = +1$. The K^+-meson represents the system $p + \tilde{\Lambda}$. The parity of the K^+-meson will be equal to $+1$ if p and $\tilde{\Lambda}$ are in ${}^3P_0$-state, and to -1 for ${}^1S_0$-state. It is easy to see that, as distinct from fermions, the parities of the boson and antiboson (for example, those of K^+ and K^-) are equal.

CHARGE PARITY OF THE BOSON

The charge parity of a system consisting of a fermion and an antifermion is determined by the requirement that the total wave function of the system be antisymmetric in all the variables: charge, spin and coordinates. The spin function has the symmetry $(-1$

corresponds to the antisymmetric function, and $+1$ to the symmetric):

$$(-1)^{1+s}, \quad \text{where } s \text{ is the spin of the system.}$$

The orbital function has the symmetry

$$(-1)^{l}, \quad \text{where } l \text{ is the orbital momentum of the system.}$$

Consequently, the charge parity is equal to

$$C = -(-1)^{l+s+1} = (-1)^{l+s}.$$

We have already mentioned that the π^0-meson consists of a proton and an antiproton in the ${}^1S_0$-state. Applying the formula $C = (-1)^{1+s}$ we find that the π^0-meson is even under charge conjugation.

It is known that the photon, being a vector particle (and not an axial one), may go over to such states of the system proton + antiproton (or electron + positron) in which $l = 0$ and $s = 1$. Hence the charge parity of the photon is odd.

The fact that the photon has an odd charge parity, together with the fact that charge parity is conserved in strong and electromagnetic interactions, leads to the impossibility of the existence, in strong and electromagnetic interactions, of processes in which an odd number of photons in the vacuum go into an even number of photons. Any matrix elements for such processes must be zero.

The theorem which states that Feynman diagrams having an odd number of external photon lines, and no other external lines, are equal to zero, is called the Furry theorem.

CHAPTER 4

WEAK INTERACTION BETWEEN LEPTONS

WHEN a process involves, besides leptons, also strongly interacting particles, the amplitude of this process is, of course, not determined by the weak interaction Lagrangian alone. However, in a number of processes the weak interaction takes place in its pure form (muon decay, neutrino–electron scattering) or is accompanied only by an electromagnetic interaction (conversion of the neutrino into three leptons in the Coulomb field of the nucleus: $\nu + Z \to e^+ + e^- + \nu + Z$ or $\nu + Z \to \mu^+ + \mu^- + \nu + Z$). The amplitudes of these processes can easily be obtained, and all characteristics can be completely calculated.

MUON DECAY

The Lagrangian containing the interaction which is responsible for muon decay $\mu \to e + \nu + \tilde{\nu}$ has the form

$$L = \frac{G}{\sqrt{2}}[(\bar{e}O_\alpha\nu)(\bar{\nu}O_\alpha\mu) + (\bar{\mu}O_\alpha\nu)(\bar{\nu}O_\alpha e)],$$

where

$$O_\alpha = \gamma_\alpha(1 + \gamma_5).$$

The first term of this expression is responsible for the μ^--decay while the second is responsible for μ^+-decay. In order to obtain the amplitude of μ^--decay it is sufficient to replace the operators $\bar{e}$, ν, $\bar{\nu}$, μ, in the first term by the spinor wave functions of the corresponding particles. In this case we obtain

$$M = \frac{G}{\sqrt{2}}(\bar{u}_e O_\alpha u_{\nu_1})(\bar{u}_{\nu_2} O_\alpha u_\mu),$$

where $\bar{u} = u^+\gamma_4$, while u_e, u_μ, u_{ν_2} and u_{ν_1} are the wave functions of the electron, muon, neutrino and antineutrino, respectively.

FIERZ RELATION

The expression

$$(\bar{u}_a O_\alpha u_b)\,(\bar{u}_c O_\alpha u_d)$$

possesses a remarkable property: it is antisymmetric under the exchanges $a \leftrightarrow c$ or $b \leftrightarrow d$. Let us see first that under such exchanges this expression preserves its structure (V–A). Indeed, it is a scalar, invariant with respect to the γ_5-transformation of any of the four constituent spinors. This property should be preserved after the exchange. But the sole operator which is not changed in multiplying from the right by γ_5 and from the left by $-\gamma_5$ is the operator $O_\alpha = \gamma_\alpha(1 + \gamma_5)$. Taking into account that after double exchange we have to return to the initial expression, we have two possibilities:

$$(\bar{u}_a O_\alpha u_b)\,(\bar{u}_c O_\alpha u_d) = \pm(\bar{u}_a O_\alpha u_d)\,(\bar{u}_c O_\alpha u_b).$$

We now show by a direct calculation that under exchange the minus sign arises.

Write $(1 + \gamma_5)\,u$ and $\bar{u}(1 - \gamma_5)$ in the form

$$(1+\gamma_5)\,u = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \chi_1 & -\chi_2 \\ -\chi_1 & +\chi_2 \end{pmatrix} = \begin{pmatrix} \varphi \\ -\varphi \end{pmatrix},$$

where $\varphi = \chi_1 - \chi_2$;

$$\bar{u}(1-\gamma_5) = (\chi_1^*,\ -\chi_2^*)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = (\chi_1^* - \chi_2^*,\ \chi_1^* - \chi_2^*) = (\varphi^*,\ \varphi^*),$$

where $\varphi^* = \chi_1^* - \chi_2^*$. Then we have

$$\begin{aligned}
&(\bar{u}_a O_\alpha u_b)\,(\bar{u}_c O_\alpha u_d) \\
&\quad = \tfrac{1}{4}[\bar{u}_a(1-\gamma_5)\gamma_\alpha(1+\gamma_5)\,u_b]\,[\bar{u}_c(1-\gamma_5)\gamma_\alpha(1+\gamma_5)\,u_d] \\
&\quad = \frac{1}{4}\left[(\varphi_a^*,\ \varphi_a^*)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \varphi_b \\ -\varphi_b \end{pmatrix}\right]\left[(\varphi_c^*,\ \varphi_c^*)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \varphi_d \\ -\varphi_d \end{pmatrix}\right] \\
&\quad\quad - \frac{1}{4}\left[(\varphi_a^*,\ \varphi_a^*)\begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}\begin{pmatrix} \varphi_b \\ -\varphi_b \end{pmatrix}\right]\left[(\varphi_c^*,\ \varphi_c^*)\begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}\begin{pmatrix} \varphi_d \\ -\varphi_d \end{pmatrix}\right] \\
&\quad = (\varphi_a^*\varphi_b)\,(\varphi_c^*\varphi_d) - (\varphi_a^*\sigma\varphi_b)\,(\varphi_c^*\sigma\varphi_d).
\end{aligned}$$

Introducing the symbols $\varphi_a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and so on, we may write this last expression in the form

$$\begin{aligned}
&(a_1^* b_1 + a_2^* b_2)\,(c_1^* d_1 + c_2^* d_2) - (a_1^* b_2 + a_2^* b_1)\,(c_1^* d_2 + c_2^* d_1) \\
&\quad + (a_1^* b_2 - a_2^* b_1)\,(c_1^* d_2 - c_2^* d_1) - (a_1^* b_1 - a_2^* b_2)\,(c_1^* d_1 - c_2^* d_2) \\
&= 2a_1^* b_1 c_2^* d_2 + 2a_2^* b_2 c_1^* d_1 - 2a_1^* b_2 c_2^* d_1 - 2a_2^* b_1 c_1^* d_2 \\
&= 2a_1^* c_2^*(b_1 d_2 - b_2 d_1) + 2a_2^* c_1^*(b_2 d_1 - b_1 d_2) \\
&= 2(a_1^* c_2^* - a_2^* c_1^*)\,(b_1 d_2 - b_2 d_1).
\end{aligned}$$

It is obvious that this expression changes the sign under the exchanges $a \leftrightarrow c$ or $b \leftrightarrow d$.

If we had anticommutating operators ψ instead of the spinors u, then we would obtain

$$(\bar{\psi}_a O_\alpha \psi_b)\,(\bar{\psi}_c O_\alpha \psi_d) = +\,(\bar{\psi}_a O_\alpha \psi_d)\,(\bar{\psi}_c O_\alpha \psi_b).$$

The relation we have just considered is a particular case of more general relations established by Fierz for an arbitrary local four-fermion interaction.

EXPRESSION FOR THE PROBABILITY

Making use of the Fierz relation, the decay amplitude can be written in the form

$$M = -\frac{G}{\sqrt{2}}(\bar{u}_e O_\alpha u_\mu)\,(\bar{u}_{\nu_2} O_\alpha u_{\nu_1}).$$

The decay probability, expressed by means of the amplitude M, has the form (see Chapter 2)

$$dW = (2\pi)^4 \frac{|M|^2}{2E_\mu} \frac{d\boldsymbol{k}}{(2\pi)^3 2E} \cdot \frac{d\boldsymbol{q}_1}{(2\pi)^3 2\omega_1} \frac{d\boldsymbol{q}_2}{(2\pi)^3 2\omega_2} \times \delta^4(k + q_1 + q_2 - p),$$

where

$k = (E, \boldsymbol{k})$ is the 4-momentum of the electron,
$q_1 = (\omega_1, \boldsymbol{q}_1)$ is the 4-momentum of the antineutrino,
$q_2 = (\omega_2, \boldsymbol{q}_2)$ is the 4-momentum of the neutrino,
$p = (E_\mu, \boldsymbol{p})$ is the 4-momentum of the muon.

The four-dimensional delta-function

$$\delta^4(k + q_1 + q_2 - p) = \delta(E + \omega_1 + \omega_2 - E_\mu)\,\delta(\boldsymbol{k} + \boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{p})$$

means that the total energy and each of the total momentum components are conserved.

APPROXIMATE ESTIMATE OF THE DECAY PROBABILITY

Before carrying out an accurate calculation let us estimate the muon decay probability. The formula for the decay rate involves the amplitude M. By virtue of the normalization of spinor that we chose, M is a dimensionless quantity. But the amplitude is proportional to the interaction constant G which possesses dimensions

($G = 10^{-5}/m^2$, where m is the mass of the nucleon), hence the numerator of the amplitude must be proportional to the square of a certain mass.

The mass that is characteristic for the muon decay is just the mass of the muon. Hence it can be assumed that the amplitude in the first approximation has the form $G\mu^2$. We substitute this into the formula for the decay rate and calculate it in the system in which the muon is at rest:

$$dW = \frac{G^2\mu^4}{(2\pi)^5 2\mu} \cdot \frac{d\mathbf{k}}{2E} \frac{d\mathbf{q}_1}{2\omega_1} \frac{d\mathbf{q}_2}{2\omega_2} \delta^4(k + q_1 + q_2 - p).$$

Now we can concentrate on the calculation of phase space.

INTEGRATION OVER THE PHASE SPACE

We first calculate the integral over phase space of two neutrinos:

$$\int \frac{d\mathbf{q}_1}{\omega_1} \frac{d\mathbf{q}_2}{\omega_2} \delta^4(q_1 + q_2 - q), \quad \text{where} \quad q = p - k.$$

This integral is a dimensionless quantity $\left(\left[\frac{d\mathbf{q}}{\omega}\right] = [m^2],\ [\delta^4(q)] = \left[\frac{1}{m^4}\right]\right)$. We shall now see that it is simply a number. How to calculate it? We write the four-dimensional delta-function as follows:

$$\delta^4(q_1 + q_2 - q) = \delta(\omega_1 + \omega_1 - \omega)\,\delta(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q}).$$

Now the integral with respect to $\mathbf{q}_2$ can be done:

$$\int \delta(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q})\, d\mathbf{q}_2 = 1.$$

As a result we get the expression:

$$\int \delta(\omega_1 + \omega_2 - \omega) \frac{d\mathbf{q}_1}{\omega_1 \omega_2}.$$

This integral is conveniently calculated in a system in which the momenta $\mathbf{q}_1$ and $\mathbf{q}_2$ are in opposite directions and have the same absolute value, i.e. in the centre-of-mass system of two neutrinos. Since the energy of the neutrino is equal to its momentum, in the δ-function we can replace $\omega_1 + \omega_2$ by $2\omega_1$.

If we take into account that

$$d\mathbf{q}_1 = q_1^2\, dq_1\, d\varphi\, d\cos\theta = \omega_1^2\, d\omega_1\, d\varphi\, d\cos\theta,$$

then further integration with respect to dq_1 is carried out trivially:

$$\int \delta(2\omega_1 - \omega)\frac{dq_1}{\omega_1\omega_2} = \int \frac{4\pi\omega_1^2\,d\omega_1}{\omega_1^2}\delta(2\omega_1 - \omega) = 2\pi.$$

As a result we obtain for the differential probability the expression:

$$dW = \frac{G^2\mu^4}{(2\pi)^4 2^4\mu}\cdot\frac{dk}{E}.$$

Now we integrate with respect to the electron momentum. The energy released in the muon decay is considerably larger than the electron mass. Therefore we shall disregard the mass of the electron in comparison with its energy and assume that, for the electron, the momentum is equal to the energy. Then we can write:

$$\int dk = \int E^2\,dE\,d\varphi\,d\cos\theta = 4\pi\int E^2\,dE.$$

We obtain

$$W = \int \frac{G^2\mu^4}{(2\pi)^4\mu}\,\frac{4\pi E^2\,dE}{2^4.E} = \frac{G^2\mu^4}{64\pi^3\mu}\int E\,dE.$$

The integral with respect to E is taken from zero to the maximum energy, which in this decay is equal to $\mu/2$. Indeed, the electron will have the maximum energy when its direction is opposite to that of the two neutrinos.

This integral is equal to

$$\int E\,dE = \tfrac{1}{2}E_{\max}^2 = \tfrac{1}{8}\mu^2.$$

Thus, for the total probability we obtain

$$W = \frac{G^2\mu^4}{2^9\pi^3}\mu.$$

The result of this approximate calculation differs little from that of an accurate one: we will see that instead of $2^9 = 512$ in the denominator we will find 192. Since $G^2\mu^4$ is a dimensionless quantity, the probability has the dimensions of mass. Thus, we obtain the correct dimensionality

$$[m] = \left[\frac{1}{t}\right].$$

From the dimensionality it can be seen that the probability of a decay of any fermion into three other fermions must always be proportional to the fifth power of the energy released in the decay. This property is due to the fact that the interaction constant has the dimensions $[G^2] = [1/m^4]$.

The lifetime τ of the μ-meson is inversely proportional to the decay rate:

$$\tau = \frac{1}{W} = \frac{192\pi^3}{G^2\mu^5}.$$

This expression is conveniently rewritten in the form

$$\tau = \frac{192\pi^3}{10^{-10}}\left(\frac{M}{\mu}\right)^5 \frac{1}{M}.$$

We express this in seconds:

$$\frac{1}{M} \quad \text{in seconds is} \quad \frac{2 \times 10^{-14}\,\text{cm}}{3 \times 10^{10}\,\text{cm/sec}} \simeq 7 \times 10^{-25}\,\text{sec}.$$

The nucleon meson mass ratio M/μ is equal to ~ 9. Hence we obtain

$$\tau \simeq \frac{192\pi^3 \times 9^5 \times 7 \times 10^{-25}}{10^{-10}} \simeq 2 \times 10^{-6}\,\text{sec}.$$

We now carry out an accurate calculation of the decay probability with the amplitude

$$M = -\frac{G}{\sqrt{2}}(\bar{u}_e O_\alpha u_\mu)(\bar{u}_2 O_\alpha u_1).$$

For clarity, we divide the calculation into several stages.

EXPRESSION FOR $|M|^2$

Since $|M|^2 = MM^*$, we must first find an expression for M^*. M represents the product of two parentheses, each of which has the form

$$(\bar{v} O_\alpha u), \quad \text{where} \quad O_\alpha = \gamma_\alpha(1 + \gamma_5).$$

To find $(\bar{v} O_\alpha u)^*$ note that

$$\begin{aligned}(\bar{v} O_\alpha u)^* &= (v^+ \gamma_4 O_\alpha u)^* = [v_i^* (\gamma_4 O_\alpha)_{ik} u_k]^* \\ &= u_k^* (\gamma_4 O_\alpha)_{ik}^* v_i = u_k^* (\gamma_4 O_\alpha)_{ki}^+ v_i = u_k^* (O_\alpha^+ \gamma_4)_{ki} v_i = \bar{u} \bar{O}_\alpha v,\end{aligned}$$

where $\bar{O}_\alpha = \gamma_4 O_\alpha^+ \gamma_4$.

If $O_\alpha = \gamma_\alpha(1 + \gamma_5)$, then

$$\bar{O}_\alpha = \gamma_4 O_\alpha^+ \gamma_4 = \gamma_4 (1 + \gamma_5)^+ \gamma_\alpha^+ \gamma_4 = (1 - \gamma_5)\gamma_\alpha = \gamma_\alpha(1 + \gamma_5) = O_\alpha.$$

It is now easy to find M^*:

$$M^* = \frac{G}{\sqrt{2}}(\bar{u}_e O_\beta u_\mu)^* (\bar{u}_2 O_\beta u_1)^* = \frac{G}{\sqrt{2}}(\bar{u}_\mu O_\beta u_e)(\bar{u}_1 O_\beta u_2).$$

Consequently,

$$|M|^2 = \frac{G^2}{2}(\bar{u}_e O_\alpha u_\mu)(\bar{u}_2 O_\alpha u_1)(\bar{u}_\mu O_\beta u_e)(\bar{u}_1 O_\beta u_2) = \frac{G^2}{2} A_{\alpha\beta} B_{\alpha\beta},$$

where

$$A_{\alpha\beta} = (\bar{u}_e O_\alpha u_\mu)(\bar{u}_\mu O_\beta u_e), \quad B_{\alpha\beta} = (\bar{u}_2 O_\alpha u_1)(\bar{u}_1 O_\beta u_2).$$

PROJECTION OPERATORS Λ^s AND Λ

We can express the product of spinor matrices of the type $u\bar{u}$ in the expressions $A_{\alpha\beta}$ and $B_{\alpha\beta}$ by means of the momentum and polarization vector of the particle. It is easy to see that for a fermion with a given polarization

$$u\bar{u} = \Lambda^s = \tfrac{1}{2}(\hat{p} + \mu)(1 - \gamma_5 \hat{s}),$$

where $\hat{p} = \gamma_\alpha p_\alpha$, $\hat{s} = \gamma_\alpha s_\alpha$, and

$$s_\mu = \begin{cases} s_4 = \dfrac{p\xi}{\mu}, \\[2ex] s = \xi + \dfrac{(p\xi)p}{\mu(\mu + E)}, \end{cases}$$

ξ is a unit vector in the direction of the polarization of the particle in the coordinate system in which the particle is at rest.

s_μ is the relativistic generalization of the vector ξ. It can easily be seen that the vector s_μ possesses the following properties: $s^2 = -1$ and $sp = 0$. We shall show that for a polarized particle $\Lambda^s = u\bar{u}$. For this purpose, consider the case when a particle at rest is polarized in the direction of the z-axis, i.e. when $\xi_x = \xi_y = 0$, $\xi_z = 1$. Then

$$\Lambda^s = \tfrac{1}{2}(\hat{p} + \mu)(1 - \gamma_5 \hat{s}) = \tfrac{1}{2}\mu(1 + \gamma_4)(1 + \gamma_5 \gamma_3)$$

$$= \frac{\mu}{2}\begin{pmatrix} 2 & & & \\ & 2 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}\begin{pmatrix} 2 & & & \\ & 0 & & \\ & & 0 & \\ & & & 2 \end{pmatrix} = 2\mu\begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix},$$

which is the same as the expression for the matrix $u\bar{u}$, obtained in the system in which the particle is at rest. But Λ^s is covariant, and, if two covariant quantities are equal in any coordinate system they are identical. Consequently, $u\bar{u}$ can indeed be replaced by Λ^s.

If we are not interested in the polarization of the particle, then a summation should be carried out over it, i.e. it is necessary to take the sum

$$\Lambda^s + \Lambda^{-s} = \tfrac{1}{2}(\hat{p} + \mu)(1 - \gamma_5 \hat{s}) + \tfrac{1}{2}(\hat{p} + \mu)(1 + \gamma_5 \hat{s})$$
$$= (\hat{p} + \mu) = \Lambda .$$

REDUCTION TO TRACES

We shall calculate $|M|^2$ for the case when a polarized muon decays and the polarization of the electron and neutrinos is not measured. For this we return to the expressions $A_{\alpha\beta}$ and $B_{\alpha\beta}$. Consider

$$B_{\alpha\beta} = (\bar{u}_2 O_\alpha u_1)(\bar{u}_1 O_\beta u_2) = \bar{u}_{2m}(O_\alpha)_{mn} u_{1n} \bar{u}_{1k}(O_\beta)_{kl} u_{2l} .$$

After we have written the indices of all the matrices, the order of the terms can be changed at will, because the indices will determine the correct order of the terms when we multiply. Therefore $B_{x\beta}$ can be rewritten in the form

$$B_{\alpha\beta} = u_{2l}\bar{u}_{2m}(O_\alpha)_{mn} u_{1n} \bar{u}_{1k}(O_\beta)_{kl}$$
$$= \Lambda_{2lm}(O_\alpha)_{mn} \Lambda_{1nk}(O_\beta)_{kl} = (\Lambda_2 O_\alpha \Lambda_1 O_\beta)_{ll} .$$

The quantity we obtained is the sum of the diagonal elements of the matrix $\Lambda_2 O_\alpha \Lambda_1 O_\beta$. This quantity is called the trace and is denoted by the symbol Sp (see p. 27).

One can present $A_{\alpha\beta}$ in an analogous way, taking into account for the muon that the operator Λ^s is involved, because we are interested in a polarized μ-meson. As a result we obtain

$$|M|^2 = \frac{G^2}{2} \operatorname{Sp}(\Lambda_e O_\alpha \Lambda_\mu^s O_\beta) \operatorname{Sp}(\Lambda_2 O_\alpha \Lambda_1 O_\beta) .$$

CALCULATION OF TRACES

Let us begin with the calculation of the simpler neutrino trace. The operator Λ for the neutrino is of the form $\hat{q}$, because the neutrino mass equals zero. For $B_{\alpha\beta}$ we obtain $B_{\alpha\beta} = \operatorname{Sp} \hat{q}_2 O_\alpha \hat{q}_1 O_\beta$, which can be rewritten in the form

$$B_{\alpha\beta} = q_{2\varrho} q_{1\sigma} \operatorname{Sp} \gamma_\varrho O_\alpha \gamma_\sigma O_\beta .$$

Substituting for the operators O_α and O_β their explicit expressions $O_\alpha = \gamma_\alpha(1 + \gamma_5)$ and $O_\beta = \gamma_\beta(1 + \gamma_5)$, we obtain

$$B_{\alpha\beta} = q_{2\varrho} q_{1\sigma} \operatorname{Sp} \gamma_\varrho \gamma_\alpha (1 + \gamma_5) \gamma_\sigma \gamma_\beta (1 + \gamma_5) .$$

If one makes use of the property $\gamma_\mu\gamma_5 = -\gamma_5\gamma_\mu$, where $\mu = 1, 2, 3, 4$, and also takes into account that $(1+\gamma_5)^2 = 2(1+\gamma_5)$, then the expression for $B_{\alpha\beta}$ can be presented in the form

$$B_{\alpha\beta} = 2q_{2\varrho}q_{1\sigma}\,\mathrm{Sp}\,\gamma_\varrho\gamma_\alpha\gamma_\sigma\gamma_\beta(1+\gamma_5).$$

Chapter 2 (p. 27) gives formulae for the calculation of traces. Applying them we obtain

$$B_{\alpha\beta} = 8q_{2\varrho}q_{1\sigma}\chi_{\varrho\sigma\varkappa\beta},$$

where

$$\chi_{\varrho\sigma\alpha\beta} = \delta_{\varrho\varkappa}\delta_{\sigma\beta} + \delta_{\varrho\beta}\delta_{\sigma\alpha} - \delta_{\varrho\sigma}\delta_{\alpha\beta} + i\varepsilon_{\varrho\sigma\alpha\beta}.$$

The expression for $A_{\alpha\beta}$ is calculated in an analogous way. Disregarding the electron mass, we get

$$\begin{aligned} A_{\varkappa\beta} &= \mathrm{Sp}\,\Lambda_e O_\alpha \Lambda^s_\mu O_\beta = \tfrac{1}{2}\mathrm{Sp}\,\hat{k}\gamma_\alpha(1+\gamma_5)(\hat{p}+\mu)(1-\gamma_5\hat{s})\times \\ &\quad \times\gamma_\beta(1+\gamma_5) = \mathrm{Sp}\,\hat{k}\gamma_\alpha(\hat{p}-\mu\gamma_5\hat{s})\gamma_\beta(1+\gamma_5) \\ &= 4k_\mu(p_\nu - \mu s_\nu)\chi_{\mu\nu\varkappa\beta}. \end{aligned}$$

For the absolute square of the amplitude we obtain

$$|M|^2 = \frac{G^2}{2}A_{\varkappa\beta}B_{\alpha\beta} = 16G^2 q_{2\varrho}q_{1\sigma}k_\mu(p_\nu - \mu s_\nu)\chi_{\varrho\sigma\varkappa\beta}\chi_{\mu\nu\varkappa\beta}.$$

Calculate now the product $\chi_{\varrho\sigma\alpha\beta}\chi_{\mu\nu\alpha\beta}$:

$$\begin{aligned} \chi_{\varrho\sigma\varkappa\beta}\chi_{\mu\nu\varkappa\beta} &= [\delta_{\varrho\varkappa}\delta_{\sigma\beta} + \delta_{\varrho\beta}\delta_{\sigma\alpha} - \delta_{\varrho\sigma}\delta_{\alpha\beta} + i\varepsilon_{\varrho\beta\sigma\alpha}]\times \\ &\quad\times[\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\alpha\nu} - \delta_{\mu\nu}\delta_{\alpha\beta} + i\varepsilon_{\mu\nu\varkappa\beta}] = 4\delta_{\mu\varrho}\delta_{\nu\sigma}. \end{aligned}$$

In finding this result we took advantage of the fact that

$$\begin{aligned} \delta_{\mu\varkappa}\delta_{\alpha\varrho} &= \delta_{\mu\varrho}, \\ \delta_{\alpha\beta}\delta_{\varkappa\beta} &= 4, \\ \varepsilon_{\varrho\sigma\varkappa\beta}\varepsilon_{\mu\nu\varkappa\beta} &= -2(\delta_{\mu\varrho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\varrho}). \end{aligned}$$

Thus, we finally have

$$|M|^2 = 64G^2(kq_2)(pq_1 - \mu sq_1).$$

INTEGRATION WITH RESPECT TO NEUTRINO MOMENTA

If it were possible to measure neutrino momenta, then the expression that we obtained could be compared with experiment. However, neutrino momenta cannot be measured, hence we have to integrate over these momenta. The corresponding decay rate is equal to

$$dW = \frac{(2\pi)^4 64 G^2 k_\alpha (p_\beta - \mu s_\beta)}{16 E_\mu (2\pi)^9} \frac{dk}{E} I_{\alpha\beta},$$

where

$$I_{\alpha\beta} = \int q_{2\alpha} q_{1\beta} \frac{d\boldsymbol{q}_1\, d\boldsymbol{q}_2}{\omega_1 \omega_2} \delta^4(q_1 + q_2 - q), \qquad q = p - k.$$

The integral $I_{\alpha\beta}$, having the dimensions of the square of momentum, must be expressed by a certain tensor depending only on q. The most general form of this tensor is

$$I_{\alpha\beta} = Aq^2 \,.\, \delta_{\alpha\beta} + Bq_\alpha q_\beta,$$

where A and B are dimensionless scalar coefficients. In order to find A and B, multiply the preceding equation first by $\delta_{\alpha\beta}$ and then by $q_\alpha q_\beta$. We obtain

$$I_{\alpha\beta}\delta_{\alpha\beta} = \pi q^2 = 4Aq^2 + Bq^2,$$

$$I_{\alpha\beta} q_\alpha q_\beta = \frac{\pi q^4}{2} = Aq^4 + Bq^4.$$

We used the fact that $q = q_1 + q_2$, $q_1^2 = q_2^2 = 0$, and made use of the result obtained in the beginning of this chapter:

$$\int \frac{d\boldsymbol{q}_1\, d\boldsymbol{q}_2}{\omega_1 \omega_2} \delta^4(q_1 + q_2 - q) = 2\pi.$$

Solving the system of equations

$$\begin{cases} 4A + B = \pi, \\ 4A + 4B = 2\pi, \end{cases}$$

we obtain $A = \pi/6$, $B = \pi/3$, whence

$$I_{\alpha\beta} = \frac{\pi}{6}(q^2\delta_{\alpha\beta} + 2q_\alpha q_\beta).$$

The expression for the decay probability, integrated with respect to neutrino momenta, is obtained in the form

$$dW = \frac{G^2}{3(2\pi)^4 \mu} k_\alpha (p_\beta - \mu s_\beta)(q^2 \delta_{\alpha\beta} + 2q_\alpha q_\beta) \frac{dk}{E},$$

$$dW = \frac{G^2 E\, dE\, d\cos\theta}{3(2\pi)^3 \mu} \{[q^2 kp + 2kqpq] - \mu[q^2 sk + 2kqqs]\}.$$

As before, we disregard the mass of the electron in comparison with its energy.

SPECTRUM OF DECAY ELECTRONS

We calculate first the part of the probability that does not depend on the spin. Taking into account the relation $q = p - k$, and assuming the muon to be at rest, we obtain

$$\begin{aligned} q^2 &= (p-k)^2 = \mu^2 - 2\mu E, \\ kq &= kp = \mu E, \\ qp &= \mu^2 - \mu E, \end{aligned}$$

whence upon integration with respect to the angle θ it follows that

$$dW = \frac{2G^2}{3(2\pi)^3\mu}[3\mu^2 - 4\mu E]\mu E^2\, dE = \frac{G^2\mu^5}{96\pi^3}(3 - 2\varepsilon)\,\varepsilon^2\, d\varepsilon.$$

Here $\varepsilon = \dfrac{E}{\mu/2}$ is the ratio of the energy E of the electron to its maximum possible energy $\mu/2$, where μ is the muon mass. In the case of a general four-fermion interaction, the electron spectrum in the muon decay is characterized by the Michel parameter ϱ:

$$W(\varepsilon)\, d\varepsilon = \frac{12}{\tau}\left[(1-\varepsilon) - \frac{2}{9}\varrho(3 - 4\varepsilon)\right]\varepsilon^2\, d\varepsilon.$$

It is easy to see that the spectrum we obtained corresponds to $\varrho = 0{\cdot}75$. Integrating dW with respect to ε from 0 to 1, we obtain for the total muon decay rate

$$\frac{1}{\tau} = W = \frac{G^2\mu^4}{192\pi^3}\mu.$$

ASYMMETRY AND POLARIZATION OF ELECTRONS

We calculate now the asymmetry of electrons in the polarized-muon decay. Taking into account that $(sp) = 0$ (this follows from the definition of s), we obtain

$$\begin{aligned} \mu[skq^2 + 2sqqk] &= \mu sk[\mu^2 - 4\mu E] \\ &= -\eta k\mu(-4\mu E + \mu^2) = \eta n\mu E(4\mu E - \mu^2), \end{aligned}$$

where η is the unit vector in the direction of polarization of the muon in the system in which the muon is at rest. For the decay probability we obtain

$$dW = \frac{G^2\mu^5}{192\pi^3}[(3 - 2\varepsilon) + \eta n(1 - 2\varepsilon)]\,\varepsilon^2\, d\varepsilon\, d\cos\theta,$$

where θ is the angle between the muon spin $\boldsymbol{\eta}$ and the vector $\boldsymbol{n}$ in the direction of the electron momentum; $\boldsymbol{n} = \boldsymbol{k}/E$, $\cos\theta = \boldsymbol{n\eta}$.

The formulae obtained refer to the case when the electron polarization is not measured. If we were interested in the dependence of the decay rate on the direction of the electron spin, we should in the expression for $A_{\alpha\beta}$ substitute Λ_e by Λ_e^s, where

$$s_4 = \frac{\boldsymbol{k\zeta}}{m}, \quad \boldsymbol{s} = \boldsymbol{\zeta} + \frac{(\boldsymbol{k\zeta})\,\boldsymbol{k}}{m(E+m)}.$$

Here m, E, $\boldsymbol{k}$ are the mass, energy and momentum of the electron, while $\boldsymbol{\zeta}$ is the unit polarization vector in the system in which the electron is at rest. Carrying out the calculations in a straightforward way and disregarding in the final result terms of the order of m/E in comparison with unity, we obtain for the μ^--decay

$$dW_- \sim (1 - \boldsymbol{\zeta n})\,[3 - 2\varepsilon + \boldsymbol{\eta n}(1 - 2\varepsilon)],$$

where $\boldsymbol{n}$ is the unit vector in the direction of the electron momentum, $\boldsymbol{\zeta}$ is the unit vector in the direction of the electron polarization, and $\boldsymbol{\eta}$ is the unit vector in the direction of the μ-meson polarization.

We see that the electron must be totally polarized longitudinally in the direction opposite to that of its motion. Carrying out analogous calculations for the μ^+-decay, we would obtain

$$dW_+ \sim (1 + \boldsymbol{\zeta n})\,[3 - 2\varepsilon - (\boldsymbol{\eta n})\,(1 - 2\varepsilon)].$$

From this formula it follows that the positron must be polarized along the direction of its motion. This result is common for the V–A-interaction. It is constructed in such a way that at $v \sim c$ particles should have left-handed polarization, whereas antiparticles should have right-handed polarization. For the neutrino $v = c$, and the weak V–A-interaction is, apparently, the only one in which they take part. Consequently the neutrino and antineutrino are polarized longitudinally: neutrinos (like the electron) have the left-handed polarization, whereas antineutrinos (like the positron) have the right-handed polarization.

Knowing this, it is easy to understand the angular distributions obtained. Consider, for example, the polarized μ^+-meson decay. When $\varepsilon \sim 1$ the neutrino and antineutrino go off in the same direction, their total spin being equal to zero. Hence the spin of the positron must be directed along the spin of the muon and, because the directions of the positron momentum and spin are the same, also the positron momentum must be directed along the muon spin:

$$W \sim 1 + \eta n.$$

When $\varepsilon \sim 0$ the neutrinos go in opposite directions. Their total spin is equal to unity. In order to carry away the muon momentum, the antineutrino must travel along the muon spin direction, whereas the neutrino and positron must go in the opposite direction:

$$W \sim 3 - \eta n.$$

For the μ^--meson an analogous reasoning holds.

Experimental data on the muon decay are in a good agreement with these consequences of the V–A-theory.

NEUTRINO-ELECTRON SCATTERING

One of the fundamental consequences the of weak interaction theory we have considered is the prediction that there must exist neutrino–electron scattering. The amplitude for neutrino–electron scattering is of the form

$$M_{\nu e} = \frac{G}{\sqrt{2}} (\bar{u}_e O_\alpha u_\nu)(\bar{u}_\nu O_\alpha u_e),$$

where u_e and u_ν are the wave functions of the electron and neutrino respectively, and

$$O_\alpha = \gamma_\alpha (1 + \gamma_5).$$

By virtue of the Fierz relation (see p. 54) u_ν and u_e can be exchanged, and we obtain

$$M_{\nu e} = -\frac{G}{\sqrt{2}} (\bar{u}_e O_\alpha u_e)(\bar{u}_\nu O_\alpha u_\nu).$$

The amplitude of the scattering of the antineutrino $\bar{\nu}$ by the electron e is obtained by replacing the neutrino bracket by its charge-conjugate (see p. 75):

$$M_{\bar{\nu} e} = -\frac{G}{\sqrt{2}} (\bar{u}_e O_\alpha u_e)\,[\bar{u}_{\bar{\nu}} \gamma_\alpha (1 - \gamma_5)\, u_{\bar{\nu}}].$$

Knowing the amplitudes of the neutrino and antineutrino scattering by the electron, it is easy to calculate cross sections for these two processes. In accordance with the results obtained on

p. 39, the scattering cross section is expressed in terms of the amplitude M in the following way:

$$d\sigma = \frac{(2\pi)^4|M|^2}{4(p_1 q_1)} \frac{dq_2}{(2\pi)^3 2\omega_2} \frac{dp_2}{(2\pi)^3 2E_2} \delta^4(p_1 + q_1 - p_2 - q_2),$$

where p_1 and p_2 are the electron 4-momenta before and after the collision, while q_1 and q_2 are the neutrino four-momenta before and after the collision.

After carrying out a simple calculation, we obtain for the total cross sections for νe and $\tilde{\nu} e$ scattering the following formulae:

$$\sigma_{\nu e} = \sigma_0 \frac{2\omega^2}{1 + 2\omega},$$

$$\sigma_{\tilde{\nu} e} = \frac{\sigma_0 \omega}{3}\left[1 - \frac{1}{(1 + 2\omega)^3}\right],$$

where $\sigma_0 = 2G^2 m^2/\pi = 8{\cdot}3 \times 10^{-45}$ cm^2, while ω is the neutrino energy in units of the electron mass.

These results (of course, to within coefficients) are easy to obtain without any calculations, simply on the basis of the dimensionality. As a matter of fact, we know that the cross section must be proportional to the square of the weak interaction constant G, which has the dimensions of $[1/m^2]^2$ while, on the other hand, the dimensions of a cross section are $[\sigma] = [1/m^2]$. Consequently, G^2 must be multiplied by the square of the energy characteristic for the νe-scattering. Such a characteristic energy is the total energy of the system $E = E_e + E_\nu$ in the centre-of-mass system. Thus, the cross section must have the form

$$\sigma \sim G^2 E^2.$$

The total energy of the system can be written in the form

$$E^2 = (p_1 + q_1)^2 = m^2 + 2p_1 q_1 + q_1^2 \approx 2q_1 p_1 = 2mE_\nu = 2m^2\omega$$

(assuming $E \gg m$) and, hence, the cross section will be proportional to the neutrino energy in the laboratory coordinate system:

$$\sigma \sim G^2 E^2 \sim G^2 m^2 \omega.$$

Thus, already from the dimensions we see that the cross section must be proportional to the square of the total energy in the centre-of-mass system and to the first power of the neutrino energy in the laboratory coordinate system.

Measurement of the scattering of the neutrino (and antineutrino) on the electron is a problem of paramount importance. The accuracy of the experiments already carried out allows one to conclude that experimentally this cross section is less than 4×10^{-43} cm^2 for neutrinos produced in neutron piles.

MUON-PAIR PRODUCTION IN THE NEUTRINO BEAM

The interaction of the muon current with itself must lead to the process of neutrino-muon scattering analogous to νe-scattering. Direct observation of this process is, in practice, impossible. Another process, the transformation of the neutrino into $\mu^+ + \mu^-$ and ν,

$$\nu \rightarrow \nu + \mu^+ + \mu^-,$$

resulting from the same interaction, is more accessible to observation. In vacuum this process is forbidden by the energy-momentum conservation law. However, in the Coulomb field of nuclei it is allowed:

$$\nu + Z \rightarrow \nu + \mu^+ + \mu^- + Z.$$

Estimates show that the detection of this process is at the limit of present experimental possibilities.

It is obviously important to detect the other, analogous, reactions:

$$\nu + Z \rightarrow \nu + e^+ + e^- + Z,$$
$$\nu + Z \rightarrow \nu + e^- + \mu^+ + Z,$$
$$\nu + Z \rightarrow \nu + e^+ + \mu^- + Z.$$

CHAPTER 5

LEPTONIC DECAYS OF STRONGLY INTERACTING PARTICLES. GENERAL PROPERTIES OF THE AMPLITUDES

PROBLEM OF STRONG INTERACTIONS

In the weak interactions that we have considered so far, no strongly interacting particles have taken part. If we wish to consider, for instance, the decay of the π^+-meson into a muon and a neutrino, then we can no longer avoid strong interactions. In the language of ordinary field theory it can be said that the decay of the π^+-meson into the muon and neutrino proceeds in two stages: first the π^+-meson turns into a proton and an antineutron, and then these particles annihilate, giving μ^+ and ν. Graphically this process has the form shown in Fig. 6. In terms of the Sakata model the proton and antineutron constituting the π^+-meson go over to $\mu + \nu$. But, one way or another, we have to do here with strong interactions.

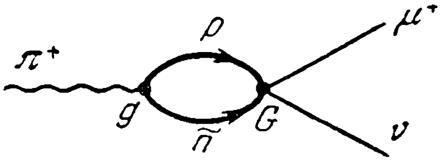

FIG. 6

Within the framework of the composite model one knows no method of calculating processes involving strongly interacting particles, hence not much use can be derived from the Sakata model in estimating one or another matrix element. But likewise, within the framework of ordinary field theory we cannot really calculate the transformation of π-mesons into nucleons and antinucleons. The lack of a theory of strong interactions thus deprives us of the possibility of calculating directly the $\pi^+ \to \mu^+ + \nu$ decay.

We encounter the same difficulty in considering the decay of the neutron into the proton, electron and neutrino. Because the neutron and proton are strongly interacting particles this process is "deformed" by the strong interaction. In the course of the decay the neutron and proton exchange π-mesons or, in terms of the composite model, nucleon-antinucleon pairs. This leads to a change in the initial value of the matrix element of the decay of "bare" nucleons, and we cannot calculate this change for lack of a theory of strong interactions.

These remarks on the decay of the π-meson and neutron refer completely to all leptonic decays of strongly interacting particles.

Therefore we shall try to find some indirect way of estimating the amplitude for such a process on the basis of the various symmetry properties possessed by weak and strong interactions.

GENERAL FORM OF THE AMPLITUDE

The interaction leading to leptonic decays of strongly interacting particles is represented as the product of the baryonic current $j_{s\alpha}$ and leptonic current $j_{l\alpha}$:

$$L_w^{sl} = j_{s\alpha}^+ j_{l\alpha} + j_{s\alpha} j_{l\alpha}^+.$$

Here $j_{l\alpha} = \bar{e}\gamma_\alpha(1 + \gamma_5)\,\nu$ or $\bar{\mu}\gamma_\alpha(1 + \gamma_5)\,\nu$ according to whether the electron decay or muon decay is considered, while $j_{s\alpha} = \bar{n}\gamma_\alpha(1 + \gamma_5)\,p$ or $\bar{\Lambda}\gamma_\alpha(1 + \gamma_5)\,p$, according to whether or not the strangeness of strongly interacting particles is conserved in this decay. (The index l stands for leptons, while s stands for strongly interacting particles.)

We have to take account of the interaction between leptons and strongly interacting particles only in the first order of perturbation theory, hence the amplitude M can be written in the form of a product:

$$M = \frac{G}{\sqrt{2}} X_\alpha l_\alpha,$$

each term of which represens a 4-vector. The quantity l_α has the form $\bar{u}_l\gamma_\alpha(1 + \gamma_5)\,u_\nu$, where u_l and u_ν are the spinor wave functions of the electron (or muon) and neutrino. The quantity X_α, which we shall call the matrix element, represents the "bare" baryonic current $j_{s\alpha}^+$ "dressed" by virtual strong interactions:

$$X_\alpha = \langle f \,|\, j_{s\alpha}^+ \,|\, i \rangle.$$

Here $|i\rangle$ and $\langle f|$ denote respectively the initial and final state of the strongly interacting particles (for β-decay, for example, $|i\rangle = |n\rangle$, while $\langle f| = \langle p|$). In the expression for X_α, although weak interactions are taken into account only in perturbation theory, the contribution of virtual strong interactions is taken completely into account. Since the current $j^+_{s\alpha}$ consists of a vector part and an axial part, then

$$X_\alpha = A_\alpha + V_\alpha .$$

Because in strong interactions the parity is conserved, the axial part of the matrix element A_α is generated by the pseudovector part of the current $j^+_{s\alpha}$, while V_α is generated by the vector part of $j^+_{s\alpha}$. We cannot calculate the value of A_α and V_α, but we know some of their properties. The essential property of A_α and V_α is that they may depend only on the sum of the momenta of the leptons, but not on each momentum separately. This is due to the fact that leptons ($e + \nu$ or $\mu + \nu$) are emitted from a single point, and simplifies substantially the determination of the form of V_α and A_α.

THREE TYPES OF MATRIX ELEMENTS

The number of different forms of leptonic decays of strongly interacting particles is not large. In essence, we shall have to deal with three basic types of processes:

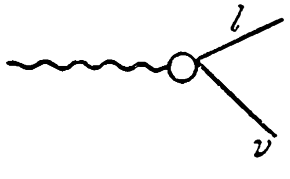

Fig. 7

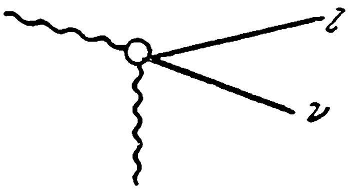

Fig. 8

1. Decay of a meson into leptons (Fig. 7), for example,

$$\pi^+ \to \mu^\pm(e^\pm) \pm \nu,$$

$$K^\pm \to \mu^\pm(e^\pm) \pm \nu.$$

Here $+\nu$ denotes emission of neutrino, while $-\nu$ denotes that of antineutrino.

2. Decay of a meson into another meson and leptons (Fig. 8):

$$\pi^+ \to \pi^0 + e \pm \nu,$$
$$K \to \pi + e + \nu,$$
$$K \to \pi + \mu + \nu.$$

3. Decay of a baryon into another baryon and leptons (Fig. 9): neutron β-decay $n \to p + e^- + \tilde{\nu}$, μ-meson capture by the proton ($\mu^- + p \to n + \nu$), leptonic decays of hyperons, for instance $\Lambda \to p + e^- + \tilde{\nu}$.

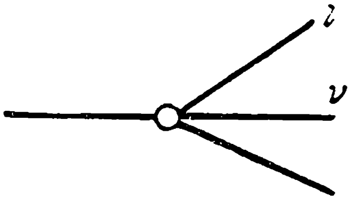

FIG. 9

The fine lines of the drawings represent leptons, the wavy lines represent mesons, and the solid lines represent baryons. The circle represents the whole set of virtual strong interactions.

DECAYS OF THE FIRST TYPE

Consider a diagram of the first type. For constructing the 4-vector X_α we have at our disposal the scalar (pseudoscalar) φ—the wave function of the meson undergoing decay—and the meson 4-momentum k_α equal to the sum of the momenta of the leptons: $k = k_l + k_\nu$ (the index l denotes e or μ). We have no choice but to write that in this case

$$X_\alpha = f\varphi k_\alpha.$$

If the meson undergoing decay is a scalar meson (i.e. φ is scalar), then $X_\alpha = V_\alpha$, $A_\alpha = 0$, and if the meson is a pseudoscalar meson (i.e. φ is pseudoscalar), then $X_\alpha = A_\alpha$, $V_\alpha = 0$. The quantity f in the expression for X_α is a number whose value we do not know. (In general, f is a function of k^2, but in this case $k^2 = \mu^2$, where μ is the mass of the meson.) One might say that, because of the presence of the unknown coefficient in the matrix element, we have not achieved much; but this is not so. As an illustration of what we get by knowing the matrix element, up to an unknown factor, consider the decays

$$\pi \to \mu + \nu \quad \text{and} \quad \pi \to e + \nu.$$

The amplitudes of these processes are of the form

$$M = f\varphi k_\alpha \bar{u}_l \gamma_\alpha (1 + \gamma_5) u_\nu = f\varphi \bar{u}_l \hat{k} (1 + \gamma_5) u_\nu$$
$$= f\varphi \bar{u}_l (\hat{k}_l + \hat{k}_\nu)(1 + \gamma_5) u_\nu,$$

where k_l is the 4-momentum of the electron or muon. Making use of the Dirac equation for l and ν

$$\bar{u}_l \hat{k}_l = m_l \bar{u}_l, \qquad \hat{k}_\nu u_\nu = 0,$$

it is easy to obtain

$$M = f m_l \varphi \bar{u}_l (1 + \gamma_5) u_\nu.$$

Therefore it turns out that the matrix element is proportional to the mass of the lepton, and, since the mass of the electron is 200 times less than that of the muon, the $\pi \to e + \nu$ decay must be less frequent by four orders in comparison with the $\pi \to \mu + \nu$ decay. The same holds also for the decays $K \to e + \nu$ and $K \to \mu + \nu$.

DECAYS OF THE SECOND TYPE

In the case when a meson decays into leptons, everything is quite simple. The case when a meson decays into another meson and leptons is not much more complicated. In this case, for constructing the matrix element, we have the wave functions and 4-momenta of the meson undergoing decay (φ_1, k_1) and those of the meson produced (φ_2, k_2). From these quantities we can construct X_α. The most general form of X_α is

$$X_\alpha = \varphi_2^+ \varphi_1 (f_1 k_{1\alpha} + f_2 k_{2\alpha}),$$

where f_1 and f_2 are scalar functions which we do not know and which, in general, are distinct. If the parities of the mesons 1 and 2 are the same, then $X_\alpha = V_\alpha$, $A_\alpha = 0$. If the parities are different, then $X_\alpha = A_\alpha$, $V_\alpha = 0$. On what do the scalar functions f_1 and f_2 depend? Only on scalar quantities. Since only the sum of the momenta of the leptons (k) enters the matrix element, from the energy-momentum conservation law $(k_1 = k + k_2)$ it follows that there are two independent 4-momenta. We shall choose k_1 and k_2 to be the independent 4-momenta. From these 4-vectors the following scalars can be constructed: k_1^2, k_2^2, $(k_1 k_2)$; however, $k_1^2 = m_1^2$, $k_2^2 = m_2^2$ and so the only scalar variable is $(k_1 k_2)$. Sometimes instead of $(k_1 k_2)$ it is more convenient to consider $k^2 = (k_1 - k_2)^2$, i.e. the square of the 4-momentum transferred to the leptons. Thus, the quantities f_1 and f_2 are functions of k^2. (Note that in the system in which the meson undergoing decay is at rest, $(k_1\, k_2) = m_1 E_2$, where E_2 is the total energy of the meson produced.)

DECAYS OF THE THIRD TYPE

Consider, finally, the matrix elements describing baryon decay. In this case, in addition to the 4-momenta k_1 and k_2 of the initial and final baryon there are also other vectors which can be constructed of the baryon spinors u_1 and $\bar{u}_2$ by means of γ-matrices.

We construct V_α for the case when the parities of the decaying and produced baryon are the same. In this case V_α is a linear combination of terms of the type $\bar{u}_2 v_\alpha u_1$, where v_α is to be formed from $k_{1\alpha}$, $k_{2\alpha}$, γ_α, $\gamma_\alpha \hat{k}_1$, $\gamma_\alpha \hat{k}_2$, $\hat{k}_1 \gamma_\alpha$, $\hat{k}_2 \gamma_\alpha$, $\hat{k}_1 \gamma_\alpha \hat{k}_2$, $\hat{k}_2 \gamma_\alpha \hat{k}_1$ and so on, while $\bar{u}_2$ and u_1 are the spinor wave functions of the two baryons.

The number of different terms of the type $\bar{u}_2 v_\alpha u_1$ can be reduced substantially by means of the Dirac equation. For example,

$$\begin{aligned} \bar{u}_2 \gamma_\alpha \hat{k}_1 u_1 &= m_1 \bar{u}_2 \gamma_\alpha u_1, \\ \bar{u}_2 \hat{k}_2 \gamma_\alpha u_1 &= m_2 \bar{u}_2 \gamma_\alpha u_1, \\ \bar{u}_2 \hat{k}_1 \gamma_\alpha u_1 &= \bar{u}_2 (-\gamma_\alpha \hat{k}_1 + 2k_{1\alpha})\, u_1 \\ &= -m_1 \bar{u}_2 \gamma_\alpha u_1 + 2\bar{u}_2 k_{1\alpha} u_1, \quad \text{etc.} \end{aligned}$$

Proceeding in the same way, we arrive at an expression with only three terms:

$$V_\alpha = \bar{u}_2 (h_1 \gamma_\alpha + h_2 k_{1\alpha} + h_3 k_{2\alpha})\, u_1 .$$

We could retain the expression in this form, but it is somewhat inconvenient, because it depends in particular on the momenta of the initial and final baryon. The expression is more conveniently presented in a form where there is dependence only on the momentum transferred to the leptons. This is easy to do by taking advantage of the fact that

$$\begin{aligned} 2k_{1\alpha} &= \gamma_\alpha \hat{k}_1 + \hat{k}_1 \gamma_\alpha, & 2k_{2\alpha} &= \gamma_\alpha \hat{k}_2 + \hat{k}_2 \gamma_\alpha; \\ 2\sigma_{\alpha\beta} k_\beta &= \gamma_\alpha \hat{k} - \hat{k} \gamma_\alpha, & 2\sigma_{\alpha\beta} &= \gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha . \end{aligned}$$

Then we obtain

$$V_\alpha = \bar{u}_2 (f_1 \gamma_\alpha + f_2 \sigma_{\alpha\beta} k_\beta + f_3 k_\alpha)\, u_1 ,$$

where $k = k_1 - k_2$ is the 4-momentum transferred to the leptons. The quantities f are functions of k^2.

The axial part of the matrix element X_α is equal to

$$A_\alpha = \bar{u}_2 [g_1 \gamma_\alpha + g_2 \sigma_{\alpha\beta} k_\beta + g_3 k_\alpha] \gamma_5 u_1 .$$

The functions g also depend only on k^2. If the parities of the initial and final baryons were different, the expressions for A_α and V_α would change places. Thus, it turns out that the matrix element X_α contains, in general, six unknown complex functions f and g.

REALITY OF THE FUNCTIONS f AND g

We shall now show that by virtue of the CP-invariance of the theory the functions f and g must be real. To prove this statement we have to consider how the expressions contained in V_α and A_α transform under the PC-transformation and Hermitian conjugation.

Under space inversion P, as we know,

$$u \rightarrow \gamma_4 u, \quad \bar{u} \rightarrow \bar{u} \gamma_4, \quad k_4 \rightarrow k_4' = k_4, \quad \boldsymbol{k} \rightarrow \boldsymbol{k}' = -\boldsymbol{k}.$$

Hence

$$\bar{u}u \rightarrow \bar{u}\gamma_4\gamma_4 u = \bar{u}u,$$

$$\bar{u}\gamma_5 u \rightarrow \bar{u}\gamma_4\gamma_5\gamma_4 u = -\bar{u}\gamma_5 u,$$

$$\bar{u}\gamma_\alpha u \rightarrow \bar{u}\gamma_4\gamma_\alpha\gamma_4 u = \begin{cases} \bar{u}\gamma_\alpha u, & \alpha = 4, \\ -\bar{u}\gamma_\alpha u, & \alpha \neq 4, \end{cases}$$

$$\bar{u}\gamma_\alpha\gamma_5 u \rightarrow \bar{u}\gamma_4\gamma_\alpha\gamma_5\gamma_4 u = \begin{cases} -\bar{u}\gamma_\alpha\gamma_5 u, & \alpha = 4, \\ \bar{u}\gamma_\alpha\gamma_5 u, & \alpha \neq 4, \end{cases}$$

$$\bar{u}\sigma_{\alpha\beta} u \rightarrow \bar{u}\gamma_4\sigma_{\alpha\beta}\gamma_4 u = \begin{cases} \bar{u}\sigma_{\alpha\beta} u, & \alpha, \beta \neq 4, \\ -\bar{u}\sigma_{\alpha\beta} u, & \begin{cases} \alpha \neq 4,\ \beta = 4, \\ \alpha = 4,\ \beta \neq 4. \end{cases} \end{cases}$$

Making use of these relations we find that under P-inversion the expressions for V_α and A_α transform in the following way:

$$\begin{aligned} V_\alpha &= \bar{u}_2 [f_1 \gamma_\alpha + f_2 \sigma_{\alpha\beta} k_\beta + f_3 k_\alpha] u_1 \\ &\rightarrow \bar{u}_2 \gamma_4 [f_1 \gamma_\alpha + f_2 \sigma_{\alpha\beta} k_\beta' + f_3 k_\alpha'] \gamma_4 u_1 \\ &= \begin{cases} \bar{u}_2 [f_1 \gamma_\alpha + f_2 \sigma_{\alpha\beta} k_\beta + f_3 k_\alpha] u_1, & \text{when} \quad \alpha = 4, \\ -\bar{u}_2 [f_1 \gamma_\alpha + f_2 \sigma_{\alpha\beta} k_\beta + f_3 k_\alpha] u_1, & \text{when} \quad \alpha = 1, 2, 3. \end{cases} \end{aligned}$$

We see (in accordance with p. 48) that the quantity V_α transforms as a polar vector. The quantity A_α transforms as an axial vector:

$$\begin{aligned} A_\alpha &= \bar{u}_2 [g_1 \gamma_\alpha + g_2 \sigma_{\alpha\beta} k_\beta + g_3 k_\alpha] \gamma_5 u_1 \\ &\rightarrow \bar{u}_2 \gamma_4 [g_1 \gamma_\alpha + g_2 \sigma_{\alpha\beta} k_\beta' + g_3 k_\alpha'] \gamma_5 \gamma_4 u_1 \\ &= \begin{cases} -\bar{u}_2 [g_1 \gamma_\alpha + g_2 \sigma_{\alpha\beta} k_\beta + g_3 k_\alpha] \gamma_5 u_1, & \text{when} \quad \alpha = 4, \\ \bar{u}_2 [g_1 \gamma_\alpha + g_2 \sigma_{\alpha\beta} k_\beta + g_3 k_\alpha] \gamma_5 u_1, & \text{when} \quad \alpha = 1, 2, 3. \end{cases} \end{aligned}$$

Under charge conjugation C

$$u \rightarrow C\tilde{\bar{u}}, \qquad \bar{u} \rightarrow -\tilde{u}C^{+},$$

$$k_1 \rightarrow k_1' = -k_1, \quad k_2 \rightarrow k_2' = -k_2,$$

$$k = k_2 - k_1 \rightarrow k' = k_1' - k_2' = k.$$

The matrix C satisfies the relations (see p. 49):

$$C^{+}C = 1, \qquad C\gamma_\alpha C^{+} = -\tilde{\gamma}_\alpha.$$

In what follows we shall make use of the fact that in our representation the matrix $C = \gamma_2\gamma_4$ is Hermitian:

$$C = C^{+}.$$

Let us see how the following expressions transform under C-conjugation:

$$\bar{u}_2 u_1 \rightarrow -\tilde{u}_2 C C\tilde{\bar{u}}_1 = -\tilde{u}_2\tilde{\bar{u}}_1 = -\bar{u}_1 u_2,$$

$$\bar{u}_2\gamma_5 u_1 \rightarrow -\tilde{u}_2 C\gamma_5 C\tilde{\bar{u}}_1 = -\tilde{u}_2\tilde{\gamma}_5\tilde{\bar{u}}_1 = -\bar{u}_1\gamma_5 u_2, \text{since}\, \tilde{\gamma}_5 = \gamma_5,$$

$$\bar{u}_2\gamma_\alpha u_1 \rightarrow -\tilde{u}_2 C\gamma_\alpha C\tilde{\bar{u}}_1 = -\tilde{u}_2(-\tilde{\gamma}_\alpha)\,\tilde{\bar{u}}_1 = \bar{u}_1\gamma_\alpha u_2,$$

$$\bar{u}_2\gamma_\alpha\gamma_5 u_1 \rightarrow -\tilde{u}_2 C\gamma_\alpha\gamma_5 C\tilde{\bar{u}}_1 = \tilde{u}_2\tilde{\gamma}_\alpha\tilde{\gamma}_5\tilde{\bar{u}}_1 = -\bar{u}_1\gamma_\alpha\gamma_5 u_2,$$

$$\bar{u}_2\sigma_{\alpha\beta}u_1 \rightarrow -\tilde{u}_2 C\sigma_{\alpha\beta}C\tilde{\bar{u}}_1 = -\tilde{u}_2\frac{C(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)\,C}{2}\bar{u}_1$$

$$= -\tilde{u}_2\frac{C\gamma_\alpha C C\gamma_\beta C - C\gamma_\beta C C\gamma_\alpha C}{2}\tilde{\bar{u}}_1$$

$$= -\tilde{u}_2\frac{\tilde{\gamma}_\alpha\tilde{\gamma}_\beta - \tilde{\gamma}_\beta\tilde{\gamma}_\alpha}{2}\tilde{\bar{u}}_1 = \bar{u}_1\frac{\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha}{2}u_2 = \bar{u}_1\sigma_{\alpha\beta}u_2.$$

Employing these relations we find that under C-conjugation the expressions for V_α and A_α transform in the following way:

$$V_\alpha = \bar{u}_2[f_1\gamma_\alpha + f_2\sigma_{\alpha\beta}k_\beta + f_3 k_\alpha]\,u_1$$

$$\rightarrow -\tilde{u}_2 C^{+}[f_1\gamma_\alpha + f_2\sigma_{\alpha\beta}k_\beta + f_3 k_\alpha]\,C\tilde{\bar{u}}_1$$

$$= \bar{u}_1[f_1\gamma_\alpha + f_2\sigma_{\alpha\beta}k_\beta - f_3 k_\alpha]\,u_2,$$

$$A_\alpha = \bar{u}_2[g_1\gamma_\alpha + g_2\sigma_{\alpha\beta}k_\beta + g_3 k_\alpha]\,\gamma_5 u_1$$

$$\rightarrow -\tilde{u}_2 C^{+}[g_1\gamma_\alpha + g_2\sigma_{\alpha\beta}k_\beta + g_3 k_\alpha]\,\gamma_5 C\tilde{\bar{u}}_1$$

$$= \bar{u}_1[-g_1\gamma_\alpha + g_2\sigma_{\alpha\beta}k_\beta - g_3 k_\alpha]\,\gamma_5 u_2.$$

We can now deduce how V_α and A_α behave under the transformation of combined inversion CP:

$$V_\alpha = \bar{u}_2[f_1\gamma_\alpha + f_2\sigma_{\alpha\beta}k_\beta + f_3k_\alpha]\,u_1$$
$$\to \begin{cases} \bar{u}_1[f_1\gamma_\alpha + f_2\sigma_{\alpha\beta}k_\beta - f_3k_\alpha]\,u_2, & \text{when} \quad \alpha = 4, \\ -\bar{u}_1[f_1\gamma_\alpha + f_2\sigma_{\alpha\beta}k_\beta - f_3k_\alpha]\,u_2, & \text{when} \quad \alpha = 1,2,3; \end{cases}$$
$$A_\alpha = \bar{u}_2[g_1\gamma_\alpha + g_2\sigma_{\alpha\beta}k_\beta + g_3k_\alpha]\,\gamma_5u_1$$
$$\to \begin{cases} \bar{u}_1[g_1\gamma_\alpha - g_2\sigma_{\alpha\beta}k_\beta + g_3k_\alpha]\,\gamma_5u_2, & \text{when } \alpha = 4, \\ -\bar{u}_1[g_1\gamma_\alpha - g_2\sigma_{\alpha\beta}k_\beta + g_3k_\alpha]\,\gamma_5u_2, & \text{when } \alpha = 1,2,3. \end{cases}$$

Consider, finally, the operation of Hermitian conjugation.

Hermitian conjugation (operation +) is carried out as follows:

$$(\bar{u}_2u_1)^+ = (u_2^+\gamma_4u_1)^+ = u_1^+\gamma_4^+u_2 = \bar{u}_1u_2,$$
$$(\bar{u}_2\gamma_5u_1)^+ = (u_2^+\gamma_4\gamma_5u_1)^+ = \bar{u}_1\gamma_4\gamma_5\gamma_4u_2 = -\bar{u}_1\gamma_5u_2,$$
$$(\bar{u}_2\gamma_\alpha u_1)^+ = \bar{u}_1\gamma_4\gamma_\alpha^+\gamma_4u_2 = \bar{u}_1\gamma_\alpha u_2,$$
$$(\bar{u}_2\gamma_\alpha\gamma_5u_1)^+ = \bar{u}_1\gamma_4\gamma_5\gamma_\alpha^+\gamma_4u_2 = \bar{u}_1\gamma_\alpha\gamma_5u_2,$$
$$(\bar{u}_2\sigma_{\alpha\beta}u_1)^+ = \bar{u}_1\gamma_4\sigma_{\alpha\beta}^+\gamma_4u_2 = -\bar{u}_1\sigma_{\alpha\beta}u_2.$$

It should be noted that in these transformations it is convenient to make use of the relation $\gamma_4\gamma_\alpha^+\gamma_4 = \gamma_\alpha$ or $\gamma_\alpha^+ = \gamma_4\gamma_\alpha\gamma_4$. Taking into account that under Hermitian conjugation the functions f and g go over to their complex-conjugates, while $k \to -k$, we find the Hermitian conjugates of expressions for A_α and V_α:

$$V_\alpha^+ = \{\bar{u}_2[f_1\gamma_\alpha + f_2\sigma_{\alpha\beta}k_\beta + f_3k_\alpha]\,u_1\}^+$$
$$= \bar{u}_1[f_1^*\gamma_\alpha + f_2^*\sigma_{\alpha\beta}k_\beta - f_3^*k_\alpha]\,u_2,$$
$$A_\alpha^+ = \{\bar{u}_2[g_1\gamma_\alpha + g_2\sigma_{\alpha\beta}k_\beta + g_3\,k_\alpha]\,\gamma_5u_1\}^+$$
$$= \bar{u}_1[g_1^*\gamma_\alpha - g_2^*\sigma_{\alpha\beta}k_\beta + g_3^*k_\alpha]\,\gamma_5u_2.$$

In a way analogous to that used above for the expressions V_α and A_α it is easy to establish the behaviour of the leptonic current $l_\alpha = \bar{u}_l\gamma_\alpha(1+\gamma_5)\,u_\nu$ under CP-inversion and Hermitian conjugation. Under CP-inversion

$$\bar{u}_l\gamma_\alpha(1+\gamma_5)\,u_\nu \to \begin{cases} \bar{u}_\nu\gamma_\alpha(1+\gamma_5)\,u_l, & \text{when} \quad \alpha = 4, \\ -\bar{u}_\nu\gamma_\alpha(1+\gamma_5)\,u_l, & \text{when} \quad \alpha = 1,2,3. \end{cases}$$

Under Hermitian conjugation

$$\bar{u}_l\gamma_\alpha(1+\gamma_5)\,u_\nu \to \bar{u}_\nu\gamma_\alpha(1+\gamma_5)\,u_l.$$

We now have all that we need to prove that the functions f and g are real. If there is no interaction between the decay products, then the sum of the amplitude M and its Hermitian conjugate M^+ can be considered as the effective Lagrangian of the decay interaction. By virtue of the weak interaction CP-invariance we have the relation

$$CP(M + M^+) = M + M^+,$$

from which

$$CPM = M^+$$

follows. Comparing CPM and M^+ and making use of the relations obtained above, we find that $f = f^*$, $g = g^*$.

Thus the combined parity conservation leads to the fact that the functions f_1, f_2, f_3 and g_1, g_2, g_3 must be real. In an analogous way it can be proved that in meson decays (decays of the second type) the functions f_1 and f_2 are real.†

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† We assumed above that the spinors u are numbers and hence commute (that they are "c-numbers"). If we assumed them to be anticommuting operators ("q-numbers"), the result would remain the same for the amplitude of the baryon decay, because X_α and l_α would change the sign simultaneously.

CHAPTER 6

STRANGENESS CONSERVING LEPTONIC DECAYS. ANALOGY WITH ELECTRODYNAMICS

CONSERVATION OF THE VECTOR CURRENT

Leptonic decays of strongly interacting particles are divided into two large groups. The first group consists of processes in which the strangeness S of strongly interacting particles does not change during the decay:

$$\Delta S = 0.$$

The second group consists of processes in which S changes:

$$\Delta S \neq 0.$$

Chapters 9, 10, 11 are devoted to processes of the second group. In this section and two subsequent ones we shall consider decays pertaining to the first group: neutron β-decay, decay of the π-meson into $e + \nu$ or into $\mu + \nu$, $\pi^+ \rightarrow \pi^0 + e^+ + \nu$ decay, and some decays of strange particles, for example: $\Sigma^+ \rightarrow \Lambda^0 + e^+ + \nu$, $\Sigma^- \rightarrow \Lambda^0 + e^- + \tilde{\nu}$ or $K^0 \rightarrow K^+ + e^- + \tilde{\nu}$. (The probability of the last decay is very low because of the smallness of the phase space.) Besides these decays there are possible decays in which photons take part along with leptons, but the strangeness remains unchanged, for example the radiative muon decay $\pi^+ \rightarrow e^+ + \nu + \gamma$, but we shall not consider such processes.

All decays in which the strangeness of strongly interacting particles does not change are due to a primary interaction which transforms the proton into the neutron, and the electron into the neutrino. The current that transforms the proton into the neutron—nucleonic current—consists of two terms: a vector and an axial term. In this section we shall consider in detail the properties of the nucleonic vector current. It turns out that there is much in common between this current and the electromagnetic one. The analogy between these currents was first pointed out by Gershtein and

Zel'dovich in 1955, when it was generally believed that β-decay is due not to the vector (V) and axial (A) variants of thc interaction but to the scalar (S) and tensor (T). Gershtein and Zel'dovich pointed out that, if the weak interaction were a vector interaction and if the corresponding current of strongly interacting particles were conserved, then the constant of this interaction would not change under the action of virtual strong interactions, as the proton electric charge does not change.

In the ordinary Yukawa scheme, in which π-mesons as well as nucleons are elementary particles and are involved in the strong interaction Lagrangian, the conserved electric current must contain a π-mesonic term. In the same way, according to Gershtein and Zel'dovich, a π-mesonic term must be also involved in the conserved vector current of weak interaction, which makes it possible to predict the constant in the π-meson β-decay.

In 1957, Feynman and Gell-Mann put forward the hypothesis of the conserved weak interaction vector current in connection with the theory of the universal V–A interaction, and showed that this hypothesis provides a simple and obvious explanation of the equality of the β-decay vector constant and muon decay constant.

As will be seen in what follows, the analogy between the strangeness conserving vector current and the electromagnetic current arises most naturally in the Sakata model. We shall begin the discussion of this analogy with a more detailed consideration of the electromagnetic properties of strongly interacting particles.

ELECTROMAGNETIC PROPERTIES OF THE PROTON

In the composite model the electromagnetic interaction of strongly interacting particles reduces to the interaction of electromagnetic field with the proton charge, because n and Λ have no electric charge. The charges of all other particles: π- and K-mesons, Σ- and Ξ-hyperons, reduce to the charge of the proton or antiproton composing them. Magnetic moments of particles arise as a secondary phenomenon; for example, the neutron magnetic moment is due to the fact that the neutron is "dressed" by a cloud of virtual proton–antiproton pairs. Only the normal magnetic moment of the proton is a primary one (see below).

Interaction of a "bare" proton with an electromagnetic field is of the form $e\bar{u}_p\gamma_\alpha u_p A_\alpha$, where A_α is the 4-potential of the electromagnetic field. Now we switch on the strong interaction and ask the question: what form will the matrix element have for the interaction of a photon and a proton "dressed" by virtual strongly interacting particles?

Our reasoning will be the same as used in explaining the general form of the β-decay matrix element. We have to construct a 4-vector of the spinors $\bar{u}_2$, u_1 and of the photon momentum q. In a preceding section we have seen that such a 4-vector consists of three terms:

$$Z_\alpha^p = e\bar{u}_2 [C_p(q^2)\gamma_\alpha - M_p(q^2) q_\beta \sigma_{\alpha\beta} - N_p(q^2) q_\alpha] u_1,$$

where $2\sigma_{\alpha\beta} = \gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha$, while C, N, M are certain real functions of q^2.

It is easy to show that $N(q^2) \equiv 0$. Indeed, under charge conjugation the term proportional to N changes sign, whereas the terms proportional to C and M do not. Consequently, the third term has a charge parity differing from that of the first and second term. Hence, the third term must be equal to zero. Moreover, there is another reason for $N(q^2)$ being equal to zero. It is known that the electric current is a conserved quantity. For this reason the product $q_\alpha Z_\alpha$ must be equal to zero. Multiply Z_α by q_α:

$$q_\alpha Z_\alpha = \bar{u}_2 [C\gamma_\alpha q_\alpha - M q_\beta \sigma_{\alpha\beta} q_\alpha - N q_\alpha q_\alpha] u_1 = \bar{u}_2 [C\hat{q} - Nq^2] u_1,$$

since $q_\beta q_\alpha \sigma_{\alpha\beta} = \frac{1}{2} q_\alpha q_\beta (\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha) = 0$. Recalling that $\hat{q} = \hat{p}_2 - \hat{p}_1$, $\bar{u}_2 \hat{p}_2 = m\bar{u}_2$, $\hat{p}_1 u_1 = m u_1$, we obtain $\bar{u}_2 \hat{q} u_1 = 0$. Consequently, $q_\alpha Z_\alpha = Nq^2 \bar{u}_2 u_1$, but because $q_\alpha Z_\alpha = 0$, $N(q^2)$ is also equal to zero. Thus, in the expression for Z_α^p there remain two terms:

$$Z_\alpha^p = e\bar{u}_2 [C_p(q^2)\gamma_\alpha - M_p(q^2) q_\beta \sigma_{\alpha\beta}] u_1.$$

The quantities C_p and M_p are called respectively the proton charge form factor and the proton magnetic form factor. When $q^2 \to 0$ the quantity $C_p(q^2)$ tends to unity, $C_p(0) = 1$, since the proton charge is equal to unity. The quantity $eM_p(q^2)$ when $q^2 \to 0$ must tend to the value of the proton anomalous magnetic moment: $eM_p(0) = 1{\cdot}7\,\mu_n$, where μ_n is the nuclear magneton.

The term "charge form factor" is relative. The point is that in the non-relativistic approximation the term $e\bar{u}\gamma_\alpha C_p(q^2)\, u \approx e\bar{u}\gamma_\alpha u$ contains not only the charge but also the normal magnetic moment of the proton. Consider a proton at rest, absorbing a photon with a small momentum q:

$$u_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\varphi_1, \qquad \bar{u}_2 = \varphi_2^*\left(1, \ -\frac{\sigma q}{2m}\right).$$

Here u_1 and $\bar{u}_2$ are respectively the spinor of the initial and final state of the proton, while φ_1 and φ_2^* are two-component spinors. We assume that $|q| \ll m$ and hence replace $E + m$ by $2m$ in the expression for $\bar{u}_2$. Now consider the term with $\alpha = 4$ in the expression $e\bar{u}_2\gamma_\alpha u_1 A_\alpha$:

$$e\varphi_2^*\left(1, \ -\frac{\sigma q}{2m}\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\varphi_1 A_4 = e\varphi_2^*\varphi_1 A_4.$$

It is seen that this term gives simply the interaction of the electrostatic potential A_4 with the charge e.

But for $\alpha = 1, 2, 3$:

$$-e\varphi_2^*\left(1, \ -\frac{\sigma q}{2m}\right)\begin{pmatrix} 0 & \sigma A \\ -\sigma A & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\varphi_1 = -e\varphi_2^*\frac{\sigma q \sigma A}{2m}\varphi_1.$$

It is easy to see that this expression is in fact the one for the interaction of the magnetic moment with magnetic field. Indeed,

$$\begin{aligned} -\frac{e}{2m}\varphi_2^*\sigma q \sigma A \varphi_1 &= -\frac{e}{2m} q_i A_k \varphi_2^* \sigma_i \sigma_k \varphi_1 \\ &= -\frac{e}{2m} q_i A_k \varphi_2^* (\delta_{ik} + i\varepsilon_{ikl}\sigma_l)\varphi_1 \\ &= -\frac{e}{2m}\varphi_2^*(i\varepsilon_{ikl} q_i A_k \sigma_l)\varphi_1 = \frac{e}{2m}\varphi_2^*\sigma\varphi_1\mathscr{H}. \end{aligned}$$

(In this derivation we made use of the Lorentz condition $q_\alpha A_\alpha = 0 = q_4 A_4 - qA$; since we assumed $q_4 = 0$, it follows that $qA = 0$. Moreover, we made use of the definition of magnetic field $\mathscr{H}_l = -i\varepsilon_{ikl} q_i A_k$.) The same result can be obtained in a different way. Denote the proton 4-momenta before and after the interaction by p_1 and p_2; $q = p_2 - p_1$, $p = p_1 + p_2$. Taking into account that $2p_\alpha = \gamma_\alpha \hat{p} + \hat{p}\gamma_\alpha$, we get

$$\begin{aligned} 2\bar{u}_2 p_\alpha u_1 &= \bar{u}_2(\gamma_\alpha \hat{p} + \hat{p}\gamma_\alpha)u_1 \\ &= \bar{u}_2[\gamma_\alpha(2\hat{p}_1 + \hat{q}) + (2\hat{p}_2 - \hat{q})\gamma_\alpha]u_1 \\ &= 4m\bar{u}_2\gamma_\alpha u_1 + \bar{u}_2(\gamma_\alpha\hat{q} - \hat{q}\gamma_\alpha)u_1. \end{aligned}$$

Whence it follows that

$$e\bar{u}_2\gamma_\alpha u_1 A_\alpha = e\bar{u}_2\frac{p_\alpha}{2m}u_1 A_\alpha - e\bar{u}_2\frac{\sigma_{\alpha\beta}q_\beta}{2m}u_1 A_\alpha.$$

When $q \to 0$ the first term gives the electrostatic interaction, because $\frac{p_4}{2m} \to 1$, $\frac{p}{2m} \to 0$; the second term represents the interaction of the magnetic moment $\frac{e}{2m}$ with the magnetic field (this is seen from comparison with the term proportional to $M(q^2)$ in the expression for Z^p). When $q^2 = 0$ we obtain the static limit of the form factors. At $q^2 < 0$ a virtual photon acts as if it penetrates the proton and "sees" the distribution of charges and virtual particle currents. The most detailed data on the dependence of the quantities C_p and M_p on q^2 have been obtained in experiments on electron–proton scattering, carried out by the Hofstadter group.

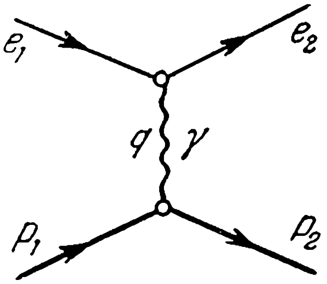

FIG. 10

In these experiments protons were "scanned" by virtual photons the source of which were electrons (see Fig. 10). At not too large values of q^2 the magnetic and charge form factors can be presented in the form

$$C_p(q^2) = 1 + \frac{q^2 r_c^2}{6},$$

$$M_p(q^2) = M_p(0)\left(1 + \frac{q^2 r_m^2}{6}\right).$$

The quantities r_c and r_m are called the electric and magnetic radii of the proton. Experiments revealed that $r_m \approx r_c \approx \frac{1}{2m_\pi}$, where m_π is the mass of the π-meson. This result is quite natural: electromagnetic radii are determined by the effective size of the pion cloud surrounding the proton. It is seen that at $q^2 = -m_\pi^2$ the effective proton charge decreases by about 4 per cent; the same holds for its magnetic moment.

ELECTROMAGNETIC PROPERTIES OF THE NEUTRON

One can describe in an analogous way the electromagnetic vertex of the neutron:

$$Z_\alpha^n = e\bar{u}_2\left[C_n(q^2)\gamma_\alpha - M_n(q^2)\sigma_{\alpha\beta}q_\beta\right]u_1.$$

Since the neutron charge equals zero, $C_n^{(0)} = 0$. The quantity M_n is expressed by means of the neutron anomalous magnetic moment: $eM_n(0) = -1{\cdot}9\mu_n$.

We now study the interaction of nucleons with photons from the standpoint of isotopic spin.

ISOTOPIC SPIN

It is known that the strong interaction is isotopically invariant. (In the Sakata model this invariance of strong interaction reduces to the fact that the proton and neutron enter the strong interaction current in a symmetric way:

$$\bar{\psi}_p\psi_p + \bar{\psi}_n\psi_n,$$

the masses of these particles being the same; see pp. 5 and 11.) It is obvious that within the framework of the isotopically invariant strong interaction it is impossible to distinguish between the proton and neutron. This reminds us of the degeneracy of states of a particle having a spin and a magnetic moment, so long as the magnetic field is switched off. (In this analogy one finds the reason for describing the isotopic spin by means of the mathematics of ordinary spin.) The proton and neutron are described by a certain function ψ which has two components, a proton and a neutron component, just as a particle with the spin $\frac{1}{2}$ has two components $+\frac{1}{2}$ and $-\frac{1}{2}$ along z-axis. If there is no interaction that can distinguish between these two states, the system is degenerate. If the electromagnetic interaction, which is different for proton and neutron, is "switched on", it removes the isotopic degeneracy, as the switching on of the magnetic field removes the spin degeneracy for a particle with spin $\frac{1}{2}$. The introduction of the two-component isotopic spinor ψ allows one to make use of all the mathematical methods for non-relativistic spin. In particular, τ-matrices, which operate on the spinor ψ, are the same as the Pauli σ-matrices:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Sometimes it is more convenient to employ not the matrices τ_1 and τ_2 but the following matrices:

$$\tau^{+} = \tfrac{1}{2}(\tau_1 \pm i\tau_2),$$

$$\tau^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau^{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

What is the advantage of the matrices τ^+ and τ^-? Consider the action of τ^+ and τ^- on ψ:

$$\tau^+\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}, \qquad \tau^+\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix},$$

$$\tau^+ n = p, \qquad \tau^+ p = 0,$$

$$\tau^-\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}, \qquad \tau^-\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix},$$

$$\tau^- n = 0, \qquad \tau^- p = n.$$

It is seen that the matrix τ^+ transforms the neutron into the proton, while τ^- transforms the proton into the neutron.

ISOTOPIC FORM FACTORS

By means of the matrices τ and spinors ψ we can write the interaction of the photon with the proton in the form

$$e\bar{\psi}_p\gamma_\alpha\psi_p A_\alpha = e\bar{\psi}\gamma_\alpha\frac{1+\tau_3}{2}\psi A_\alpha .$$

It is seen that the electromagnetic current consists of two terms:

$\bar{\psi}\gamma_\alpha\psi$ (isotopic scalar S), and

$\bar{\psi}\gamma_\alpha\tau_3\psi$ (isotopic vector V).

(It should be noted that τ and γ_α operate in different spaces and hence can be interchanged.)

When $q^2 = 0$ we can write

$$C_p(0) = \frac{C_S(0)+C_V(0)}{2} = 1 \quad \text{and} \quad C_n(0) = \frac{C_S(0)-C_V(0)}{2} = 0.$$

Whence $C_S(0) = C_V(0) = 1$.

Analogous relations can be written also when $q^2 \neq 0$:

$$C_p(q^2) = \frac{C_S(q^2)+C_V(q^2)}{2}, \qquad C_n(q^2) = \frac{C_S(q^2)-C_V(q^2)}{2}.$$

The same relations are valid for magnetic form factors:

$$M_p(q^2) = \frac{M_S(q^2)+M_V(q^2)}{2}, \qquad M_n(q^2) = \frac{M_S(q^2)-M_V(q^2)}{2}.$$

In interpreting the experimental results it is more convenient to make use of isotopic vector and scalar form factors and not of proton and neutron form factors.

Now that we have the necessary mathematical framework we can proceed to the consideration of a very profound analogy existing in the Sakata model—the analogy between electrodynamics and the strangeness conserving vector weak interaction.

RELATIONSHIP BETWEEN WEAK AND ELECTROMAGNETIC FORM FACTORS

The isovector part of the electromagnetic current is of the form $\bar{\psi}\gamma_\alpha\tau_3\psi$.

The vector part of the weak current with $\Delta S = 0$ is of the form

$$j_{n_\alpha}^{(V)} = \bar{\psi}_p\gamma_\alpha\psi_n = \bar{\psi}\gamma_{\alpha\tau^+\psi}.$$

The analogy between these two currents is striking: the isovector part of the electromagnetic current and the weak nucleonic vector current represent the components of the same isotopic vector. This allows all strangeness conserving interactions involving the vector current, i.e. all interactions due to the current $j_n^{(V)}$, to be put in correspondance with electrodynamic interactions. The isotopic invariance can be stated in an obvious way as the isotropy of isotopic space. In isotopic space different projections of the same vector are equivalent. Rotating isotopic coordinates we can obtain τ_1 or τ_2 instead of τ_3. Since the strong interaction is isotopically invariant, while the currents $\bar{\psi}\gamma_\alpha\tau_3\psi$ and $\bar{\psi}\gamma_\alpha\tau^+\psi$ are the components of the same isotopic vector, the matrix elements of these currents, resulting from virtual strong interactions, will be the same (to within immaterial numerical factors).

MATRIX ELEMENT OF THE $n \leftrightarrow p$ TRANSITION

As we shall see in Chapter 7, the vector amplitude of the neutron β-decay consists of three terms:

$$M = \frac{G}{\sqrt{2}}\bar{u}_p[f_1(q^2)\gamma_\alpha - f_2(q^2)\sigma_{\alpha\beta}q_\beta - f_3(q^2)q_\beta]u_n l_\alpha.$$

(The sign minus of f_2 and f_3 is connected with the fact that $q = -k$; see pp. 73 and 80.) Analogous amplitudes describe the muon capture by proton $\mu^- + p \to n + \nu$ and processes of the type

$$\nu + n \to e + p,$$

$$\nu + n \to \mu + p,$$

$$e^- + p \to n + \nu.$$

By virtue of the analogy with electrodynamics the matrix element of the weak transition $n \leftrightarrow p$,

$$f_1(q^2)\gamma_\alpha - f_2(q^2)\sigma_{\alpha\beta}q_\beta - f_3(q^2)\, q_\alpha,$$

must be equal to the nucleon isovector electromagnetic form factor

$$C_V(q^2)\gamma_\alpha - M_V\sigma_{\alpha\beta}q_\beta.$$

This can be put, symbolically, in the form of the equation (Fig. 11):

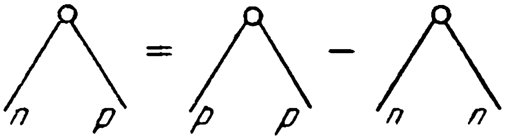

FIG. 11

Comparing these expressions it is easy to obtain expressions for f_1, f_2 and f_3:

$$f_1(q^2) = C_V(q^2),$$
$$f_2(q^2) = M_V(q^2),$$
$$f_3(q^2) = 0.$$

Verification of these relations is of great interest.

We consider them in more detail.

VECTOR COUPLING CONSTANT

The equality $f_1(q^2) = C_V(q^2)$ leads to a number of remarkable consequences, one of the most important being

$$f_1(0) = C_V(0) = 1.$$

We have already mentioned that the equalities $C_V(0) = 1$ and $C_S(0) = 1$ express the fact that the electric charges of nucleons are equal to their "bare" charges. Compare the proton and positron. In the case of the positron the electric charge is a "bare" point charge, whereas in the case of proton the electric charge is "dressed", "diffuse", because of strong interactions. None the less the charges of the proton and positron are exactly the same. This is a consequence of the conservation of the electromagnetic current.

Returning to weak interactions we compare the muon decay $\mu \rightarrow e + \nu + \bar{\nu}$ with the neutron β-decay. Muon decay is due only to weak interaction. Hence this decay is characterized by the "bare" weak interaction constant G. In the neutron decay the term γ_α is multiplied not by G but by Gf_1, where f_1 takes account of the contribution of virtual strong interactions. But the analogy with electrodynamics allows one to state that $f_1(0)$ must be equal to unity: the weak vector "charge" of the nucleon must be equal to its "bare charge". Thus, the vector constant of β-decay does not change or, as is sometimes said, is not renormalized.

Experimentally the muon decay constant and the vector constant of the β-decay agree to within a few per cent. It should be noted that this agreement should not necessarily be absolute, since virtual photons violate the isotopic invariance of the strong interaction. The analogy between the weak vector current and the electromagnetic current is valid to within electromagnetic corrections.

In the preceding text we have assumed that it is $f_1(0)$ that enters the matrix element of the neutron β-decay. This approximation appears to be very good, because the energy released in the neutron β-decay ($\sim$ 1 MeV) is substantially lower than that at which the nucleon structure manifests itself. Similar to $C_p(q^2)$ the quantity $C_V(q^2)$ and, consequently, also $f_1(q^2)$ are characterized by the radius r_V of the order of $1/2m_\pi$, where m_π is the pion mass:

$$C_V(q^2) = 1 + \frac{q^2 r_V^2}{6}.$$

Therefore even in the low-energy muon capture by the proton, where $|\boldsymbol{q}| \sim 100$ MeV/c, the value of f_1 must be only 2 per cent lower than unity.

The dependence of the form factor $f(q^2)$ on q^2 can be studied in reactions of the type $\tilde{\nu} + p \to \mu^+ + n$ and others involving high-energy neutrinos.

"WEAK MAGNETISM"

We pass on to the equation

$$f_2(q^2) = M_V(q^2).$$

When $q^2 = 0$ it follows that

$$f_2(0) = \frac{3{\cdot}6\mu_n}{e} = \frac{1{\cdot}8}{m_p}.$$

Because in the matrix element of the $n \leftrightarrow p$ transition $f_2(0)$ is multiplied by q, in the neutron β-decay this term is not large. It must be substantially larger in the muon capture. Since $M_V \sigma_{\alpha\beta} q_\beta$ represents the anomalous magnetic moment, the term $f_2 \sigma_{\alpha\beta} q_\beta$ is called "weak magnetism". Extending the analogy between the weak and electromagnetic magnetism, one can interrelate certain types of β and electromagnetic transitions in nuclei (for example, in the triplet boron–carbon–nitrogen, as was done by Gell-Mann).

"EFFECTIVE SCALAR"

Test of the equality $f_3 = 0$ resulting from the analogy with electrodynamics is difficult in processes involving electrons, because in these processes the contribution of the term $f_3 q_\alpha$, even if it were not equal to zero, would all the same be very small. Indeed, it can easily be seen that the contribution of this term is proportional to the lepton mass multiplied by the matrix element of the "effective scalar" interaction:

$$-\bar{u}_p f_3 q_\alpha u_n \bar{u}_l \gamma_\alpha (1 + \gamma_5) u_\nu = f_3 m_l \bar{u}_p u_n \bar{u}_l (1 + \gamma_5) u_\nu .$$

In μ-capture this term would be larger.

We shall end this discussion of the vector part of the nucleon matrix element and consider matrix elements of other particles and, in particular, π-mesons, resulting from the vector current.

INTERACTION OF THE π^+-MESON WITH THE PHOTON

We know that all electromagnetic interactions of strongly interacting particles in the composite model are due to the interaction of the proton with the electromagnetic field. We also know that, in spite of the presence of the strong interaction between the proton and antineutron in the π^+-meson, its charge is exactly equal to that of the proton. The charges of the different particles are the same. This is a remarkable fact, resulting from the electromagnetic current conservation. We can make a direct use of it and apply it at once to weak interactions.

The interaction of the π^+-meson with the electromagnetic field can be written in the following way:

$$A_\alpha Z_\alpha^\pi = e A_\alpha \varphi^+ \varphi (k_{2\alpha} + k_{1\alpha}),$$

where A_α is the 4-potential, k_1 and k_2 are the 4-momenta of the π^+-meson before and after the interaction, φ^+, φ are the wave functions of the meson, while e is the electric charge.

Why does the quantity Z_α^π have such a form? The spin of the π-meson is equal to zero. Hence the 4-vector Z_α^π must be proportional to a linear combination of k_2 and k_1:

$$Z_\alpha^\pi = e\varphi^+\varphi[a(k_{1\alpha} + k_{2\alpha}) + b(k_{1\alpha} - k_{2\alpha})],$$

where a and b are scalar functions. Since the electromagnetic current is conserved, we have

$$(k_{1\alpha} - k_{2\alpha})\, Z_\alpha^\pi = 0.$$

Whence we find

$$(k_1^2 - k_2^2)\, a + (k_1 - k_2)^2\, b = 0;$$

since $k_1^2 = k_2^2 = m_\pi^2$, $b = 0$. Consequently,

$$Z_\alpha^\pi = ea\varphi^+\varphi(k_{1\alpha} + k_{2\alpha}).$$

If the normalization of the wave functions φ is chosen to be $1/\sqrt{(2k_0)}$, where k_0 is the energy of the meson, then for a meson at rest and a static field A_4 we obtain

$$Z_4^\pi = ea(0)\frac{2m_\pi}{2m_\pi} = e,$$

whence it follows that $a(0) = 1$. The quantity a is a function of q^2, where $q = k_2 - k_1$. The function $a(q^2)$ can be investigated, for example, in pion-electron scattering.

$\pi^+ \rightarrow \pi^0 + e^+ + \nu$ DECAY

Making use of the analogy with electrodynamics, one can find the form that the matrix element for the $\pi^+ \rightarrow \pi^0 + e^+ + \nu$ decay must have. In fact it is readily seen that this matrix element must be

$$G\varphi_2^+\varphi_1(k_{1\alpha} + k_{2\alpha})\,\bar{u}_e\gamma_\alpha(1 + \gamma_5)\,u_\nu.$$

We have assumed that the form factor $a(q^2)$ is unity, because the energy released in this decay is small ($\sim 4{\cdot}6$ MeV) in comparison with the π-meson mass.

We shall now explain why, in the expression for the matrix element, there is a coefficient G instead of $(1/\sqrt{2})\,G$ as in nucleon β-decay. For this purpose we write the electric current of nucleons and π-mesons:

$$e\bar{\psi}\frac{1 + \tau_3}{2}\psi + e\varphi^+T_3\varphi.$$

(We have shown only the isotopic structure of the interaction.)

The π-meson is a triplet in isotopic space; hence, as distinct from nucleon, it will be represented by a column vector containing three components:

$$\pi^+ = \begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad \pi^0 = \begin{pmatrix}0\\1\\0\end{pmatrix}, \qquad \pi^- = \begin{pmatrix}0\\0\\1\end{pmatrix}.$$

This isotopic vector is acted upon by the 3×3-matrices $\boldsymbol{T}$, as the two-component spinor ψ is acted upon by τ-matrices:

$$T_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}, \qquad T_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix},$$

$$T_3 = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}.$$

The matrices $\boldsymbol{T}$ satisfy the relation $\boldsymbol{T} \times \boldsymbol{T} = i\boldsymbol{T}$. The matrix T_3 is the matrix of the electric charge. The interaction with the electromagnetic field is directly proportional to T_3.

Consider now the vector part of the electromagnetic current of nucleons and π-mesons:

$$e\left(\bar{\psi}\frac{\tau_3}{2}\psi + \varphi^+ T_3 \varphi\right).$$

Analogously one can write

$$\bar{\psi}\frac{\tau_1}{2}\psi + \varphi^+ T_1 \varphi$$

and

$$\bar{\psi}\frac{\tau_2}{2}\psi + \varphi^+ T_2 \varphi.$$

Multiplying this last expression by $+i$ and combining it with the preceding one, we obtain

$$\bar{\psi}\frac{\tau_1 + i\tau_2}{2}\psi + \varphi^+(T_1 + iT_2)\varphi = \bar{\psi}\tau^+\psi + \varphi^+ T^+ \varphi$$

$$= \bar{p}n + \sqrt{2}\varphi^+ + \varphi_0 + \sqrt{2}\varphi_0^+\varphi_-.$$

Recall that

$$\tau^+ = \frac{\tau_1 + i\tau_2}{2} = \begin{pmatrix}0&1\\0&0\end{pmatrix}, \qquad T^+ = T_1 + iT_2 = \sqrt{2}\begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix},$$

$$\tau^- = \frac{\tau_1 - i\tau_2}{2} = \begin{pmatrix}0&0\\1&0\end{pmatrix}, \qquad T^- = T_1 = iT_2 = \sqrt{2}\begin{pmatrix}0&0&0\\1&0&0\\0&1&0\end{pmatrix}.$$

Thus, the pion matrix element contains the additional coefficient $\sqrt{2}$ as compared with the nucleon matrix element. The same result is easy to obtain in an elementary way, if it is recalled that $\pi^+ = p\bar{n}$, while $\pi^0 = (1/\sqrt{2})(p\bar{p} - n\bar{n})$. The $\pi^+ \to \pi^0 + e^+ + \nu$ decay takes place on account of two processes: the proton decay $p \to n + e^+ + \nu$ and the antineutron decay $\bar{n} \to \bar{p} + e^+ + \nu$. The amplitude of each of these processes is proportional to $G/\sqrt{2}$; since they have different signs, and the $\pi^+ \to \pi^0$ transition involves the difference between them, the total amplitude of the $\pi^+ \to \pi^0$ transition is proportional to $2G/(\sqrt{2}\sqrt{2}) = G$. Thus, the amplitude of the $\pi^+ \to \pi^0 + e^+ + \nu$ decay has the form

$$G\varphi_0^+\varphi_+(k_\alpha^+ + k_\alpha^0)\,\bar{u}_\nu\gamma_\alpha(1 + \gamma_5)\,u_e.$$

Knowing the amplitude one can calculate all characteristics of the $\pi^+ \to \pi^0 + e^+ + \nu$ decay and, in particular, its decay rate (see Chapter 8).

ANALOGY WITH ELECTRODYNAMICS — A CONSEQUENCE OF THE MINIMAL MODEL

The analogy with electrodynamics has allowed us to derive a number of conclusions. The main one is that the vector constant of the neutron decay is equal to the "bare" weak interaction constant. This conclusion of the non-renormalizability of the vector constant of the strangeness conserving weak interaction, as well as the other ones considered in this and in the preceding chapter, is a consequence of the Sakata model, and is associated with the fact that the nucleonic vector current in the Sakata model is automatically conserved.†

In other theories, in which there is not just one pair of elementary particles of the same strangeness but several multiplets of elementary particles, the conserved current (if we wish to have such a current) has to be made "artificially". Indeed, let π-mesons, K-mesons, Σ-hyperons, Ξ-hyperons be assumed to be elementary and to possess a primary weak interaction with leptons. Consider the weak interaction of nucleons with leptons. Since the proton spends part of its time in the form of $n + \pi^+$ or $\Lambda + K^+$, or $\Sigma^0 + K^+$ and so on, for each of these particles the constants of the primary weak interaction must be chosen in such a way that the contribution of all these virtual particles does not change the nucleonic vector constant.

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† It is easy to convince oneself of the conservation of the nucleonic vector current in the Sakata model, if its divergence $\dfrac{\partial}{\partial x_\alpha}(\bar{p}\gamma_\alpha n)$ is calculated, assuming the Lagrangian of the isotopically invariant strong interaction to be four-fermion or to result from neutral intermediate meson exchange, as is the case, for example, in the vecton model (see p. 31).

It is seen that in a theory in which all elementary particles are actually elementary, the conservation of the vector current is not only not forced but appears to be an improbable accident. In the Sakata model (and, in a more general way, in the universal model) this conservation is unavoidable. Therefore, if experiment should contradict even one of the consequences of current conservation, it would mean that the minimal model is not valid and that all strongly interacting particles cannot be described as the eigenstates of a system of three interacting baryon fields.

CHAPTER 7

STRANGENESS CONSERVING LEPTONIC DECAYS. ISOTOPIC PROPERTIES OF THE NUCLEONIC CURRENT

NUCLEONIC CURRENT — ISOVECTOR

The analogy between the strangeness conserving vector current and the electric current, considered in the preceding chapter, is based on two properties of the vector current: first, it is conserved, second, it is an isotopic vector. The strangeness conserving axial current is not conserved and it is not similar to the electric current which has the form $\bar{\psi}\gamma_\alpha\psi$ and not $\bar{\psi}\gamma_\alpha\gamma_5\psi$. However, the axial current, like the vector current, is an isotopic vector: it is produced in the Sakata model by $p \leftrightarrow n$ transition. This isotopic property allows one to deduce certain selection rules, the so-called selection rules of isotopic parity.

ISOTOPIC ROTATION AND CHARGE CONJUGATION

In order to explain the isotopic parity we have to return again to strong interactions and consider in more detail the two operations: charge conjugation and rotation in isotopic spin space.

We have already mentioned that the strong interaction conserves charge parity and that it is isotopically invariant. When a system obeys two conservation laws at the same time, then there may arise additional selection rules, which would not result from either of these laws alone.

T_2-ROTATION AND NUCLEONS

Consider the rotation in isotopic spin space by 180° about the second axis (see Fig. 12). We shall denote this operation by T_2. We study the behaviour of isotopic spinors under this operation.

Pick a nucleon isospinor ψ:

$$\psi = \begin{pmatrix}1\\0\end{pmatrix} \text{ for proton}$$

and

$$\psi = \begin{pmatrix}0\\1\end{pmatrix} \text{ for neutron.}$$

Under T_2-rotation

$$\psi \rightarrow \psi' = i\tau_2\psi,$$
$$\bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi}(-i\tau_2).$$

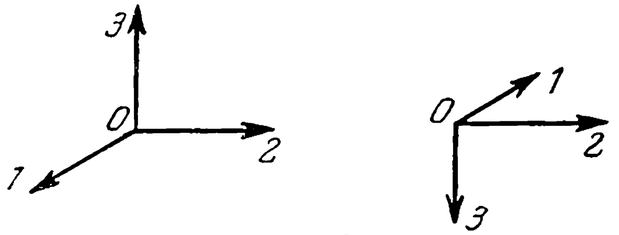

Fig. 12

If we note that

$$\tau_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \text{while} \quad \tau_2^2 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},$$

then it is easily seen that the isotopic scalar $\bar{\psi}\psi$ under T_2-rotation will go into itself.

Furthermore

$$i\tau_2\begin{pmatrix}1\\0\end{pmatrix} = -\begin{pmatrix}0\\1\end{pmatrix}, \qquad i\tau_2\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix},$$

$$(1, 0)(-i\tau_2) = -(0, 1), \quad (0, 1)(-i\tau_2) = (1, 0).$$

Thus, under the operation T_2

$$p \rightarrow -n, \qquad n \rightarrow p,$$
$$\bar{p} \rightarrow -\bar{n}, \qquad \bar{n} \rightarrow \bar{p}.$$

T_2-ROTATION AND π-MESONS

How do the π-mesons behave under T_2-rotation? We recall that

$$\pi^+ = \bar{n}p, \quad \pi^0 = \frac{\bar{p}p - \bar{n}n}{\sqrt{2}}, \quad \pi^- = \bar{p}n.$$

Here, and in what follows, to avoid introduction of additional notation ($\tilde{p}$ and $\tilde{n}$), we denote the antiproton by $\bar{p}$ and the proton by p (similarly for the neutron). Since the symbol $\bar{p}$ stood for proton creation and antiproton annihilation, it is as if we had written the wave functions of the annihilating particles.

Knowing how nucleonic wave functions behave under T_2-rotation, we obtain

$$\pi^+ = p\bar{n} \rightarrow -n\bar{p} = -\pi^-,$$
$$\pi^- = n\bar{p} \rightarrow -p\bar{n} = -\pi^+,$$
$$\pi^0 = \frac{\bar{p}p - \bar{n}n}{\sqrt{2}} \rightarrow -\frac{\bar{p}p - \bar{n}n}{\sqrt{2}} = -\pi^0.$$

It is seen that under T_2-rotation the wave function of the π^0-meson goes over into itself with a minus sign. This means that the π^0-meson has a definite (odd) parity under T_2-transformation.

The π^+- and π^--mesons, under T_2-rotation, do not go over into themselves but into each other. Consequently, they do not have any definite T_2-parity.

G-TRANSFORMATION

We now add to the operation T_2 the charge conjugation operation C, under which

$$\pi^+ \rightarrow \pi^-, \qquad \pi^- \rightarrow \pi^+, \qquad \pi^0 \rightarrow \pi^0.$$

It is easy to see that under the simultaneous action of T_2 and C

$$\pi^+ \rightarrow -\pi^+, \qquad \pi^- \rightarrow -\pi^-, \qquad \pi^0 \rightarrow -\pi^0.$$

The product of the operations C and T_2 is denoted by the letter G:

$$G = CT_2 = T_2C.$$

It is seen that all three π-mesons have odd G-parity. In the literature, the G-parity is sometimes called isotopic parity.

LEE–YANG THEOREM

The fact that the pion wave function has a definite isotopic parity allows one to establish selection rules using this quantum number for systems of π-mesons.

Indeed, since strong interaction conserves both charge parity and isotopic spin, it also conserves G-parity. Therefore the G-parity of a system of strongly interacting particles cannot change. Whence it follows that an odd number of π-mesons can only go over into an odd one, while an even number of π-mesons can go over only into an even one. In other words, a diagram containing only π-meson external lines is different from zero only in the case when the number of these lines is even. This theorem, formulated by Lee and Yang, reminds one of the Furry theorem for photons.

The Lee–Yang theorem is valid not only when all external meson lines correspond to real mesons but also when some of them (or all) correspond to virtual mesons. For example, by virtue of this theorem the following diagram (Fig. 13) must be equal to zero, because its upper vertex contains an odd number of π-meson "tails".

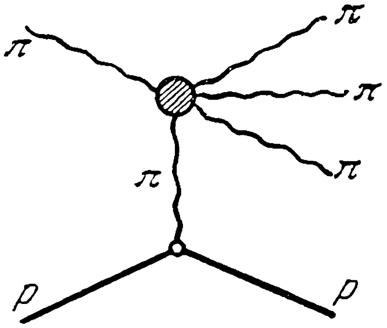

FIG. 13

If a diagram contains, in addition to π-meson external lines, external lines of other particles, for example those of nucleons, then the rule is, in general, inapplicable.

G-PARITY OF THE NUCLEON?

This is connected with the fact that such particles as $K, N, \Lambda, \Sigma, \Xi$ have no definite isotopic parity.

Under G-transformation

$$
\begin{array}{ll}
p \to -\bar{n}, & n \to \bar{p}, \\
\bar{p} \to -n, & \bar{n} \to p, \\
\Lambda \to \bar{\Lambda}, & \Sigma^+ \to -\bar{\Sigma}^-.
\end{array}
$$

It is easy to see that, in order for a particle to have a definite G-parity, its baryon charge and strangeness must be equal to zero, because charge conjugation changes the sign of these quantum numbers whereas T_2-rotation does not.

G-PARITY OF THE NUCLEON–ANTINUCLEON SYSTEM

Although nucleon and antinucleon have no definite G-parity, the system of a nucleon and an antinucleon has a definite G-parity which is determined by the values of the spin s, orbital momentum l

and isotopic spin T of the system (recall that π-meson is a bound state of the nucleon–antinucleon system with $s = 0$, $l = 0$, $T = 1$).

In Chapter 3 we have seen that the charge parity of the proton–antiproton system is equal to

$$C = (-1)^{l+s}.$$

This is also true of the charge parity of the system of a neutron and an antineutron. We now consider the isotopic states of the system of a nucleon and an antinucleon.

As a result of strong interaction the systems $\bar{p}p$ and $\bar{n}n$ may transform into each other. But there are linear combinations of the states $\bar{p}p$ and $\bar{n}n$, which are conserved by strong interaction:

$$\frac{1}{\sqrt{2}}(\bar{p}p + \bar{n}n) = \frac{1}{\sqrt{2}}\bar{\psi}\psi, \qquad T = 0, \qquad T_3 = 0,$$

$$\frac{1}{\sqrt{2}}(\bar{p}p - \bar{n}n) = \frac{1}{\sqrt{2}}\bar{\psi}\tau_3\psi, \quad T = 1, \qquad T_3 = 0.$$

The first state is an isotopic scalar (singlet). The second one is a component of an isotopic vector, i.e. of an isotopic triplet. The remaining two components of this triplet are

$$\bar{p}n = \bar{\psi}\tau^+\psi, \qquad T = 1, \qquad T_3 = -1,$$

$$\bar{n}p = \bar{\psi}\tau^-\psi, \qquad T = 1, \qquad T_3 = +1.$$

Under T_2-rotation a state of the nucleon–antinucleon system with $T = 0$, $T_3 = 0$ will go into itself with a plus sign whereas the state with $T = 1$, $T_3 = 0$ will go into itself with a minus sign. Consequently, the T_2-parity of the nucleon–antinucleon system with $T_3 = 0$ is equal to $(-1)^T$.

For a neutral system of a nucleon and an antinucleon we can make use of the definition $G = CT_2$. Then we obtain

$$G = (-1)^{l+s+T}.$$

Having obtained this formula for the neutral component with $T_3 = 0$ we can, taking advantage of isotopic invariance, extend it to charged states of the nucleon–antinucleon system ($\bar{p}n$, $n\bar{p}$), for which $T_3 = \pm 1$.

G-PARITY AND ANNIHILATION

The formula obtained allows one to establish selection rules for the π-meson annihilation of nucleon and antinucleon (Fig. 14). The G-parity of the $\bar{N}N$-system equals $(-1)^{l+s+T}$, while that of a system consisting of π-mesons equals $(-1)^n$. Since strong interaction conserves G-parity, the equality

$$(-1)^{l+s+T} = (-1)^n$$

must be fulfilled in the annihilation. Thus, in the annihilation of a slow ($l = 0$) antineutron in hydrogen ($T = 1$) in a state with

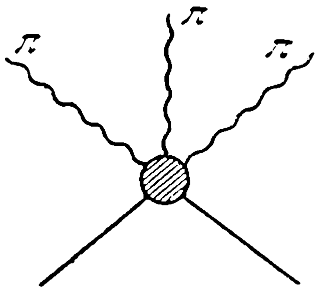

FIG. 14

$s = 1$ only an even number of π-mesons may be created ($n = 2, 4, \ldots$), while in a state with $s = 0$ only an odd number of π-mesons may arise ($n = 3, 5, \ldots$).

G-PARITY OF THE ω^0-MESON

From G-parity conservation one obtains at once the selection rules for the decay of the neutral vector meson ω^0: the G-parity of the ω^0-meson, which has an isotopic spin zero, while its charge parity is odd (as in the case of the photon), must also be odd. Consequently, such a meson may decay into three, but not into two, π-mesons:

$$\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0,$$
$$\omega^0 \not\rightarrow \pi^+ + \pi^-.$$

This last decay may go only by the simultaneous action of the strong and electromagnetic interactions. (The electromagnetic interaction is not isotopically invariant and, consequently, does not conserve G-parity.)

G-PARITY AND β-DECAY

We have sufficiently digressed in this consideration of the strong interaction. Let us return now to the weak interaction, and consider the neutron β-decay. We have written the matrix element for β-decay by now many times. It represents the sum of two terms, a vector and an axial vector:

$$V_\alpha = \bar{u}_p[f_1\gamma_\alpha - f_2\sigma_{\alpha\beta}q_\beta - f_3 q_\alpha]\, u_n,$$
$$A_\alpha = \bar{u}_p[g_1\gamma_\alpha - g_2\sigma_{\alpha\beta}q_\beta - g_3 q_\alpha]\,\gamma_5 u_n.$$

The matrix element V_α represents the free weak vector current v_α "dressed" by virtual strong interactions ($v_\alpha = \bar{p}\gamma_\alpha n$), while A_α represents the "dressed" axial current a_α ($a_\alpha = \bar{p}\gamma_\alpha\gamma_5 n$).

It is easy to see that the free currents v_α and a_α have definite (and different) G-parities. In order to convince oneself of this, note that the β-decay

$$n \rightarrow p + e^- + \nu$$

is equivalent to the annihilation process

$$n + \bar{p} \rightarrow e^- + \nu.$$

The current $\bar{p}\gamma_\alpha n$ describes annihilation from states with an odd value of $l + s$, while the current $\bar{p}\gamma_\alpha\gamma_5 n$ describes that from states with an even value of $l + s$. (It should be noted that under charge conjugation the vector γ_α and the axial vector $\gamma_\alpha\gamma_5$ behave differently.) Since strong interactions, "dressing" initial currents, are isotopically invariant and conserve charge parity, the equalities

$$\left.\begin{aligned} G(V_\alpha) &= G(v_\alpha), \\ G(A_\alpha) &= G(a_\alpha) \end{aligned}\right\} \qquad (*)$$

must be fulfilled.

How do different terms of V_α and A_α behave under G-transformation? Note that under the isotopic rotation T_2 all these terms behave in the same way, because they are components of an isotopic vector. The distinction between them is associated with the fact that they behave in different ways under charge conjugation.

As we have seen in Chapter 3, under charge conjugation C the scalar (1), pseudoscalar (γ_5) and axial ($\gamma_\alpha\gamma_5$) do not change sign; while the vector (γ_α) and tensor ($\gamma_\alpha\gamma_\beta$) do (all this in the theory of q-numbers).

Consequently, the terms

$$\bar{u}_p\gamma_\alpha u_n, \qquad \bar{u}_p\sigma_{\alpha\beta}q_\beta u_n, \qquad \bar{u}_p\sigma_{\alpha\beta}q_\beta\gamma_5 u_n$$

have even G-parity, while the terms

$$\bar{u}_p\gamma_\alpha\gamma_5 u_n, \qquad \bar{u}_p q_\alpha u_n, \qquad \bar{u}_p q_\alpha\gamma_5 u_n$$

have odd G-parity. Whence and from the equalities (*) it follows that $f_3 = 0$ and $g_2 = 0$. The fact that f_3 is zero is not a new result for us: in Chapter 6 we have seen that f_3 ("effective scalar") must be equal to zero by virtue of the vector current conservation. Now it turns out that the effective scalar is "twice-forbidden".

At the same time weak magnetism is not forbidden by the G-parity conservation. On the other hand, in the expression for A_α the term proportional to g_3 ("effective pseudoscalar") is allowed, while the term proportional to g_2 is forbidden. In analogy with weak magnetism, this last term could be called a "weak dipole", because it reminds one of the electric dipole moment. The G-parity conservation in strong interactions requires that the weak dipole should be forbidden, just as the combined parity conservation (CP) forbids the particle to have an electric dipole moment.

OTHER DECAYS

All of these remarks on β-decay refer, of course, also to the matrix element of μ-capture. The fact that the initial weak currents v_α and a_α have a definite G-parity allows one to establish the relationship between the matrix elements of the decays

$$\Sigma^+ \to \Lambda^0 + e^+ + \nu \quad \text{and} \quad \Sigma^- \to \Lambda^0 + e^- + \tilde{\nu}.$$

These matrix elements must be the same.

Unfortunately, neutron β-decay, μ-capture and $\Sigma^\pm \to \Lambda^0 + e^\pm \pm \nu$ decays serve only for verifying that the free currents v_α, a_α actually represent isotopic vectors with a definite (but different) G-parity. These processes cannot prove anything about whether or not the currents contain terms representing isotopic tensors of a higher rank. This is associated with the fact that the interaction with $\Delta T \geqq 2$ cannot give any contribution to the transitions $(T = \frac{1}{2}) \to (T = \frac{1}{2})$, $(T = 1) \to (T = 0)$. Such an interaction might occur, for example, in the $(T = 1) \to (T = 1)$ transitions.

CHAPTER 8

STRANGENESS CONSERVING LEPTONIC DECAYS. CALCULATIONS OF CONCRETE PROCESSES

WE NOW proceed with the consideration of leptonic decays in which the strangeness of the strongly interacting particles before and after the decay remains the same.

The form of the matrix elements for various decays has been defined above. By means of these matrix elements and the calculation technique described in the example of muon decay (see Chapter 4) we are going to calculate the characteristics of a number of decays.

π_{e3}-DECAY ($\pi^+ \to \pi^0 + e^+ + \nu$)

The matrix element of the π_{e3}-decay has the form

$$M = G\varphi_0^+\varphi_+(k_\alpha^0 + k_\alpha^+)\,\bar{u}_\nu\gamma_\alpha(1 + \gamma_5)\,u_e,$$

where φ_0, k_α^0 and φ_+, k_α^+ are the wave functions and 4-momenta, respectively, of π^0- and π^+-mesons, while $\bar{u}_\nu\gamma_\alpha(1 + \gamma_5)\,u_e$ is the leptonic current.

Consider the decay in a coordinate system in which the disintegrating π^+-meson is at rest. The energy released is $\Delta = m_{\pi^+} - m_{\pi^0} = 139{\cdot}6 - 135 = 4{\cdot}6$ MeV. Therefore the π^0-meson produced in the decay of the π^+-meson can be assumed to be practically at rest, and this means that its 4-momentum has only the energy component $E_0 = m_{\pi^0}$, $\boldsymbol{k}_0 = 0$. Therefore,

$$\varphi_0^+\varphi_+(k_\alpha^0 + k_\alpha^+) = \begin{cases} \varphi_0^+\varphi_+(E_0 + E_+) \approx \varphi_0^+\varphi_+ \times 2m_\pi, & \text{if } \alpha = 4, \\ 0, & \text{if } \alpha \neq 4, \end{cases}$$

and, hence,

$$M = G\,.\,2m_\pi\bar{u}_\nu\gamma_4(1 + \gamma_5)\,u_e\varphi_0^+\varphi_+.$$

The probability of the π_{e3}-decay is equal to

$$W = \frac{(2\pi)^4}{(2\pi)^9} \int \frac{d\mathbf{p}_\nu \, d\mathbf{p}_e \, d\mathbf{k}_0}{2E_\nu 2E_e 2m_\pi} \, \frac{\overline{|M|^2}}{2m_\pi} \delta^4(k_+ - k_0 - p_\nu - p_e).$$

We calculate $\overline{|M|^2}$ (the bar denotes summing over the polarization of ν and e):

$$\begin{aligned}
\frac{\overline{|M|^2}}{4m_\pi^2 G^2} &= \overline{|\bar{u}_\nu \gamma_4 (1 + \gamma_5) u_d|^2} \\
&= \mathrm{Sp}(\hat{p}_e + m_e) \gamma_4 (1 + \gamma_5) \hat{p}_\nu \gamma_4 (1 + \gamma_5) \\
&= 2\mathrm{Sp}\, \hat{p}_e \gamma_4 p_\nu \gamma_4 (1 + \gamma_5) = 2\mathrm{Sp}\, \hat{p}_e \gamma_4 \hat{p}_\nu \gamma_4 \\
&= 2\mathrm{Sp}\, \hat{p}_e \gamma_4 (E_\nu \gamma_4 - \mathbf{p}_\nu \boldsymbol{\gamma}) \gamma_4 = 2\mathrm{Sp}\, \hat{p}_e (E_\nu \gamma_4 + \mathbf{p} \boldsymbol{\gamma}) \\
&= 8(E_e E_\nu + \mathbf{p}_e \mathbf{p}_\nu) = 8E_e E_\nu (1 + \mathbf{v}_e \mathbf{v}_\nu).
\end{aligned}$$

Substituting this result into W:

$$\begin{aligned}
W &= \frac{1}{(2\pi)^5} G^2 \times \\
&\times 8 \int E_e E_\nu (1 + \mathbf{v}_e \mathbf{v}_\nu) \frac{d\mathbf{p}_e}{2E_e} \frac{d\mathbf{p}_\nu}{2E_\nu} d\mathbf{k}_0 \delta^4(k_+ - k_0 - p_\nu - p_e) \\
&= \frac{2G^2}{(2\pi)^5} \int (1 + \mathbf{v}_e \mathbf{v}_\nu) \, d\mathbf{p}_e \, d\mathbf{p}_\nu \, \delta(E_\nu + E_e - \Delta),
\end{aligned}$$

$$\Delta = m_{\pi^+} - m_{\pi^0} = 4.6\,\text{MeV}.$$

We now calculate the integral over phase space, disregarding the electron mass ($|\mathbf{p}_e| = E_e$):

$$\int (1 + \mathbf{v}_e \mathbf{v}_\nu) \, d\mathbf{p}_e \, d\mathbf{p}_\nu \, \delta(E_\nu + E_e - \Delta) = \int d\mathbf{p}_e \, d\mathbf{p}_\nu \, \delta(E_\nu + E_e - \Delta).$$

Since the term $\mathbf{v}_e \mathbf{v}_\nu$ is linear in the cosine, on integration it will drop out:

$$\begin{aligned}
&\int d\mathbf{p}_e \, d\mathbf{p}_\nu \, \delta(E_e + E_\nu - \Delta) \\
&\quad = (4\pi)^2 \int |\mathbf{p}_e|^2 \, d|\mathbf{p}_e| \, |\mathbf{p}_\nu|^2 \, d|\mathbf{p}_\nu| \, \delta(E_\nu + E_e - \Delta) \\
&\quad = (4\pi)^2 \int E_e^2 \, dE_e E_\nu^2 \, dE_\nu \, \delta(E_\nu + E_e - \Delta) \\
&\quad = (4\pi)^2 \int E_e^2 \, dE_e (\Delta - E_e)^2 \\
&\quad = (4\pi)^2 \Delta^5 \int_0^1 x^2 (1 - x)^2 \, dx = \frac{(4\pi)^2 \Delta^5}{30}.
\end{aligned}$$

Finally we obtain for the total decay probability

$$W = \frac{2G^2(4\pi)^2\Delta^5}{(2\pi)^5 30} = \frac{G^2\Delta^5}{30\pi^3}.$$

Taking account of the finiteness of the mass of the electron ($m_e/\Delta \neq 0$) and π-meson ($\Delta/m_\pi \neq 0$) will give about a 10 per cent correction to this expression:

$$W = \frac{G^2\Delta^5}{30\pi^3}\left(1 - \frac{5m_e^2}{\Delta^2} - \frac{3}{2}\frac{\Delta}{m_\pi}\right).$$

The probability for the $\pi^+ \to \pi^0 + e^+ + \nu$ decay that we calculate amounts to about 10^{-8} that of the $\pi \to \mu + \nu$ decay.

Note that in this decay we knew, in essence, all the relevant data. We knew the matrix element accurately by virtue of the vector current conservation and the smallness of the energy released in the decay (see Chapter 6). Hence the value of the decay rate is predicted quite unambiguously by theory, and the validity of the theory can be checked experimentally.

Measurement of the relative probability for the $\pi^+ \to \pi^0 + e^+ + \nu$ decay, which was suggested many years ago by Zel'dovich, is of great interest, and recent experiments done at Dubna and CERN in 1962 give results in good agreement with theoretical predictions based on the electrodynamic analogy.

$\pi_{\mu 2}$-DECAY ($\pi^+ \to \mu^+ + \nu$) AND π_{e2}-DECAY ($\pi^+ \to e^+ + \nu$)

In the matrix element describing these processes, we do not know the form factor arising from strong interactions. Therefore, for the matrix element we have to write:

$$M = \frac{G}{\sqrt{2}} f\varphi k_\alpha \bar{u}_\nu \gamma_\alpha (1 + \gamma_5) u_l,$$

where φ is the wave function of the π-meson, while k is its 4-momentum (the index l denotes μ or e), $k = p_l + p_\nu$. The value of f is the same for $\pi_{\mu 2}$- and π_{e2}-decays. To explain this, note that f is due to virtual strong interactions and to the axial vector weak interaction (Fig. 15). The value of f is determined by the properties of the dashed block of the drawing. Since leptons are emitted at one point (locally) and since the weak interaction is universal, f does not depend on the nature of leptons emitted but only on the square of the 4-momentum transferred to the leptons. However, in our case and for $\pi_{\mu 2}$- and π_{e2}-decays the square of the transferred 4-momentum is m_π^2. Consequently, the constant f is the same for the two decays. This allows us, although we cannot calculate the rate of each decay in particular, to predict their ratio.

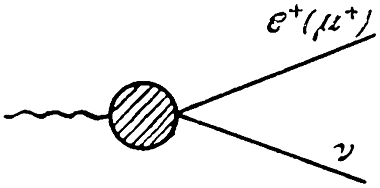

FIG. 15

In order to simplify the matrix element, make use of the Dirac equation

$$k_\alpha \bar{u}_\nu \gamma_\alpha (1+\gamma_5)\,u_l = \bar{u}_\nu(\hat{p}_\nu + \hat{p}_l)\,(1+\gamma_5)\,u_l = m_l \bar{u}_\nu (1-\gamma_5)\,u_l.$$

Whence

$$M = \frac{f m_l G}{\sqrt{2}}\varphi \bar{u}_\nu (1-\gamma_5)\,u_l.$$

Now calculate the decay rate:

$$W = \frac{(2\pi)^4}{(2\pi)^6}\int \frac{dp_l\,dp_\nu}{2E_l\,2E_\nu}\,\frac{\overline{|M|^2}}{2m_\pi}\,\delta^4(k-p_l-p_\nu)$$

$$= \frac{(2\pi)^4 G^2 f^2 m_l^2}{(2\pi)^6 2m_\pi\,.\,2}\int \frac{dp_l\,dp_\nu}{2E_l\,2E_\nu}\,\delta^4(k-p_l-p_\nu)\,\overline{|\bar{u}_\nu(1-\gamma_5)\,u_l|^2},$$

$$\overline{|\bar{u}_\nu(1-\gamma_5)u_l|^2} = \mathrm{Sp}\,\hat{p}_\nu(1-\gamma_5)\,(\hat{p}_l + m_l)\,(1+\gamma_5) = 8(p_l p_\nu).$$

Here the bar denotes the sum over the spin states l and ν. Thus for W we find

$$W = \frac{G^2 f^2 m_l^2 8 p_l p_\nu}{4\pi^2\,.\,16 m_\pi}\int \frac{dp_l\,dp_\nu}{E_l E_\nu}\,\delta^4(k-p_l-p_\nu)$$

$$= \frac{G^2 f^2 m_l^2\,.\,8 p_l p_\nu}{64\pi^2 m_\pi}\int \frac{dp_l}{E_l E_\nu}\,\delta(E_l + E_\nu - m_\pi).$$

Integration over the phase space of the lepton presents no difficulty ($|p_l| = |p_\nu| = E_\nu = p$):

$$\int \frac{dp_l}{E_l E_\nu}\,\delta(E_l + E_\nu - m_\pi) = \int \frac{4\pi p^2\,dp}{p E_l}\,\delta(p + E_l - m_\pi)$$

$$= \frac{4\pi p^2}{p E_l \dfrac{d(p+E_l)}{dp}} = 4\pi \frac{p}{m_\pi}.$$

Note that from $(p_l + p_\nu)^2 = m_\pi^2$ it follows that

$$2p_l p_\nu = m_\pi^2\left(1 - \frac{m_l^2}{m_\pi^2}\right).$$

The decay rate is equal to

$$W = \frac{G^2 f^2 m_\pi}{8\pi} m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2.$$

The constant f in this case has the dimensions of mass

$$[f] = [m].$$

Comparing this expression with the experimentally measured probability of the π-meson decay, we get $f \approx m_\pi$.

The ratio of the probabilities of decays into $e + \nu$ and $\mu + \nu$ does not depend on the unknown function f and is equal to

$$R = \frac{w(e+\nu)}{w(\mu+\nu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 \approx 1{\cdot}3 \times 10^{-4}.$$

For a long time experimentalists could not detect π_{e2}-decay, and its absence was considered as one of the main difficulties for the theory of the universal V–A-interaction. This decay was, however, eventually detected in the autumn of 1958. At present, experiments give a value of the ratio R in a good agreement with the theoretical one.

POLARIZATION OF MUONS IN THE $\pi_{\mu 2}$-DECAY

Since the spin of the π^+-meson is equal to zero, while the neutrino is polarized antiparallel to its momentum, the μ^+-meson must also be polarized antiparallel to its own momentum (see Fig. 16, where long arrows denote the momenta, while short arrows denote the polarization of the particles).

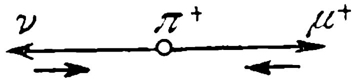

FIG. 16

In an analogous way, μ^--meson in the decay of π^--meson must be polarized longitudinally along the momentum. The same holds, of course, also for e^+ and e^- in the $\pi^\pm \to e^\pm \pm \nu$ decays. Such a polarization of e and μ can be called "forced"; as we shall see later on, in the β-decay, where there are no strict kinematical restrictions, e^- are polarized mainly antiparallel to their momentum.

The results of measurement of the polarization of muons arising from the decay of π-mesons are in agreement with theoretical predictions.

NEUTRON β-DECAY

We now pass over to the neutron decay $n \to p + e^- + \tilde{\nu}$. The amplitude for this decay is equal to

$$M = \frac{G}{\sqrt{2}} (V_\alpha + A_\alpha) \bar{u}_l \gamma_\alpha (1 + \gamma_5) u_\nu,$$

where, as was shown above (Chapters 5–7),

$$V_\alpha = \bar{u}_p (f_1 \gamma_\alpha - f_2 \sigma_{\alpha\beta} q_\beta) u_n,$$

$$A_\alpha = \bar{u}_p (g_1 \gamma_\alpha - g_3 q_\alpha) \gamma_5 u_n.$$

If one disregards "weak magnetism" (f_2) and the "effective pseudoscalar" (g_3), which give a small contribution to the β-decay, and denotes $g_1/f_1 = \alpha$, then the amplitude of the neutron β-decay can be presented in the form

$$M = \frac{G}{\sqrt{2}} \bar{u}_p \gamma_\alpha (1 + \alpha\gamma_5) u_n \bar{u}_e \gamma_\alpha (1 + \gamma_5) u_\nu.$$

The coefficient of γ_α in the baryon current is equal to unity as a consequence of the vector current conservation. The coefficient α of $\gamma_\alpha \gamma_5$, resulting from strong interaction, is a certain quantity which we cannot calculate. For the "bare" interaction it is equal to 1. For the real neutron, as is known from experiment, $\alpha = 1{\cdot}25$ (see below). If the theory of the weak interaction we are presenting is accepted, then from experiments on the neutron β-decay we infer, in essence, only the value α. We cannot, as yet, explain why α is so close to unity.

Calculate the neutron decay rate.

In these calculations the proton and neutron can be considered as non-relativistic particles, because the released energy is only $Q \approx 1{\cdot}3$ MeV. This considerably simplifies the work.

The four-dimensional spinor has the form $u = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$, where φ and χ are two-component spinors. When $|p| \ll m$, the component χ

tends to zero, because $\chi = \frac{\sigma p}{E + m}\varphi$. Therefore the four-component spinors of the proton and neutron go over to the two-component ones:

$$\bar{u}_p = (\varphi_p^+, 0), \qquad u_n = \begin{pmatrix}\varphi_n \\ 0\end{pmatrix}.$$

Consider the non-relativistic limit of the expressions contained in the matrix element:

$$\bar{u}_p \gamma_4 u_n = (\varphi_p^+, 0)\begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix}\varphi_n \\ 0\end{pmatrix} = \varphi_p^+ \varphi_n,$$

$$\bar{u}_p \gamma u_n = (\varphi_p^+, 0)\begin{pmatrix}0 & \sigma \\ -\sigma & 0\end{pmatrix}\begin{pmatrix}\varphi_n \\ 0\end{pmatrix} = 0,$$

$$\bar{u}_p \gamma_4 \gamma_5 u_n = -(\varphi_p^+, 0)\begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}\varphi_n \\ 0\end{pmatrix} = 0,$$

$$\bar{u}_p \gamma \gamma_5 u_n = -(\varphi_p^+, 0)\begin{pmatrix}0 & \sigma \\ -\sigma & 0\end{pmatrix}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}\varphi_n \\ 0\end{pmatrix} = -\varphi_p^+ \sigma \varphi_n.$$

As a result the matrix element assumes the form

$$M = \frac{G}{\sqrt{2}}\{\varphi_p^+ \varphi_n \bar{u}_e \gamma_4 (1 + \gamma_5)\, u_\nu + \alpha \varphi_p^+ \sigma \varphi_n \,.\, \bar{u}_e \gamma (1 + \gamma_5)\, u_\nu\}.$$

We have the sum of two terms—a vector and an axial term. In the vector term only the time-like components differ from zero, while in the axial term only the space-like components differ from zero.

We now find the square of the matrix element, summed over the spin states of the proton, electron and neutrino and averaged over the spin states of the neutron:

$$\overline{|M|^2} = \frac{G^2}{2}\Big\{\overline{|\varphi_p^+ \varphi_n \cdot \bar{u}_e \gamma_4 (1 + \gamma_5)\, u_\nu|^2} + \alpha^2\, \overline{|\varphi_p^+ \sigma \varphi_n \cdot \bar{u}_e \gamma (1 + \gamma_5)\, u_\nu|^2}\Big\}.$$

The interference term, in averaging, turns out to be zero. We calculate $\overline{|\varphi_p^+ \varphi_n|^2}$. The projection operator $\Lambda = \hat{p} + m$ in the non-relativistic approximation reads

$$\Lambda = m(1 + \gamma_4) = m\begin{pmatrix}I + I & 0 \\ 0 & I - I\end{pmatrix} = 2m\begin{pmatrix}I & 0 \\ 0 & 0\end{pmatrix}.$$

Here 1 is a unit 4 × 4-matrix, while I is a unit 2 × 2-matrix. If we had considered polarized particles, the operator Λ_s would contain σ-matrices.

Summing over the spin states of the proton and averaging over the spin states of the neutron give ($m_p = m_n = m$):

$$\overline{|\varphi_p^+\varphi_n|^2} = \tfrac{1}{2}\mathrm{Sp}(2m_p I\,.\,2m_n\,.\,I) = (2m)^2,$$

$$\overline{\varphi_p^+\sigma_i\varphi_n\varphi_n^+\sigma_k\varphi_p} = \tfrac{1}{2}\mathrm{Sp}\,2m\sigma_i\,.\,2m\sigma_k = (2m)^2\,\delta_{ik},$$

where $i, k = 1, 2, 3$. Here the bar denotes summing over the proton polarizations and averaging over the neutron polarizations. Making use of these relations it is easy to obtain

$$\overline{|[\bar{u}_e\gamma_4(1+\gamma_5)\,u_\nu]\,(\varphi_p^+\varphi_n)|^2} = 32m^2(E_e E_\nu + p_e p_\nu),$$

$$\begin{aligned}
&\overline{|\bar{u}_e\gamma_i(1+\gamma_5)\,u_\nu\varphi_p^+\sigma_i\varphi_n|^2}\\
&\quad = (2m)^2\,\mathrm{Sp}\,\hat{p}_\nu\gamma_i(1+\gamma_5)\,(\hat{p}_e+m)\,\gamma_i(1+\gamma_5)\\
&\quad = 2(2m)^2\,\mathrm{Sp}\,\hat{p}_\nu\gamma_i\hat{p}_e\gamma_i = 32m^2(3E_\nu E_e - p_e p_\nu);\\
&\qquad\qquad\qquad\qquad\qquad\qquad i = 1, 2, 3.
\end{aligned}$$

We substitute these results into the matrix element

$$\begin{aligned}
\overline{|M|^2} &= \frac{32m^2G^2}{2}\{E_\nu E_e + p_e p_\nu + \alpha^2(3E_eE_\nu - p_e p_\nu)\}\\
&= 16m^2G^2E_\nu E_e\{(1+3\alpha^2) + v_e v_\nu(1-\alpha^2)\}.
\end{aligned}$$

Thus, it turns out that the electron–neutrino correlation has the form

$$1 + \lambda v_e\cos\theta_{e\nu}, \quad \text{where} \quad \lambda = \frac{1-\alpha^2}{1+3\alpha^2}.$$

For $\alpha = 1{\cdot}25$† $\lambda \cong -0{\cdot}08$. The decay rate is equal to

$$W = \frac{(2\pi)^4}{(2\pi)^9}\int\frac{\overline{|M|^2}}{2m_n}\,\frac{dp_\nu}{2E_\nu}\,\frac{dp_e}{2E_e}\,\frac{dp_p}{2E_p}\,\delta^4(p_n - p_p - p_e - p_\nu).$$

After integration with respect to the angles, we get

$$W = \frac{G^2(4\pi)^2}{(2\pi)^5}\,(1+3\alpha^2)\int|p_e|^2\,|p_\nu|^2\,d|p_e|\,d|p_\nu|\,\delta(E_\nu + E_e - \Delta).$$

Therefore, for the electron spectrum we obtain

$$dW \sim (E_e^2 - m_e^2)^{\frac{1}{2}}(\Delta - E_e)^2\,E_e dE_e.$$

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† According to C. S. Wu, $\alpha = 1{\cdot}15 \pm 0{\cdot}04$ (note added in 1964).

To obtain the total rate one has to take integral over the electron spectrum.

We calculated this integral to be $\Delta^5/30$ for the case when the electron mass can be disregarded (see the $\pi^+ \to \pi^0 + e^+ + \nu$ decay). However, as the kinetic energy released in the β-decay is comparable with the electron mass, including effects of this mass leads to a substantial lowering of the decay rate. Accurate calculations give

$$W = \frac{G^2 \Delta^5}{60\pi^3} (1 + 3\alpha^2)\, 0{\cdot}47 .$$

Attention is drawn to the following. If it is assumed that there is no axial interaction ($\alpha = 0$), and if the electron mass is disregarded, then we obtain a decay rate two times lower than that of the π_{e3}-decay of the π^+-meson (at the same Δ's; see p. 102). This corresponds to the fact that the coefficient in the matrix element of the decay of π-meson is larger than that in the β-decay by a factor of $\sqrt{2}$.

Thus, we have calculated the spectrum of β-decay electrons, electron–neutron correlation and total β-decay probability.

The most accurate values of α are obtained from comparison of the theoretical β-decay rate with the experimental data on the neutron lifetime, obtained by the Spivak group ($\tau = 1013 \pm 29$ sec). If the value of α is determined from other experiments, then data on the neutron lifetime can be used to prove that the vector constant in the β-decay is not renormalized ($f_1 = 1$).

POLARIZATION OF ELECTRONS

To calculate the polarization of electrons arising from β-decay, replace $u_e \bar{u}_e$ by $\Lambda_s = \frac{1}{2}(\hat{p}_e + m_e)(1 + \hat{s}_e \gamma_5)$ (see Chapter 4). It can easily be seen that in the product $(1 + \gamma_5)(\hat{p}_e + m_e)(1 + \hat{s}_e \gamma_5)$ only the quantity $\hat{p}_e - m_e \hat{s}_e$ will give a non-zero contribution to the trace.

If we are interested in the spectrum of non-polarized electrons, then only the fourth component of the vector p_e enters the result: $p_{e4} = E_e$. If we are interested in the spectrum of polarized electrons, then p_{e4} is to be replaced by $p_{e4} - m s_{e4}$. Taking into account that $s_{e4} = p_e \xi_e / m_e$, where p_e is the electron momentum, while ξ_e is the unit vector of the electron polarization determined in the system in which the electron is at rest (see p. 59), we see that the probability of a decay with emission of a polarized electron is proportional to the quantity

$$\left(E_e - m_e \frac{p_e \xi_e}{m_e}\right) = E_e(1 - v_e \xi_e).$$

This result means that in the β-decay the electrons are poarized antiparallel to their motion (left-handed screw) and that the degree of their polarization is equal to the electron velocity v_e (in units of c). Indeed, the number of right-handed polarized electrons is proportional to $1 - v_e$, while that of the left-handed ones is proportional to $1 + v_e$. Consequently, the degree oft left-handed polarization is equal to

$$\frac{(1 + v_e) - (1 - v_e)}{(1 + v_e) + (1 - v_e)} = v_e.$$

DECAY OF THE POLARIZED NEUTRON

If a disintegrating neutron is completely polarized, then the electron angular distribution must be of the form

$$1 + v_e \varrho_e \cos \varphi_e,$$

while the neutrino angular distribution must be of the form

$$1 + \varrho_\nu \cos \varphi_\nu,$$

where

$$\varrho_e = -\frac{2(\alpha^2 - \alpha)}{1 + 3\alpha^2},$$

$$\varrho_\nu = \frac{2(\alpha^2 + \alpha)}{1 + 3\alpha^2},$$

$\varphi_e(\varphi_\nu)$ being the angle between the neutron spin and electron (neutrino) momentum. These formulae are easy to obtain by means of the standard calculation technique. If it is assumed that $\alpha = 1{\cdot}25$, then we obtain $\varrho_e \approx -0{\cdot}08$, $\varrho_\nu \approx 1$. Experimental data are in agreement with these figures.

By virtue of the combined parity conservation a correlation of the type $\eta[p_e \times p_\nu]$ cannot be present, where η is the vector of the neutron polarization and p_e and p_ν are the momenta of the electron and neutrino (see pp. 45–46). Experiment confirms this conclusion.

μ-CAPTURE BY THE PROTON

The amplitude of the process $\mu^- + p \to n + \nu$ is equal to

$$M = \frac{G}{\sqrt{2}} (V_\alpha + A_\alpha)\, \bar{u}_\nu \gamma_\alpha (1 + \gamma_5)\, u_\mu,$$

$$V_\alpha = \bar{u}_n [f_1 \gamma_\alpha - f_2 \sigma_{\alpha\beta} q_\beta]\, u_p,$$

$$A_\alpha = \bar{u}_n [g_1 \gamma_\alpha - g_3 q_\alpha]\, \gamma_5 u_p.$$

On the basis of data on the electromagnetic structure of nucleons it can be concluded that the values of f_1 and f_2 differ from analogous quantities for the neutron β-decay by not more than 1–2 per cent. The same is apparently valid for g_1.

For a rough estimate, we disregard the dependence of the form factor on energy, and also terms corresponding to weak magnetism and the effective pseudoscalar. Then the amplitude assumes the form

$$M = \frac{G}{\sqrt{2}} [\bar{u}_n \gamma_\alpha (1 + \alpha\gamma_5)\, u_p]\, [\bar{u}_\nu \gamma_\alpha (1 + \gamma_5)\, u_\mu].$$

The cross section calculated by means of this amplitude is equal to

$$\sigma = \frac{G^2 m_\mu^2}{2\pi v} (1 + 3\alpha^2),$$

where m_μ is the muon mass, while v is the muon velocity.

We now calculate the probability of μ^--meson capture from the K-shell of the μ-atom. As is known, the probability of any reaction is equal to the product of the cross section σ and the flux j:

$$W = \sigma j = \sigma \varrho v,$$

where ϱ is the density of the μ-wave. In the μ-atom the density of μ^--mesons near the nucleus is equal to $1/\pi a^3$, where a is the Bohr radius of the μ-atom ($a = 1/m_\mu e^2$, and $e^2 = 1/137$).

Taking this into account, we obtain for muon capture in an atom of mesohydrogen

$$W = \frac{G^2 m_\mu^2}{2\pi v} (1 + 3\alpha^2)\, v \frac{m_\mu^3 e^6}{\pi} = \frac{G^2 m_\mu^5 e^6}{2\pi^2} (1 + 3\alpha^2).$$

The ratio of this probability to that of the muon decay is equal to

$$\frac{W(\mu^- + p \to n + \nu)}{W(\mu^- \to e^- + \nu + \tilde{\nu})} = \frac{e^6 (1 + 3\alpha^2)\, 192\pi^3}{2\pi^2} \cong 6 \times 10^{-4}.$$

It can be shown that the corrections due to the effective pseudoscalar (g_3) and weak magnetism (f_2) amount to several tenths for muon capture by the proton. The correction due to weak magnetism can be estimated quite precisely, while for the correction due to the effective pseudoscalar only an order of magnitude can be predicted. If the theory we have presented is valid, then we know fairly accurately f_1, g_1 and f_2 from data on the neutron β-decay and from the experiments on the electromagnetic structure of nucleons. Therefore experiments on muon capture should make it possible to determine the value of the effective pseudoscalar g_3.

DECAYS OF STRANGE PARTICLES

Strangeness conserving decays of strange particles possess low probabilities, since the energy released in these decays is usually very small. Therefore these decays cannot compete with strangeness changing decays of strange particles, in which the energy released is large.

The $K^0 \to K^+ + e^- + \tilde{\nu}$ decay is analogous to the $\pi^+ \to \pi^0 + e^+ + \nu$ decay. Its matrix element is of the form

$$\frac{G}{\sqrt{2}} \varphi_+^+ \varphi_0 (k_\alpha^+ + k_\alpha^0)\, \bar{u}_e \gamma_\alpha (1 + \gamma_5)\, u_\nu,$$

while its decay rate is equal to

$$W = \frac{G^2 \Delta^5}{60\pi^3}.$$

For $\Delta = m_{K^0} - m_{K^+} = 3{\cdot}7$ MeV we obtain $W \sim 0{\cdot}1\ \text{sec}^{-1}$. The probability of the $\Sigma^- \to \Sigma^0 + e^- + \tilde{\nu}$ decay, the "most readily observed" of similar β-decays within the Σ-triplet, is 10^{-9} of the total probability of the Σ^--hyperon decay.

The most probable strangeness conserving decays of strange particles are the following:

$$\Sigma^{\pm} \to \Lambda^0 + e^{\pm} \pm \nu.$$

The form of the matrix elements of these decays depends on P—the relative parity of Σ and Λ. If $P = +1$, then

$$V_\alpha = \bar{u}_\Lambda [\gamma_\alpha f_1 + \sigma_{\alpha\beta} k_\beta f_2 + k_\alpha f_3]\, u_\Sigma,$$

$$A_\alpha = \bar{u}_\Lambda [\gamma_\alpha g_1 + \sigma_{\alpha\beta} k_\beta g_2 + k_\alpha g_3]\, \gamma_5 u_\Sigma.$$

If $P = -1$, then V_α and A_α are interchanged.

Because the nucleon current has a definite G-parity (more precisely, GP-parity) the functions f and g in the $\Sigma^+ \to \Lambda + e^+ + \nu$ decay are the same as in the $\Sigma^- \to \Lambda^0 + e^- + \tilde{\nu}$ decay. From the vector current conservation it follows that $k_\alpha V_\alpha = 0$ and that

$$f_1 = -\frac{f_3 k^2}{M_\Sigma - M_\Lambda}, \quad \text{if} \quad P = +1,$$

$$g_1 = \frac{g_3 k^2}{M_\Sigma + M_\Lambda}, \quad \text{if} \quad P = -1.$$

This leads to the fact that $V_\alpha \to 0$ as $k \to 0$ and, consequently, V_α is small in comparison with A_α. It should be noted that, by virtue of the analogy between the weak vector current and electrodynamic current, V_α is the same as the matrix element of the electromagnetic conversion decay

$$\Sigma^0 \to \Lambda^0 + e^+ + e^-.$$

Retaining only dominant terms in the matrix element, it is easy to obtain

$$W_{\Sigma^\pm \to \Lambda^0 + e^\pm \pm \nu} = \frac{3G^2 \Delta_+^5 g_1^2}{60\pi^3}, \quad \text{if} \quad P = +1,$$

$$W_{\Sigma^\pm \to \Lambda^0 + e^\pm \pm \nu} = \frac{G^2 \Delta_+^5 f_1^2}{60\pi^3}, \quad \text{if} \quad P = -1.$$

If we assume tentatively that $g_1 = 1$, then for $P = +1$ we obtain $W_{\Sigma^+} \cong 0{\cdot}7 \times 10^6\,\text{sec}^{-1}$, $W_{\Sigma^-} \simeq 1{\cdot}1 \times 10^6\,\text{sec}^{-1}$. The difference between these probabilities is associated with that of the masses of the Σ^+- and Σ^--hyperons and, consequently, of the corresponding values Δ_+ and Δ_-.

Making use of data on the lifetime of the $\Sigma^\pm$-hyperons, it is easy to find that the fraction of the $\Sigma \to \Lambda$ decays must amount to

$\sim 0{\cdot}5 \times 10^{-4}$ for Σ^+-hyperon, and
$\sim 1{\cdot}8 \times 10^{-4}$ for Σ^--hyperon.

The ratio $W_{\Sigma^+ \to \Lambda^0}/W_{\Sigma^- \to \Lambda^0} \approx 0{\cdot}6$; it does not depend on the assumed value of g_1 or f_1.

Measurement of this ratio would be of great interest, since it would allow a check on the hypothesis that the strangeness conserving current has a definite GP-parity.†

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† For data on the probabilities of $\Sigma \to \Lambda\, e\, \nu$ decays see Table 8 on p. 220 (note added in 1964).

CHAPTER 9

STRANGENESS CHANGING LEPTONIC DECAYS. GENERAL PROPERTIES

WE NOW proceed to a new chapter dealing with the leptonic decays of strange particles.

In Chapter 1 the scheme of the weak interactions was described. According to it, the four currents $\bar{e}\nu$, $\bar{\mu}\nu$, $\bar{n}p$, $\bar{\Lambda}p$ interact with one another either directly or via certain intermediate vektor-bosons. The interaction of these currents gives rise to all weak processes (Fig. 17).

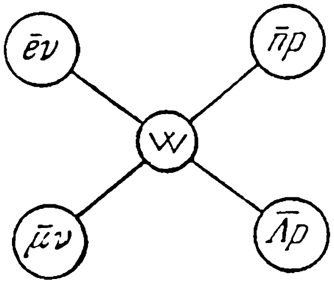

FIG. 17

To start with, we considered interactions involving only leptons (muon decay and neutrino–electron scattering). Then strangeness conserving decays of strongly interacting particles were studied. These decays resulted from the interaction of the current $\bar{n}p$ with the currents $\bar{e}\nu$ and $\bar{\mu}\nu$. We proceed now to decays resulting from the interaction of strange particles with leptons.

In our model the only current that can lead to strangeness non-conservation in strange-particle decays is the strange current $\bar{\Lambda}p$, hence all strangeness changing decays of strange particles must be determined by the current $\bar{\Lambda}p$. It should be noted that not every leptonic strange-particle decay is due to the strange current $\bar{\Lambda}p$, but only those in which the strangeness of the strongly interacting particles changes. For example, the $\Sigma^{\pm} \rightarrow \Lambda^0 + e^{\pm} \pm \nu$ decay, which we considered earlier, is due to the nucleonic current $\bar{n}p$.

HYPERON DECAYS

To what decays does an interaction of the current $\bar{\Lambda}p$ with leptonic currents lead? It is quite obvious that there will be the transition $\Lambda^0 \to p + e^- + \tilde{\nu}$. This β-decay of the Λ^0-hyperon is due to the currents $\bar{e}\nu$ and $\bar{\Lambda}p$. The currents $\bar{\mu}\nu$ and $\bar{\Lambda}p$ will give the decay $\Lambda^0 \to p + \mu^- + \tilde{\nu}$. The presence of these leptonic decays of Λ^0-particle is already sufficient to give rise to leptonic decays of all other hyperons.

Consider the β-decay of Σ-hyperon. As a result of strong interaction, the Σ-hyperon may go over to $\pi^- + \Lambda^0$: $\Sigma^- \xrightarrow{s} \pi^- + \Lambda^0$ (s denotes strong interaction).

As the Λ-particle may decay into the proton, electron and antineutrino, the sequence of transformations will continue:

$$\Sigma^- \xrightarrow{s} \Lambda^0 + \pi^- \xrightarrow{w} e^- + \tilde{\nu} + p + \pi^-$$

(w denotes weak interaction).

As a result of the strong interaction, π^- may get absorbed by a proton:

$$\Sigma^- \xrightarrow{s} \Lambda^0 + \pi^- \xrightarrow{w} e^- + \tilde{\nu} + p + \pi^- \to e^- + \tilde{\nu} + n.$$

This sequence of reactions can be presented graphically by Fig. 18.

Thus, knowing that there exists the β-decay of Λ-hyperon, and that the Λ-hyperon is connected by strong interactions with the Σ^--hyperon, we find that the Σ^--hyperon must have the decay $\Sigma^- \to n + e^- + \tilde{\nu}$; also the decay $\Sigma^- \to n + \mu^- + \tilde{\nu}$. The sequence of transformations in the β-decay of the cascade hyperon is easy to construct:

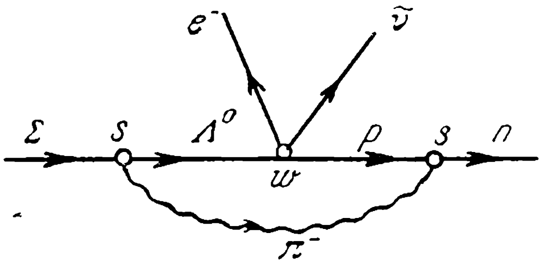

FIG. 18

$$\Xi^- \xrightarrow{s} K^- + \Lambda^0 \xrightarrow{w} K^- + p + e^- + \tilde{\nu} \xrightarrow{s} \Lambda^0 + e^- + \tilde{\nu}.$$

If $K^- + p$ form instead of a Λ^0 a Σ^0, then there will arise the decay

$$\Xi^- \to \Sigma^0 + e^- + \tilde{\nu}.$$

The chains describing these decays are presented graphically in Fig. 19.

Analogously one has the muon decays of the Ξ^--hyperon

$$\Xi^- \rightarrow \Lambda^0(\Sigma^0) + \mu^- + \tilde{\nu}$$

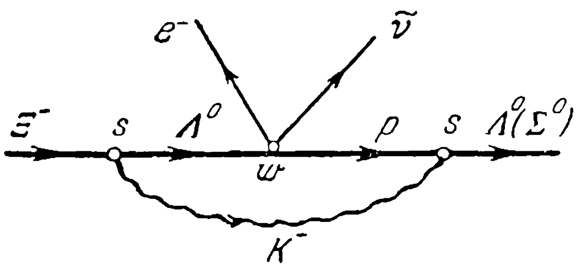

FIG. 19

and the decays of the neutral cascade hyperon

$$\Xi^0 \rightarrow \Sigma^+ + e^-(\mu^-) + \tilde{\nu}.$$

Thus, the combination of weak and strong interactions results in the leptonic decay of all hyperons. In the same way the presence of the leptonic decay of Λ-particle leads to leptonic decays of K-mesons.

K-DECAYS

Consider the K^+-meson. As a consequence of the strong interaction it may become $p + \tilde{\Lambda}$. The virtual $\tilde{\Lambda}$-particle may decay as a result of the weak interaction:

$$K^+ \xrightarrow{s} p + \tilde{\Lambda} \xrightarrow{w} e^+(\mu^+) + \nu + \tilde{p} + p.$$

For virtual protons and antiprotons there are a number of possibilities.

1. The proton and antiproton annihilate completely and no strongly interacting particle remains. Then the decays $K_{e2}(K^+ \rightarrow e^+ + \nu)$ and $K_{\mu 2}(K^+ \rightarrow \mu^+ + \nu)$ arise. The K_{e2}-decay has so far not beeen observed, while the $K_{\mu 2}$-decay contributes 60 per cent of all decays of positive K-mesons.

2. The proton and antiproton become a π^0-meson. Then the decays K_{e3}^+ $(K^+ \rightarrow e^+ + \nu + \pi^0)$ and $K_{\mu 3}^+(K^+ \rightarrow \mu^+ + \nu + \pi^0)$ arise.

The probability of each of these amounts to about 4 per cent of the total probability of the decay of the positive K-meson.

3. The proton and antiproton annihilate into two π-mesons. Then the following decays arise:

$$K^+ \to e^+ + \nu + 2\pi^0,$$
$$K^+ \to e^+ + \nu + \pi^+ + \pi^-,$$
$$K^+ \to \mu^+ + \nu + 2\pi^0,$$
$$K^+ \to \mu^+ + \nu + \pi^+ + \pi^-.$$

These decays are called the K_{e4}^+ and $K_{\mu 4}^+$. First of these has recently been observed.† Because of the smallness of the phase space the K_{e4}^+-decay rate is expected to be, according to a theoretical estimation, of the order of $(10^{-3}$–$10^{-4})$ of the K_{e3}-decay rate. The $K_{\mu 4}$-decay rate is expected to be an order of magnitude lower than the K_{e4}-decay rate.

It should be mentioned that, in principle, the virtual proton and antiproton may even go over into three π-mesons, but such a K_{e5}-decay is rather unlikely and, probably, will not be observed soon.

Decays of the K^--meson can also be considered in an analogous way.

Here the distinction consists only in the fact that the K^--meson becomes, as a result of strong interactions, the antiproton + Λ-hyperon. Then, as a consequence of the weak interaction these particles become the antiproton + proton + e^- + $\tilde{\nu}$.

In the same way the decay of the neutral K^0-meson will take place:

$$K^0 \xrightarrow{s} n + \tilde{\Lambda} \to e^+(\mu^+) + \nu + n + \tilde{p}.$$

The pair $n + \tilde{p}$ possesses a charge, hence the K_{e2}^0 and $K_{\mu 2}^0$ decays are forbidden. However, the K_{e3}^0, $K_{\mu 3}^0$, K_{e4}^0, $K_{\mu 4}^0$ decays should occur.

Thus, it is seen that in order to account for a number of strangeness changing leptonic decays of strange particles it is quite sufficient that the pair $\tilde{\Lambda}p$ interacts with leptons. Moreover, the existence of some still undetected decays is predicted with absolute certainty.

The choice of Λ-hyperon as the only representative of the strange particles in the strangeness changing current puts very strict restrictions upon leptonic decays of strange particles. What are they?

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† At present about 100 cases of K_{e4}-decays are registered; see p. 221 and Table 9 on p. 221 (note added in 1964).

$|\Delta S| = 1$ RULE

Since in our current either the Λ goes over to the proton or the proton goes over the to Λ, strangeness is changed only by unity: $|\Delta S| = 1$.

This means that there cannot exist leptonic decays in which the strangeness of strongly interacting particles changes (to first order in the weak coupling constant) by an amount larger than unity, for example, by two.

The decay of the cascade hyperon $\Xi^- \rightarrow n + e^- + \tilde{\nu}$ would be such a decay.

The strangeness of the cascade hyperon is minus 2, while that of the neutron is 0; it is seen that strangeness in this decay would be changed by two, whereas our current allows strangeness to be changed only by unity.

The same holds for the decay of the neutral cascade hyperon $\Xi^0 \rightarrow p + e^- + \tilde{\nu}$ and to the decays with μ-meson emission: $\Xi^- \rightarrow n + \mu^- + \tilde{\nu}$, $\Xi^0 \rightarrow p + \mu^- + \tilde{\nu}$.

All these decays are forbidden, if the current $(\bar{\Lambda} p)$ is the only strange current.

$\Delta Q = \Delta S$ RULE

In this model there is one more selection rule. In the $\Lambda \rightarrow p$ transition the charge of strongly interacting particles increases by unity ($\Delta Q = +1$). Strangeness in this decay also increases by unity ($\Delta S = +1$). Consequently, $\Delta Q = \Delta S$. In the $\bar{\Lambda} \rightarrow \bar{p}$ transition the change in the charge is $\Delta Q = -1$, while that of the strangeness is $\Delta S = -1$, and so again $\Delta Q = \Delta S$.

Thus, in our scheme the weak interaction obeys the rule

$$\Delta Q = \Delta S.$$

This leads to a number of rules in leptonic decays of strange particles. In particular, the Σ^+-hyperon decays $\Sigma^+ \rightarrow n + e^+(\mu^+) + \nu$ are forbidden.

As can easily be seen, for these decays $\Delta Q = -1$, while $\Delta S = +1$ and, consequently, $\Delta Q \neq \Delta S$. This forbiddenness is particularly interesting in connection with the fact that the analogous Σ^--hyperon decays $\Sigma^- \rightarrow n + e^-(\mu^-) + \tilde{\nu}$ are allowed by the $\Delta Q = \Delta S$ rule and must take place (see the beginning of this chapter).

It should be noted that the decay

$$\Sigma^+ \to p + \pi^+ + e^- + \tilde{\nu}$$

does not contradict the $\Delta Q = \Delta S$ rule.

However, because of smallness of the phase space this decay is strongly suppressed.

In an analogous way it follows that the decays

$$K^0 \to \pi^- + e^+(\mu^+) + \nu \quad \text{and} \quad \bar{K}^0 \to \pi^+ + e^-(\mu^-) + \tilde{\nu},$$

for which $\Delta Q = \Delta S$, are allowed, whereas the decays

$$\bar{K}^0 \to \pi^- + e^+(\mu^+) + \nu \quad \text{and} \quad K^0 \to \pi^+ + e^-(\mu^-) + \tilde{\nu},$$

for which $\Delta Q = -\Delta S$, are forbidden. We shall return to these matters later on, when the properties of neutral K-mesons are discussed (see Chapter 15).

Applying the $\Delta Q = \Delta S$ rule to the K_{e4} decays it is readily seen that the decays

$$K^+ \to \pi^+ + \pi^- + e^+(\mu^+) + \nu, \qquad (*)$$

$$K^+ \to 2\pi^0 + e^+(\mu^+) + \nu \qquad (**)$$

are allowed, whereas the decay

$$K^+ \to 2\pi^+ + e^-(\mu^-) + \tilde{\nu}$$

is forbidden.

In connection with this prediction the detection of K_{e4}-decays is of great interest. In emulsions, the decays (*) and (**) should look like anomalous τ-decays, because three charged particles are produced in them. It would be very interesting to establish that the decay (*) exists and that the decay (**) is forbidden. The fact that they are similar to each other may lower substantially the possible systematic error in estimating their relative probability. The same goes for the corresponding $K_{\mu4}$-decays. Finally, from the $\Delta Q = \Delta S$ rule it follows that the decay $\Xi^0 \to \Sigma^- + e^+ + \nu$ is forbidden, whereas the decay $\Xi^0 \to \Sigma^+ + e^- + \tilde{\nu}$ is allowed.

It should be noted that the $\Delta Q = \Delta S$ rule is strict to within corrections involving the weak interaction. Virtual electromagnetic interactions cannot violate it, because electromagnetic interactions conserve strangeness. The same refers to the $|\Delta S| = 1$ rule.

$\Delta T = \frac{1}{2}$ RULE

The rule which we are going to discuss is valid to within electromagnetic corrections, because it refers not to the change in strangeness but to that in isotopic spin, and electromagnetic interactions do not conserve isotopic spin.

From the fact that the isotopic spin of the Λ is zero, while that of proton is $\frac{1}{2}$, it follows that the current $\bar{\Lambda}p$ changes isotopic spin by $\frac{1}{2}$. The same conclusion can be expressed as follows: the current $\bar{\Lambda}p$ is an isotopic spinor. From this property of the strange current, experimentally observable inferences can be derived; in particular, one can interrelate the probabilities of the K_{e3} and $K_{\mu 3}$ decays for charged and neutral K-mesons.

Let us interrelate the probabilities of the decays

$$K^+ \to e^+ + \nu + \pi^0, \qquad K^0 \to e^+ + \nu + \pi^-.$$

For convenience denote the pair $e^+ + \nu$ by L^+. Then

$$K^+ \to L^+ + \pi^0, \qquad K^0 \to L^+ + \pi^-.$$

The fact that isotopic spin in these decays changes by $\frac{1}{2}$ can be thought of, but only as a figment of imagination, as if L had an isotopic spin of $\frac{1}{2}$ which it carries away. If L is an isotopic spinor, then beside the component L^+ there must be a component L^0. But the introduction of the L^0-component is mathematical fiction, and no physical meaning can be ascribed to it!

Introducing the idea of the isotopic spinor L, we can assume that in the reactions

$$K^+ \to L^+ + \pi^0 \quad \text{and} \quad K^0 \to L^+ + \pi^-$$

the isotopic spin is conserved.

In order to obtain the ratio between the probabilities of these decays, we shall make use not of the usual mathematical methods but of a method proposed by Shmushkevich, which enables one to obtain in an easy way the simplest isotopic relations.

Consider a sample consisting of K-mesons. Let the number of K^+-mesons in the sample be equal to that of K^0-mesons. It can be said that in this case the sample is isotopically unpolarized. The K-mesons of the sample will decay according to four different channels:

$$(1)\ K^+ \to \pi^+ + L^0, \qquad (3)\ K^0 \to \pi^- + L^+,$$
$$(2)\ K^+ \to \pi^0 + L^+, \qquad (4)\ K^0 \to \pi^0 + L^0.$$

By virtue of the mirror symmetry of the reactions on the same line the reaction rates are

$$W_1 = W_3, \qquad W_2 = W_4.$$

As the sample is isotopically unpolarized, while the interaction, after the introduction of the L^0, is isotopically invariant, the decay products must be isotopically unpolarized.

This means that there must be the same number of the π^+-, π^-- and π^0-mesons.

Whence we find at once that $W_1 = W_3 = W_2 + W_4$.

Taking advantage of the fact that $W_2 = W_4$, we can rule out fictitious reactions containing L^0, and obtain the relation between W_2 and W_3:

$$W(K^0_{e3}) = 2\,W(K^+_{e3}).$$

We have just obtained the relation between the K_{e3}-decays of the neutral and charged K-meson. An analogous relation, of course, is valid also for $K_{\mu3}$-decays.

When we discuss K^0_1- and K^0_2-mesons we shall return to these relations, but even here it should be stressed that an accurate measurement of the probabilities of neutral K_{e3}- and $K_{\mu3}$-decays would be of great interest.

Analogous relations can be established between the decays

$$\Xi^- \rightarrow \Sigma^0 + e^- + \tilde{\nu} \quad \text{and} \quad \Xi^0 \rightarrow \Sigma^+ + e^- + \tilde{\nu}.$$

The ratio of the probabilities of these processes must also be equal to $\frac{1}{2}$.

VECTOR CURRENT $\bar{\Lambda}p$ IS NOT CONSERVED

We now proceed to the question as to whether or not the vector part of the current $\bar{\Lambda}p$ is conserved. The vector current $\bar{n}p$ conservation law allowed a number of important conclusions on the strangeness conserving vector interaction to be derived (equality of the constants of the β- and μ-decay, prediction of the $\pi^+ \rightarrow \pi^0$ decay rate, weak magnetism). In fact, it is easy to see that the vector current $\bar{\Lambda}p$ is not conserved. This follows from the difference between strong interactions of the Λ-hyperon and proton. We observe that even the difference in the properties of the proton and neutron, which is of electromagnetic nature, leads to the fact that conservation of the vector current $\bar{n}p$ is not exact, but holds only to within electromagnetic corrections. However, the properties of the Λ-hyperon and proton differ to a much greater extent than those of the neutron and proton. Their masses differ by about 170 MeV, and not by 1·3 MeV—the neutron-proton mass difference. These particles have different isotopic spins (0 for Λ, and $\frac{1}{2}$ for p) and different strong interactions. All this provides grounds for assuming the vector current $\bar{\Lambda}p$ to have a divergence substantially differing from zero. Non-conservation of the vector current $\bar{\Lambda}p$ deprives us of a number of inferences which we have had in the case of the current $\bar{n}p$. First of all, we can no longer expect that the vector constant of the Λ-hyperon β-decay will remain unchanged under the action of virtual strong interactions. In order to see this in more detail, let us return to the neutron β-decay.

Why does the vector constant of the neutron β-decay not change?

In calculating the decay of a "dressed" neutron we should, in principle, take into account three types of strong interactions. First, we must take into account the fact that the neutron interacts before it disintegrates; it emits and absorbs virtual particles. Secondly, we have to take into account that the emitted proton is also strongly interacting. Finally, the neutron may emit virtual particles which will be absorbed by the proton.

Thus, there are three types of graphs (Fig. 20). In these graphs, the blocks representing the strong interaction are hatched.

The fact that the neutron and proton pertain to the same isotopic multiplet and have the same strong interactions results in a cancellation among the contributions of all the three diagrams, and the "dressed" constant of the vector interaction turns out to be equal to the "bare" one. In the case when the strong interaction of the particles before and after decay is different there is no reason for expecting compensation of the contributions of the three diagrams.

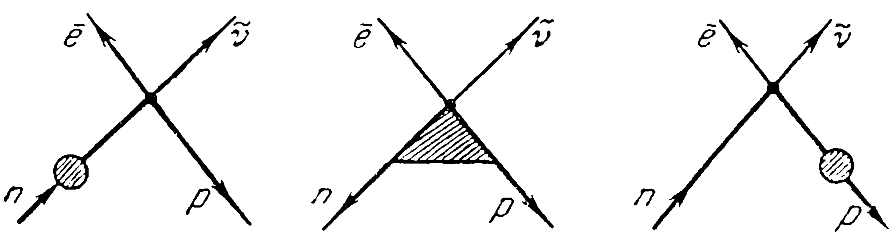

FIG. 20

The strong interactions of the Λ-hyperon and the proton are essentially different. Hence we have no grounds for expecting the

Λ-hyperon β-decay to have the same constant as the neutron β-decay; we know that strong interactions may change very substantially the constant of the Λ-hyperon β-decay. There is nothing surprising in this change.

Thus, in the case of the strangeness changing decay of strange particles, we do not have a vector current conservation law and, hence, we have no analogy with electrodynamics. Moreover, in this case, we do not have G-symmetry. This is because under the G-transformation the pair $\bar{n}p$ goes over to itself and, consequently, may have a definite G-parity, while the pair $\bar{\Lambda}p$ has no definite G-parity, because under the G-transformation it goes over not into itself but into the pair $\bar{n}\Lambda$.

HYPERON-DECAY PROBABILITIES

The absence of the G-symmetry and vector current conservation law leads to the fact that both "weak dipole" and "effective scalar" terms are allowed in the matrix elements of hyperon β-decays. Therefore the hyperon β-decay is described not by four terms, as in the case of the neutron, but by six terms. Such a decay as $\Lambda \rightarrow p + e^- + \bar{\nu}$ is described by matrix elements having the form ($q = p_e + p_\nu = p_\Lambda - p_p$)

$$M = \frac{G}{\sqrt{2}}(V_\alpha + A_\alpha)\,\bar{u}_e\gamma_\alpha(1 + \gamma_5)\,u_\nu,$$

$$V_\alpha = \bar{u}_p[f_1\gamma_\alpha + f_2\sigma_{\alpha\beta}q_\beta + f_3 q_\alpha]\,u_\Lambda,$$

$$A_\alpha = \bar{u}_p[g_1\gamma_\alpha + g_2\sigma_{\alpha\beta}q_\beta + g_3 q_\alpha]\,\gamma_5 u_\Lambda.$$

As to the six unknown coefficients f and g, we know only that by virtue of the combined parity conservation they are real. Since we do not know the value of these coefficients, we cannot deduce the probability of the Λ-hyperon β-decay. Therefore any theoretical prediction of the hyperon β-decay rate is for the present out of the question.

It has frequently been stated that there are contradictions between the theoretical hyperon decay probability and experiment. What does this contradiction mean? Since we are unable to calculate any of these six coefficients, the most reasonable thing that can be done is to eliminate terms proportional to the momentum transferred, i.e. to assume that $f_2 = f_3 = g_2 = g_3 = 0$.

Further assume that $f_1 = g_1 = L$, although, speaking strictly, we have no grounds for this.

Thereupon we reduce the matrix element to the form

$$M = \frac{G}{\sqrt{2}} \bar{u}_p \gamma_\alpha (1 + \gamma_5) u_\Lambda \bar{u}_e \gamma_\alpha (1 + \gamma_5) u_\nu .$$

We know that this matrix element is only a guess, but we can at least use it to calculate the decay probability. Decay probabilities obtained in this way are called "theoretical probabilities". Standard calculation gives for the "theoretical probability" the formula

$$W = \frac{G^2 \Delta^5 C}{15\pi^3}.$$

In this formula, Δ is the maximum electron energy, $\Delta = (M_Y^2 - M_N^2)/2M_Y$, where M_Y is the hyperon mass, while M_N is the nucleon mass. The dimensionless coefficient C takes account of the recoil. When $\Delta \to 0$, $C \to 1$, while when $\Delta \to M_Y/2$, $C \to 2{\cdot}5$. Thus, C does not change strongly from decay to decay.

In the limiting case $\Delta \to 0$ our formula is the same as that for the neutron β-decay rate

$$W_n = \frac{G^2 \Delta^5 (1 + 3\alpha^2)}{60\pi^3},$$

if in the latter we put $\alpha = 1$ (α is the ratio of the axial to vector constant).

In the limiting case $\Delta \to M_Y/2$ the formula coincides with that of the μ-meson decay rate:

$$W_\mu = \frac{G^2 \mu^5}{192\pi^3}.$$

By means of our "theoretical formula", it is easy to calculate that the β-decay rate relative to the total decay rate should amount to 1·5 per cent for the Λ^0-hyperon ($\Lambda \to p + e^- + \tilde{\nu}$), 5·8 per cent for Σ^--hyperon ($\Sigma^- \to n + e^- + \tilde{\nu}$), and 6 per cent for Ξ^--hyperon ($\Xi^- \to \Lambda^0 + e^- + \tilde{\nu}$). What has so for been observed is only about fifteen $\Lambda \to p + e^- + \tilde{\nu}$ decays per 17,000 hyperons, and a few $\Sigma^- \to n + e^- + \tilde{\nu}$ decays per 5000 Σ^--hyperons. Since there are many uncertainties in the effectiveness of observing hyperon β-decays, the probabilities of these decays cannot be estimated accurately.

Experimentalists, however, give the following estimates for leptonic decays of hyperons:

$$\Lambda \to p + e^- + \tilde{\nu} \approx 0{\cdot}2 \text{ per cent},$$
$$\Sigma^- \to n + e^- + \tilde{\nu} \approx 0{\cdot}1 \text{ per cent}.$$

It is seen that the "theoretical" and experimental estmates are in considerable disagreement.†

What does this disagreement indicate? Either that renormalization is very essential, or that the hypothesis of universality of the weak interaction is incorrect and the constant of interaction with leptons for strange currents several times lower than for the nucleonic current.

UNITARY SYMMETRY AND WEAK INTERACTION

We have stressed above that the first possibility seems to be very likely, if the difference between the strong interactions of the nucleon and Λ-hyperon is taken into account. However, if this difference is disregarded, then renormalization effects in strangeness changing leptonic decays should be the same as in strangeness conserving leptonic decays. In the approximation of unitary symmetry (see Chapter 1) the strong interactions of Λ, n and p are strictly the same.

Consequently, within the framework of unitary symmetry the fact that leptonic decays of hyperons and (as will be seen in what follows) K-mesons are suppressed cannot be accounted for by the renormalization effects of strong interaction and requires for its explanation the assumption that the universality of the weak interaction constant should be abandoned (hypothesis of restricted universality).

We shall not discuss here this possibility.‡

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† For experimental data on leptonic decays of hyperons see Table 8 on p. 220 (note added in 1964).

‡ For a detailed consideration of the weak interaction in terms of unitary symmetry see Chapter 20 (note added in 1964).

CHAPTER 10

STRANGENESS CHANGING LEPTONIC DECAYS. K-DECAYS

In the preceding chapter we considered the isotopic properties of the strangeness changing weak interaction, and estimated the probability of hyperon β-decay. We shall try now to find out what data confirm that this interaction is of the form V–A, and what experiments are necessary to obtain the data that are missing.

Basic data on the strangeness conserving interactions were obtained from experiments on the neutron β-decay. However, for the strangeness changing interaction an analogous procedure is not adequate, because the hyperon β-decay is an extremely rare event.

The K-meson decays $K_{\mu 2}$, K_{e3} and $K_{\mu 3}$ are the main source of information on strangeness changing interactions.

$K_{\mu 2}$-DECAY

The $K^+_{\mu 2}$-decay ($K^+ \rightarrow \mu^+ + \nu$) amounts to about 60 per cent of all decays of positive K-mesons. It is analogous to the $\pi_{\mu 2}$-decay ($\pi^+ \rightarrow \mu^+ + \nu$). The $K^+_{\mu 2}$-decay rate is calculated in the same way as the $\pi_{\mu 2}$-decay rate, and is equal to

$$W_{K^+_{\mu 2}} = \frac{G^2 f_K^2}{8\pi} m_\mu^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2,$$

where G is the weak interaction constant, m_μ, m_K are the masses of the muon and kaon, while f_K is an unknown constant due to the strong interaction.

First of all, we find from experiment the ratio of f_K to the corresponding π-meson constant. The ratio of the $K^+_{\mu 2}$ to $\pi^+_{\mu 2}$-decay rate is equal to

$$\frac{W(K^+ \rightarrow \mu^+ + \nu)}{W(\pi^+ \rightarrow \mu^+ + \nu)} = \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K}{m_\pi} \left[\frac{1 - \left(\frac{m_\mu}{m_K}\right)^2}{1 - \left(\frac{m_\mu}{m_\pi}\right)^2}\right]^2 \approx 17{\cdot}5 \left(\frac{f_K}{f_\pi}\right)^2.$$

From experiment $W_{K^+_{\mu 2}} \cong 5 \times 10^7\,\text{sec}^{-1}$, $W_{\pi^+_{\mu 2}} \cong 4 \times 10^7\,\text{sec}^{-1}$ and, consequently, $W_{K^+_{\mu 2}} : W_{\pi^+_{\mu 2}} \approx 1{\cdot}25$, whence it follows that

$$\left(\frac{f_K}{f_\pi}\right)^2 \approx \frac{1}{14}.$$

Thus, f_K^2 is an order of magnitude lower than f_π^2. This is consistent with the fact noted before, that hyperon β-decays are suppressed. The decay of K^+-meson is undoubtedly connected with that of the hyperon. The point is that the decay of K^+-meson can be thought of in the following way (Fig. 21): as a result of the strong interaction, K^+ goes over to $p + \bar{\Lambda}$ (vertex 1), and $p + \bar{\Lambda}$ on account of the weak interaction go over to $l^+ + \nu$ (vertex 2). Therefore if vertex 2 is suppressed, it can be assumed that the entire sequence will be suppressed. Unfortunately, we cannot say anything more definite, because for this a theory of strong interactions would be necessary.

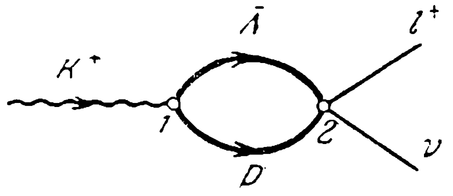

FIG. 21

We know that the current $\bar{\Lambda}p$ is the sum of two currents—an axial and a vector current—like other weak currents. Which of these currents is seen in the $K_{\mu 2}$-decay? The answer to this question depends on the parity of the K-meson. If the parity of the K-meson is odd, then the $K_{\mu 2}$-decay is due to the axial current, and if the parity is even, the decay is due to the vector current. In the Sakata model the parity of the K-meson is defined as the product of the intrinsic parities of the $\bar{\Lambda}$-hyperon and nucleon constituting the K-meson and their orbital parities. We can always choose the parity of the Λ-particle to be equal to that of the nucleon: $P_\Lambda = P_N$. This can be done because in strangeness conserving strong interactions Λ does not go over to nucleon. In weak interactions Λ goes over to nucleon, but in weak interactions parity is not conserved. Therefore the parity of the Λ-particle relative to the nucleon can be chosen in an arbitrary way. When $P_\Lambda = P_N = +1$, $P_K = -1$ if $p + \bar{\Lambda}$ are in a ${}^1S_0$-state, while if $p + \bar{\Lambda}$ are in a ${}^3S_0$-state, then $P_K = +1$.

The parity of the K-meson relative to the Λ-particle and nucleon is most likely to be odd, i.e. the K-meson like the π-meson is a pseudoscalar particle.[†] In this case in the $K_{\mu 2}$-decay an axial interaction "operates". However, all the results obtained in this section would not be changed if the intrinsic parity of the K-meson is even and if the vector interaction is responsible for the $K_{\mu 2}$-decay.

Thus, we shall assume that the axial interaction is responsible for $K_{\mu 2}$-decay. To what degree do available experimental data confirm this? For a K-meson with an odd parity the $K_{\mu 2}$-decay could be due, beside to the axial, only to a pseudoscalar term. (In the case of even parity, one would have to choose between vector and scalar. Tensor interactions cannot lead to the $K_{\mu 2}$-decay, no matter what the parity of the K-meson is.) An indication that the interaction of leptons with the strange current is the same as with the nucleonic current is the fact that μ^+-mesons in the $K_{\mu 2}$-decay have a complete left-handed polarization, like μ^+-mesons in the $\pi_{\mu 2}$-decay.

K_{e2}-DECAY

Perhaps the weightiest argument in favour of the statement that the interaction in the $K_{\mu 2}$-decay is actually an axial is the fact that no K_{e2}-decay has so far been observed. If indeed an axial interaction takes place here, as in the case of π_{e2}- and $\pi_{\mu 2}$-decays, the ratio of the decays $W(K \to e + \nu)/W(K \to \mu + \nu)$ must be proportional to

$$\frac{m_e^2}{m_\mu^2} \frac{\left[1 - \left(\dfrac{m_e}{m_K}\right)^2\right]^2}{\left[1 - \left(\dfrac{m_\mu}{m_K}\right)^2\right]^2} \approx \frac{1}{40{,}000}.$$

For a pseudoscalar interaction this ratio would not contain the factor $\left(\dfrac{m_e}{m_\mu}\right)^2$ and would be of the order of unity. To verify that there is no pseudoscalar in leptonic decays of strange particles, a precise measurement of the K_{e2}-decay rate appears to be fundamental.

The universal scheme is certainly very fine, but there is no complete evidence that it is actually realized in nature and, for the present, it is not checked at a number of points—the above is one of these. Experiments have not yet proved that the interaction of leptons with strange particles is the same form as that with nucleons, although there are indications that this may be so.

[†] At present it is experimentally proved that the K-meson is a pseudoscalar particle (note added in 1964).

Moreover, measurement of the ratio $K_{e2}/K_{\mu 2}$ is important for another reason. Assume that the μ-meson has a certain anomalous interaction with strange particles. Then the ratio $K_{e2}/K_{\mu 2}$ may change. So, measurement of the K_{e2}-decay rate might establish whether or not the μ-meson has any anomalous interactions with strange particles. Such experimental data are for the present very indefinite and give

$$\frac{W(K^+ \rightarrow e^+ + \nu)}{W(K^+ \rightarrow \mu^+ + \nu)} < 0{\cdot}01 .$$

K_{e3}-DECAY

We pass on to a discussion of what is known about the vector interaction in strangeness changing leptonic decays of strange particles. If the axial interaction leads to the leptonic decay of *K*-mesons in which they disintegrate into leptons, the vector interaction produces a decay where the *K*-meson gives rise to a π-meson along with leptons, i.e. the vector interaction should lead to the K_{e3}-decay $(K^+ \rightarrow \pi^0 + e^+ + \nu)$ and $K_{\mu 3}$-decay $(K^+ \rightarrow \pi^0 + \mu^+ + \nu)$ and to analogous decays of neutral *K*-mesons.

In what follows we shall consider in detail the K_{e3}-decay, because it is much easier to consider than the $K_{\mu 3}$-decay. We shall write its matrix element, calculate the decay rate and see that this decay, like the ones considered above, is suppressed in comparison with the corresponding decay of the π-meson $(\pi^+ \rightarrow \pi^0 + e^+ + \nu)$. Then we shall consider the spectra of the secondary particles arising from the K_{e3}-decay.

We begin with the matrix element of the K_{e3}-decay:

$$M = \sqrt{2}\, G g(E_\pi)\, \varphi_\pi \varphi_K p_{K\alpha} \bar{u}_\nu \gamma_\alpha (1 + \gamma_5)\, u_e .$$

We see that this matrix element depends only on one unknown function $g(E_\pi)$. All the rest is known. Such a matrix element is obtained from the following considerations. In general, the matrix element depends on two arbitrary functions f_1 and f_2, contained in the quantity V_α (see p. 72):

$$V_\alpha = f_1 p_{K\alpha} + f_2 p_{\pi\alpha} .$$

Multiplying the 4-vector V_α by the leptonic current $\bar{u}_e\gamma_\alpha(1+\gamma_5)\,u_\nu$ and taking into account that $p_\pi = p_K - q$ and $q_\alpha\bar{u}_e\gamma_\alpha(1+\gamma_5)\,u_\nu = m_e\bar{u}_e(1+\gamma_5)\,u_\nu = 0$ (disregarding the electron mass), we obtain our matrix element. The function g depends on q^2, but, because $q^2 = (p_K - p_\pi)^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi$, we can write g as a function of E_π instead of q^2. (It is easy to see that $2g = f_1 + f_2$.)

With the matrix element, we can now calculate in a standard way any characteristic of the K_{e3}-decay that interests us. The K_{e3}-decay probability is equal to

$$dW = \frac{(2\pi)^4}{(2\pi)^9}\int \frac{d\mathbf{p}_e}{2E_e}\,\frac{d\mathbf{p}_\nu}{2E_\nu}\,\frac{d\mathbf{p}_\pi}{2E_\pi}\,\frac{\overline{|M|^2}}{2E_K}\,\delta^4(p_K - p_\pi - p_e - p_\nu),$$

where the bar denotes summing over the spin states of the leptons.

$$\begin{aligned}\overline{|M|^2} &= 2G^2g^2p_{K\alpha}p_{K\beta}\,\mathrm{Sp}\,\hat{p}_e\gamma_\alpha(1+\gamma_5)\,\hat{p}_\nu\gamma_\beta(1+\gamma_5)\\ &= 2G^2g^2p_{K\alpha}p_{K\beta}\,.\,2\,.\,\mathrm{Sp}\,\hat{p}_e\gamma_\alpha\hat{p}_\nu\gamma_\beta(1+\gamma_5)\\ &= 4G^2g^2p_{K\alpha}p_{K\beta}p_{e\varrho}p_{\nu\tau}\,\mathrm{Sp}\,\gamma_\varrho\gamma_\alpha\gamma_\tau\gamma_\beta\\ &= 16G^2g^2p_{K\alpha}p_{K\beta}p_{e\varrho}p_{\nu\tau}(\delta_{\varrho\alpha}\delta_{\tau\beta} + \delta_{\alpha\tau}\delta_{\beta\varrho} - \delta_{\varrho\tau}\delta_{\alpha\beta})\\ &= 16G^2g^2p_{e\varrho}p_{\nu\tau}(2p_{K\varrho}p_{K\tau} - p_K^2\delta_{\varrho\tau})\\ &= 16G^2g^2p_{e\alpha}p_{\nu\beta}(2p_{K\alpha}p_{K\beta} - p_K^2\delta_{\alpha\beta}).\end{aligned}$$

PION SPECTRUM

We calculate first the π-meson spectrum. For this it is necessary to integrate with respect to the momenta of e and ν:

$$\int p_{e\alpha}p_{\nu\beta}\frac{d\mathbf{p}_e\,d\mathbf{p}_\nu}{E_eE_\nu}\delta^4(p_e + p_\nu - q), \quad \text{where} \quad q = p_K - p_\pi.$$

We calculated this integral in considering the muon decay (p. 62):

$$\int p_{e\alpha}p_{\nu\beta}\frac{d\mathbf{p}_e\,d\mathbf{p}_\nu}{E_eE_\nu}\delta^4(p_e + p_\nu - q) = \frac{\pi}{6}(2q_\alpha q_\beta + q^2\delta_{\alpha\beta}).$$

If we employ this integral and take into account that the integration over the angles of the π-meson gives 4π, then we can write

$$dW = \frac{G^2g^2 4\pi^2p^2\,dp\,[2p_{K\alpha}p_{K\beta} - p_K^2\delta_{\alpha\beta}]\,[2q_\alpha q_\beta + q^2\delta_{\alpha\beta}]}{(2\pi)^5E_\pi E_K 6},$$

where p denotes the absolute value of the π-meson 3-momentum: $p = |\mathbf{p}_\pi|$.

Multiply the expressions in brackets:

$$[2p_{K\alpha}p_{K\beta} - p_K^2\delta_{\alpha\beta}]\,[2q_\alpha q_\beta + q^2\delta_{\alpha\beta}] = 4[(p_K q)^2 - p_K^2 q^2]$$
$$= 4[(p_K, p_K - p_\pi)^2 - p_K^2(p_K - p_\pi)^2]$$
$$= 4[(p_K p_\pi)^2 - p_K^2 p_\pi^2] = 4(m_K^2 E_\pi^2 - m_K^2 m_\pi^2) = 4m_K^2 p^2.$$

If we substitute this into the expression for the decay rate and take advantage of the fact that $p\,dp = E_\pi\,dE_\pi$, then we obtain

$$dW = \frac{m_K G^2 g^2 p^3\,dE_\pi}{12\pi^3}.$$

Note that in the expression for the π-meson spectrum there is the unknown function $g(E_\pi)$.

We can assume that the function $g(E_\pi)$ does not vary strongly for the π-meson energy range involved in the K_{e3}-decay. The presence of the function g is associated with virtual strong interactions, for example with loops of the type shown in Fig. 22.

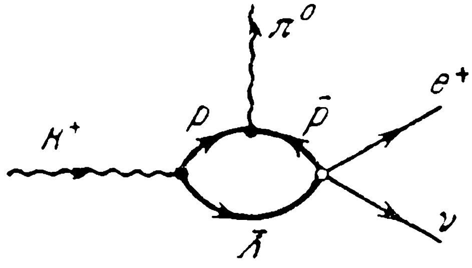

FIG. 22

Virtual baryons in this loop have masses substantially larger than the energy of the π-meson. Hence it can be assumed that the integral with respect to the momenta of these particles will not depend very strongly on the energy of the emitted π-meson.

If the function g varies slightly, then the spectrum of π^0-mesons may be used for finding out if in the K_{e3}-decay there is a vector coupling and not a scalar or a tensor. Both these types give spectra which have a peak and decrease as the maximum π-meson energy is approached.

TOTAL K_{e3}-DECAY PROBABILITY

If it is assumed that $g(E_\pi)$ is a constant, then the total K_{e3}-decay rate can be calculated. We calculate it disregarding the π-meson mass. Then the integral is trivial and the total probability is equal to

$$W = \frac{G^2 g^2 m_K^5}{768\pi^3}.$$

Including the π-meson mass leads to an additional suppression factor of 0·6, and the K_{e3}-decay rate reads

$$W = \frac{G^2 g^2 m_K^5}{768\pi^3} \,.\, 0{\cdot}6.$$

Having this expression, we can calculate the value of g^2, making use of the experimental K_{e3}-decay rate ($W_{K_{e3}} \approx 3{\cdot}4 \times 10^6 \text{ sec}^{-1}$). It turns out that $g^2 \approx 2{\cdot}5 \times 10^{-2}$. In the π_{e3}-decay ($\pi^+ \to \pi^0 + e^+ + \nu$) the corresponding coefficient equals two. Thus, the K_{e3}-decay is suppressed at least by an order of magnitude in comaprison with strangeness conserving decays.†

ELECTRON SPECTRUM

Consider the electron spectrum in the K_{e3}^+-decay. To find it, we have to integrate, the differential probability with respect to the neutrino and pion momenta

$$dW = \frac{G^2 g^2 (2p_{K\alpha} p_{K\beta} - p_K^2 \delta_{\alpha\beta}) p_{e\alpha} p_{\nu\beta}}{(2\pi)^5 E_K} \, \frac{d\mathbf{p}_e \, d\mathbf{p}_\nu \, d\mathbf{p}_\pi}{E_e E_\nu E_\pi}$$
$$\times \, \delta^4(p_\pi + p_e + p_\nu - p_K).$$

We study the integral

$$\int p_{\nu\beta} \frac{d\mathbf{p}_\nu \, d\mathbf{p}_\pi}{E_\nu E_\pi} \delta^4(p_\nu + p_\pi - r), \quad \text{where} \quad r = p_K - p_e.$$

This integral is a vector and can depend only on r, hence

$$\int \frac{d\mathbf{p}_\nu \, d\mathbf{p}_\pi}{E_\nu E_\pi} p_{\nu\beta} \delta^4(p_\nu + p_\pi - r) = A r_\beta.$$

Now we have to find the scalar quantity A. We multiply the left-hand and right-hand part of the equality by r_β and, taking into account $2rp_\nu = r^2 - m_\pi^2$, we obtain

$$A = \frac{r^2 - m_\pi^2}{2r^2} \int \frac{d\mathbf{p}_\nu \, d\mathbf{p}_\pi}{E_\nu E_\pi} \delta^4(p_\nu + p_\pi - r).$$

Calculation of the remaining integral is extremely simple to carry out in the system in which $\mathbf{r} = 0$:

$$\int \frac{d\mathbf{p}_\nu \, d\mathbf{p}_\pi}{E_\nu E_\pi} \delta^4(p_\nu + p_\pi - r) = \int \frac{d\mathbf{p}_\nu}{E_\nu E_\pi} \delta(E_\nu + E_\pi - r_4)$$
$$= 4\pi \frac{p_\nu r}{r^2} = 2\pi \frac{r^2 - m_\pi^2}{r^2}.$$

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† If the unitary symmetry (in the sense we have been using it) of strong interactions and the universality of weak interaction operated, then g^2 would be equal to $\frac{1}{2}$.

As a result we obtain

$$\int \frac{dp_\nu\, dp_\pi}{E_\nu E_\pi} \delta^4(p_\nu + p_\pi - r)\, p_{\nu\beta} = \pi \frac{(r^2 - m_\pi^2)^2}{r^4} r_\beta .$$

If this expression is substituted into the formula for the differential probability, then

$$dW = \frac{G^2 g^2}{(2\pi)^5 E_K} (2p_{K\alpha} p_{K\beta} - p_K^2 \delta_{\alpha\beta})\, p_{e\alpha} \frac{\pi(r^2 - m_\pi^2)^2}{r^4} r_\beta \frac{dp_e}{E_e} .$$

Making use of $r = p_K - p_e$ and passing to a system in which the disintegrating K^+-meson is at rest ($E_K = m_K$), we find

$$r^2 = m_K^2 - 2m_K E_e = 2m_K \left(\frac{m_K}{2} - E_e \right),$$

$$r^2 - m_\pi^2 = 2m_K(w_e - E_e),$$

where $w_e = \frac{m_K^2 - m_\pi^2}{2m_K}$ is the maximum energy of the decay electron. Taking into account

$$\begin{aligned}(2p_{K\alpha} p_{K\beta} - p_K^2 \delta_{\alpha\beta})\, p_{e\alpha} r_\beta &= 2(p_K p_e)(p_K r) - p_K^2 (p_e r) \\ &= m_K^2 E_e (m_K - 2E_e),\end{aligned}$$

we finally obtain the following expression for the electron spectrum in the K_{e3}-decay:

$$dW = \frac{G^2 g^2 m_K}{4\pi^3} \frac{(w_e - E_e)^2}{\left(\frac{m_K}{2} - E_e \right)} E_e^2\, dE_e .$$

In order to get an approximate idea of the character of this spectrum, we may disregard the mass of the π-meson in comparison with that of the K-meson. Then

$$w_e = \frac{m_K^2 - m_\pi^2}{2m_K} \approx \frac{m_K}{2} \quad \text{and} \quad dW = \frac{G^2 g^2 m_K}{4\pi^3} (w_e - E_e)\, E_e^2\, dE_e .$$

It is easy to find the maximum of this distribution. It occurs at

$$E_e = \tfrac{2}{3} w_e .$$

Thus, it is seen that the electron spectrum must have its maximum in the high energy region.

NEUTRINO SPECTRUM

Since in the calculations we disregarded the electron mass, the neutrino spectrum will be completely equivalent to the electron spectrum.

In the K_{e3}-decay of charged K-mesons it is difficult to measure the neutrino spectrum. In the K_{e3}-decay of neutral K-mesons the neutrino spectrum may be easier to measure than the spectrum of π-mesons or electrons. This comes about because in the K_{e3}-decay of neutral K-mesons we can easily find the energy of the ν in the rest system of the K^0-meson by measuring the $\pi^\pm$ and $e^\mp$ momenta. As a matter of fact,

$$(p_K - p_\nu)^2 = (p_\pi + p_e)^2.$$

Considering the left-hand side in the rest system of the K^0-meson, and the right-hand side in the laboratory coordinate system, we find

$$m_K^2 - 2m_K E_\nu^0 = m_\pi^2 + 2E_\pi E_e - 2p_\pi p_e.$$

Thus, knowing the momentum distribution of $\pi^\pm$ and $e^\mp$ in the laboratory coordinate system, one can construct the neutrino spectrum in the K_{e3}^0-decay in the rest system of the K-meson.

If the $\Delta T = \frac{1}{2}$ rule, considered above, is valid, then the neutrino spectrum in the K_{e3}^0-decay should coincide with that in the K_{e3}^+-decay (and, consequently, also with the electron spectrum, because we disregard the electron mass). This statement does not depend on the possible form of the function $g(E_\pi)$ and hence it would be of great interest to check it.

CHAPTER 11

STRANGENESS CHANGING LEPTONIC DECAYS. K_{e3}- AND $K_{\mu 3}$-DECAYS *(continued)*

THE characteristics of the K_{e3}-decay, considered in the preceding chapter, depend essentially on the form of the function $g(E_\pi)$ which contributes a multiplicative factor in the expression for the matrix element:

$$M = \sqrt{2}\, G g \varphi_K \varphi_\pi p_{K\alpha} \bar{u}_\nu \gamma_\alpha (1 + \gamma_5)\, u_e .$$

The function g is due to virtual strong interactions. We calculated the electron and pion spectra assuming the function $g(E_\pi)$ to be a constant.

In order to find out experimentally whether the K_{e3}-decay actually results from a vector interaction, it is necessary to have certain criteria which do not depend on assumptions (even if they are plausible) about the form of the function $g(E_\pi)$. Therefore it is interesting to investigate those characteristics of the K_{e3}-decay which do not depend on the form of the function $g(E_\pi)$. There are several such characteristics. The first of these is the polarization of electrons in the K_{e3}-decay.

ELECTRON POLARIZATION

Since we disregard the electron mass, and the electron and neutrino enter the leptonic bracket in a symmetric way, we can take both of them to be two-component particles. The two-component nature of a particle with $m = 0$ is expressed mathematically by multiplying its four-component Dirac wave function by the projection operator $(1 + \gamma_5)$. In this case the particle (electron) must be polarized in the same way as the neutrino, i.e. must have a left-handed polarization, while the antiparticle (positron) must be polarized like the antineutrino, i.e. must have a right-handed polarization.

The polarization of electrons in the K_{e3}-decay is analogous to that in the neutron β-decay $(-v/c)$, but electrons in the K_{e3}-decay have $v \cong c$, hence their polarization is practically equal to -1.

There are very rare cases in which the electron polarization in the K_{e3}-decay is not equal to -1. These are when the π-meson carries away no energy, while the electron and antineutrino go off in opposite directions. The picture in these cases reminds one of the K_{e2}- and π_{e2}-decays where the electrons have a "forced" (right-handed) polarization determined by the antineutrino longitudinal polarization and by the angular momentum conservation law. These cases are rather unlikely; they would be strictly forbidden if the electron mass were equal to zero.

Thus, electrons in the K_{e3}^{-}-decay should basically have a left-handed polarization, while positrons in the K_{e3}^{+}-decay should have a right-handed one. Experimental observation of K_{e3}^{-}-decays is made difficult by the fact that K^{-}-mesons are mainly captured and do not decay, so that in reality one has to do here with measurement of the positron polarization in the K_{e3}^{+}-decay. This experiment is of great interest in connection with the fact that the two other interaction types—the scalar (S) and the tensor (T)—which might be able to cause the K_{e3}-decay, would give positrons with a left-handed polarization (under the condition that the neutrino in the K_{e3}^{+}-decay has, as before, a left-handed polarization).

ELECTRON SPECTRUM AT GIVEN PION ENERGY

Another characteristic of the K_{e3}-decay, which does not depend on any assumptions about the function g, is the electron spectrum at a fixed π-meson energy. Since the function g depends only on E_{π}, by fixing E_{π} we pick a single value of the function g. Therefore in the expression for the electron spectrum at a fixed π-meson energy the quantity g is in the form of an irrelevant common factor.

In Chapter 10, the formula for the differential probability of the K_{e3}-decay was obtained:

$$dW = \frac{G^2 g^2}{(2\pi)^5 m_K} [2(p_e p_K)(p_\nu p_K) - (p_e p_\nu) p_K^2] \frac{dp_e \, dp_\pi \, dp_\nu}{E_e E_\pi E_\nu} \times$$
$$\times \, \delta^4(p_e + p_\nu + p_\pi - p_K).$$

We integrate over the neutrino phase space. Then the expression for the probability will become

$$dW = \frac{G^2 g^2}{(2\pi)^5 m_K}[2E_e E_\nu m_K^2 - m_K^2(E_e E_\nu - p_e p_\nu)]\frac{dp_e\, dp_\pi}{E_e E_\pi E_\nu} \times$$
$$\times\, \delta(E_e + E_\pi + E_\nu - m_K) = \frac{G^2 g^2}{(2\pi)^5} m_K [E_e E_\nu + p_e p_\nu]\frac{dp_e\, dp_\pi}{E_e E_\pi E_\nu} \times$$
$$\times\, \delta(E_e + E_\nu + E_\pi - m_K).$$

The scalar product $p_e p_\nu$ in this expression is easy to express in terms of E_e and E_π. As a matter of fact, in the rest system of the K-meson

$$p_e + p_\nu = -p_\pi.$$

Squaring this:

$$p_e^2 + p_\nu^2 + 2p_e p_\nu = p_\pi^2,$$

whence

$$p_e p_\nu = \frac{p_\pi^2 - p_e^2 - p_\nu^2}{2},$$

where

$$p_e^2 = E_e^2, \quad p_\pi^2 = E_\pi^2 - m_\pi^2,$$
$$p_\nu^2 = E_\nu^2 = (m_K - E_\pi - E_e)^2.$$

From this it is easy to obtain

$$E_e E_\nu + p_e p_\nu = \tfrac{1}{2}[p_\pi^2 - (E_\nu - E_e)^2] = \tfrac{1}{2}[p_\pi^2 - (m_K - E_\pi - 2E_e)^2].$$

Integrate now over the angles of the electron and π-meson:

$$\int \frac{dp_e dp_\pi}{E_e E_\pi E_\nu}\delta[E_e + E_\pi + \sqrt{(p_e^2 + p_\pi^2 + 2p_e p_\pi)} - m_K]$$
$$= \frac{8\pi^2 |p_e|^2\, |p_\pi|^2\, d|p_e|\, d|p_\pi|}{E_e E_\pi E_\nu \dfrac{|p_e|\,|p_\pi|}{E_\nu}} = 8\pi^2 dE_e dE_\pi.$$

Finally we obtain

$$dW = \frac{G^2 g^2 m_K}{8\pi^3}[p_\pi^2 - (m_K - E_\pi - 2E_e)^2]\, dE_\pi dE_e.$$

The energy of the electron at a fixed energy of the π-meson varies within the range

$$\frac{m_K - E_\pi - |p_\pi|}{2} = E_e^{\min} \leqq E_e \leqq E_e^{\max} = \frac{m_K - E_\pi + |p_\pi|}{2}.$$

The electron spectrum becomes zero at $E_e = E_e^{\min}$ as well as at $E_e = E_e^{\max}$. It has a peak at $E_e = \frac{m_K - m_\pi}{2}$ (Fig. 23).

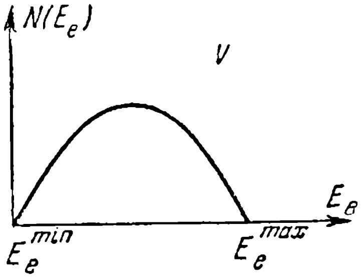

FIG. 23

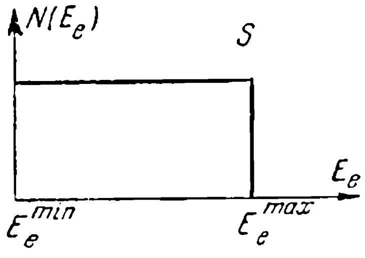

FIG. 24

For the scalar coupling the electron spectrum would have the shape shown in Fig. 24, while for the tensor it would have the shape given in Fig. 25. Interference of the scalar and tensor gives an asymmetric distribution (Fig. 26).

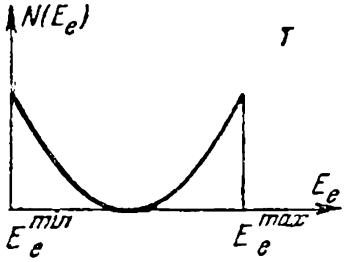

FIG. 25

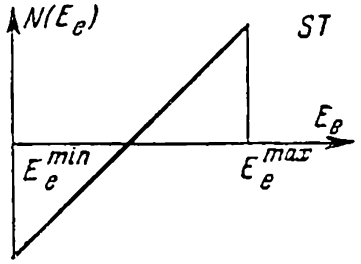

FIG. 26

Thus, a measurement of the electron spectrum at a fixed pion energy would allow the interaction in the K_{e3}-decay to be determined quite unambiguously.

DALITZ PLOT FOR K_{e3}-DECAY

However, because one has to select events with a given pion energy, it is difficult to get enough of them. It is natural to ask: can any use be made of decays with different (but known!) energies of the π-mesons, treating the data in such a way that the results do not depend on the form of the function $g(E_\pi)$? This problem was considered in detail by Kobzarev.

The energy distribution of the products of three-body decays are conveniently presented on a diagram of the type of the Dalitz plot (see p. 153), representing an equilateral triangle. In the equilateral triangle the sum of the distances from an arbitrary point A inside the triangle to its legs is a constant equal to the height of the triangle. It can easily be seen that this statement is valid, if one connects the point A with the vertices of the triangle (Fig. 27)

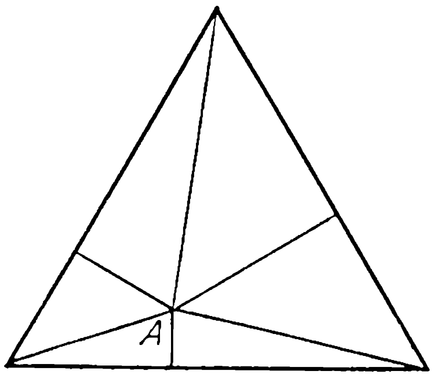

FIG. 27

and notes that the sum of the areas of the three formed triangles is equal to the area of the initial triangle.

If one constructs a triangle the height of which (in a certain scale) is equal to the energy released in a three-body decay, then to any given configuration of the decay products there will correspond a point inside the triangle, the distances from this point to the

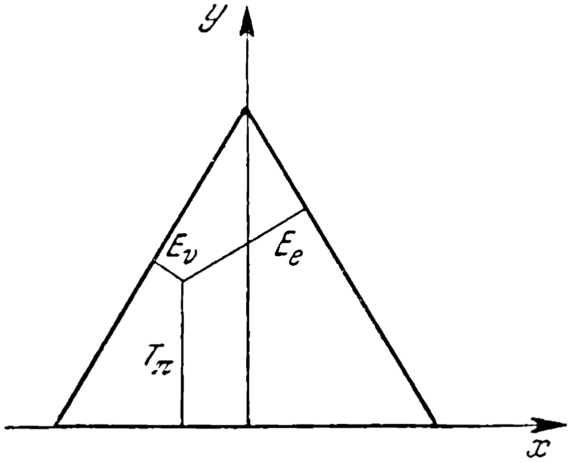

FIG. 28

legs of the triangle being equal to the energies of the particles arising from the decay. The energy conservation law is in this case taken into account automatically. If there were only the energy conservation law and no momentum conservation law, then, in observing K_{e3}-decays, we would fill in by points the entire triangle.

The momentum conservation law singles out a certain region inside the triangle.

For the K_{e3}-decay the corresponding diagram has the form shown in Fig. 28. Here $T_\pi = E_\pi - m_\pi$ is the kinetic energy of the π-meson.

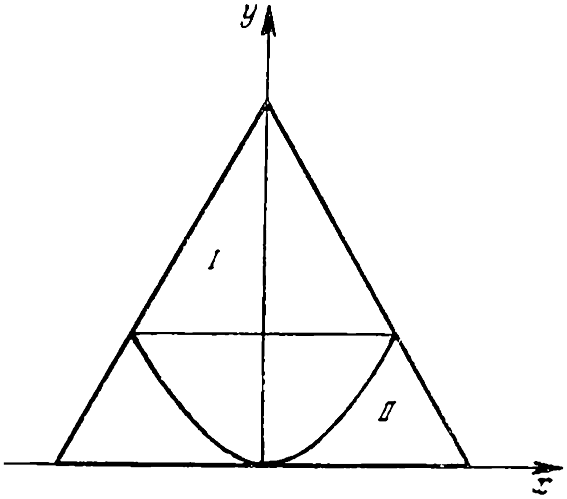

FIG. 29

The height of the triangle equals $L = m_K - m_\pi$, while $y = T_\pi$, $x = \dfrac{E_\nu - E_e}{\sqrt{3}}$.

Draw in this diagram the region allowed by the momentum conservation law (Fig. 29). This region is bounded by lines I and II.

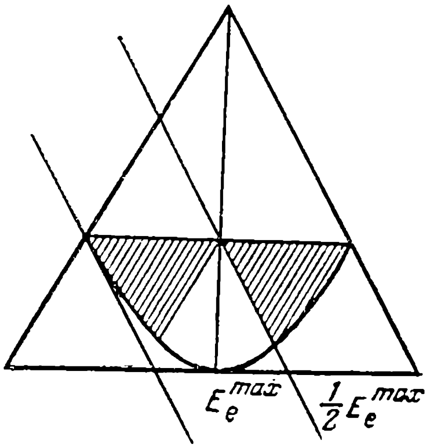

FIG. 30

Line I corresponds to the maximum energy of the π-meson $y = y_{\max} = \dfrac{(m_K - m_\pi)^2}{2m_K}$, while line II corresponds to $3x^2 = y^2 + 2m_\pi y$ (see p. 154).

Examining the diagram it is easy to see that the electron spectrum, for any form of the function $g(E_\pi)$, should have the following property: the total number of electrons with an energy in the range $0 \leqq E_e \leqq \frac{1}{2}E_e^{\max}$ must be lower than that of electrons with an energy in the range $\frac{1}{2}E_e^{\max} \leqq E_e \leqq E_e^{\max}$. This conclusion follows from the symmetry of the diagram (Fig. 30) with respect to the vertical axis (the identity of the neutrino and electron spectrum). It should be noted that such a property is possessed not only by the vector coupling but also by the pure scalar or tensor variant.

From this it is seen that the symmetry is violated only by the interference of the scalar and tensor.

"SLIDING RAY" DIAGRAM

If the diagram considered is somewhat modified by plotting on the vertical axis not the kinetic energy of the π-meson $E_{\pi\text{kin}}$ but its momentum $|p_\pi|$, $y' = |p_\pi|$, and on the horizontal axis $x' = |E_\nu - E_e|$, then the allowed region assumes a simpler form (Fig. 31).

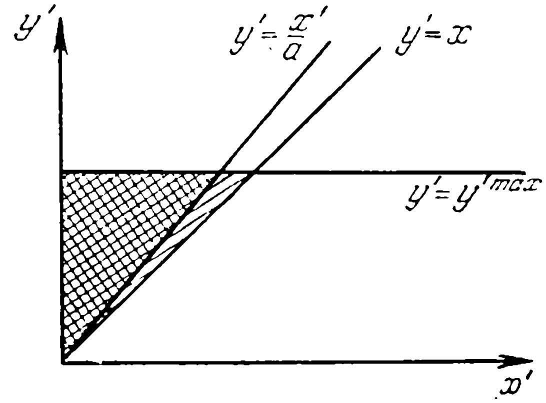

FIG. 31

It is easy to see that the energy distribution

$$dW \sim [p_\pi^2 - (E_e - E_\nu)^2]\, dE_\pi\, dE_e,$$

which we obtained earlier, will in the new variables assume the form

$$dW \sim [y'^2 - x'^2]\, dx'\, dy'.$$

Let us now draw in our diagram the ray $x' = ay'$, where $0 \leqq a \leqq 1$. At $a = 0$ and $a = 1$ this ray coincides with the boundaries of the allowed region. We calculate the ratio of the number of points to the left of the ray to the total number of points. This ratio is equal to

$$R = \frac{3}{2}\left(a - \frac{a^3}{3}\right).$$

The result does not depend on the form of the function $g(E_\pi)$. For the scalar coupling it would be equal to $R_S = a$, while for the tensor it would amount to $R_T = a^3$.

From the aforesaid it follows that, if the distribution $W(a)\,da$ is constructed, where

$$a = \frac{E_\nu - E_e}{p_\pi},$$

then this distribution will have the form

$$W(a)\,da = \tfrac{3}{2}(1 - a^2)\,da \quad \text{for} \quad V,$$
$$W(a)\,da = da \quad \text{for} \quad S,$$
$$W(a)\,da = 3a^2 da \quad \text{for} \quad T.$$

(We choose the normalization $\int\limits_0^1 W(a)\,da = 1$.) This distribution will depend on the sign of a only in the case where there is interference of S- and T-couplings.

$K_{\mu 3}$-DECAY

As distinct from the K_{e3}-decay, which depends only on one unknown function, in the $K_{\mu 3}$-decay there are two such functions (see p. 72):

$$M = \sqrt{(2)}\, G\varphi_K\varphi_\pi [g p_{K\alpha} - f q_\alpha]\, \bar{u}_\nu \gamma_\alpha (1 + \gamma_5)\, u_\mu .$$

Here q is the sum of the 4-momenta of the leptons. Making use of the Dirac equation, the term proportional to f is easy to bring into the form of a "scalar interaction"

$$-f m_\mu \bar{u}_\nu (1 - \gamma_5)\, u_\mu .$$

In the case of K_{e3}-decay this term was proportional to m_e, and was disregarded. In the case of $K_{\mu 3}$ it cannot be disregarded, because m_μ is not small. It is very important to stress that, by virtue of the universality of the weak interaction, the functions f and g, contained in the expression for the matrix element of the $K_{\mu 3}$-decay, must be equal identically to those contained in the expression for the matrix element of the K_{e3}-decay. The fact that all characteristics of the $K_{\mu 3}$-decay (π-meson spectrum, spectrum and polarization of muons, and so on) depend essentially on two unknown functions f and g, makes the interpretation of experimental data more difficult than in the case of the K_{e3}-decay. Assuming the functions f and g to be constant and carrying out a standard (but rather lengthy) calculation, one can compute the total probability of the $K_{\mu 3}$-decay. It turns out to be

$$W_{\mu 3} \cong \frac{G^2 m_K^5}{768\pi^3}[0{\cdot}5g^2 - 0{\cdot}2fg + 0{\cdot}05f^2].$$

As shown on p. 131, the probability of the K_{e3}-decay is equal to

$$W_{e3} \cong \frac{G^2 m_K^5 0{\cdot}6g^2}{768\pi^3}.$$

(It should be stressed once more that in deriving these formulae we disregarded the dependence of the functions f and g on the energy of the π-meson.) It is known from experiment that the probabilities of the $K_{\mu 3}^+$- and K_{e3}^+-decays are about the same.

Equating the probabilities that we calculated, we can find the ratio f/g:

$$0{\cdot}6g^2 \sim 0{\cdot}5g^2 - 0{\cdot}2fg + 0{\cdot}05f^2.$$

This equation has two solutions:

$$\frac{f}{g} \sim 4{\cdot}5, \quad \frac{f}{g} \sim -0{\cdot}5.$$

The muon energy spectrum is particularly sensitive to the magnitude of f/g at low energies. For muon kinetic energies up to 30 MeV, the spectra corresponding to the two possible values of f/g differ by a factor of about 3. At large energies these spectra are approximately the same. The muon polarization is even more sensitive to the magnitude of f/g.

The solution $f/g \sim 4{\cdot}5$ corresponds to a predominant scalar interaction, while the solution $f/g \sim -0{\cdot}5$ corresponds to predominant vector interaction. If the vector interaction is predominant, then the muons must have predominently a left-handed polarization (like electrons in the K_{e3}-decay and neutron β-decay). If the scalar interaction predominates, muons must have a right-handed polarization.† Thus, determining the sign of the muon polarization (which is easy to do by measuring the asymmetry of the muon decay) one can establish the value of the parameter f/g.

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† This refers to muons in the $K_{\mu 3}^-$-decay. In the $K_{\mu 3}^+$-decay, muons have a right-handed polarization for the vector interaction, and left-handed one for the scalar interaction.

The tentative values of f/g given above may get changed rather drastically if the probabilities of the K_{e3}- and $K_{\mu 3}$-decays, which are at present known only with rather large uncertainties, get defined more precisely. Thus, if it should turn out that $W_{K_{e3}} \sim 1{\cdot}5\, W_{K_{\mu 3}}$, then $f/g \sim \frac{1}{2}$.

It should be noted that if f were equal to $\frac{1}{2}g$, the matrix element of the K_{e3}- and $K_{\mu 3}$-decays would be proportional to $p_K + p_\pi$ and would be analogous to the matrix element of the π_{e3}-decay, which is proportional to $p_{\pi^+} + p_{\pi^0}$. Such an analogy should hold if the strong interaction of the Λ-hyperon and nucleon were the same (unitary symmetry).†

Concluding our consideration of strange-particle leptonic decays, we stress that these decays are a rare case in which there are a number of strict theoretical predictions that may be verified experimentally in a relatively straightforward way. Therefore their experimental investigation is one of the most important tasks in the physics of elementary particles.

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† In considering K_{e3}- and $K_{\mu 3}$-decays in terms of unitary symmetry it is convenient to write V_α in the form

$$V_\alpha = f_+ p_\alpha + f_- q_\alpha = f_+ (p_\alpha + \xi q_\alpha),$$

where

$$p = p_K + p_\pi, \quad q = p_K - p_\pi, \quad \xi = f_-/f_+.$$

It is easy to see that $g = f_+$, $f/g = \dfrac{1-\xi}{2}$, and, consequently, that to the value $f/g \sim \frac{1}{2}$ there corresponds $\xi \sim 0$. Experimental data on K_{e3}- and $K_{\mu 3}$-decays are in agreement with $\xi \sim 0$; see p. 256 (note added in 1964).

CHAPTER 12

NON-LEPTONIC DECAYS OF STRANGE PARTICLES. QUALITATIVE CONSIDERATION. θ- AND τ-DECAYS

ALL processes that we have so far considered involve leptons. We first considered the interaction between leptons, and then that of leptons with the nucleonic current. Finally, we studied the interaction of leptons with the strange current.

INTERACTION OF THE $\bar{n}p$ AND $\bar{\Lambda}p$ CURRENTS

According to the scheme of universal weak interaction the nucleonic and strange current should interact not only with leptons but also with each other. We will now try to find out what the interaction of the currents $\bar{n}p$ and $\bar{\Lambda}p$ with each other will give.

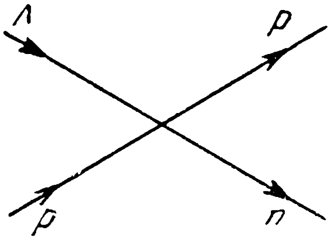


FIG. 32

The interaction of these currents can be presented graphically (Fig. 32).

Here Λ turns into a proton, whereas the proton colliding with the Λ becomes a neutron. The two upper lines correspond to the transition $\Lambda \to p$, while the two lower ones correspond to the transition $p \to n$. The interaction of the currents $\bar{n}p$ and $\bar{\Lambda}p$ has the form

$$(\bar{n}p)(\bar{\Lambda}p)^+ + (\bar{\Lambda}p)(\bar{n}p)^+ = (\bar{n}p)(\bar{p}\Lambda) + (\bar{\Lambda}p)(\bar{p}n).$$

The graph shown above corresponds to the first term. However, along with the scattering processes the same interaction gives rise also to other processes. We have taken advantage repeatedly of the fact that the particle annihilation is equivalent to the antiparticle creation. If there is the process $\Lambda + p \rightarrow n + p$, then there must also be the process $\Lambda \rightarrow \tilde{p} + n + p$.

Thus, we have an interaction which transforms Λ into the proton, antiproton and neutron.

Λ-HYPERON DECAY

How, by means of this interaction, do the ordinary Λ-hyperon mesonic decays $\Lambda^0 \rightarrow p + \pi^-$ and $\Lambda^0 \rightarrow n + \pi^0$ take place? There are many ways for this to happen. For example, assume the proton and antiproton, interacting strongly with each other, become a π^0-meson. Then we shall have the graph shown in Fig. 33.

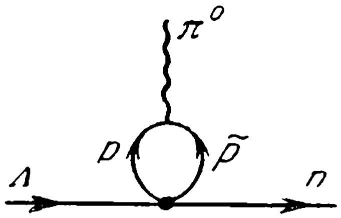


FIG. 33

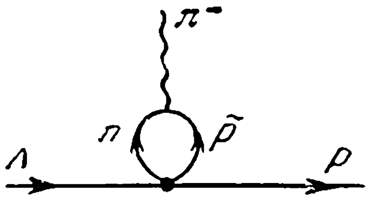


FIG. 34

One can assume the antiproton to be combined not with a proton but with a neutron; then we have a new graph (Fig. 34).

Among the simplest diagrams are those in which the proton and antiproton annihilate without forming any π-meson and where the nucleon loop is closed (Fig. 35).

As a result the Λ becomes a virtual neutron. (It is easy to see that the transition of the Λ-particle to a real neutron contradicts the energy-momentum conservation law: $p_\Lambda^2 = m_\Lambda^2 \neq p_n^2 = m_n^2$.) Since the isotopic spin of the Λ is zero, while that of the neutron is $\frac{1}{2}$, the change in the isotopic spin for such a diagram is equal to $\Delta T = \frac{1}{2}$.

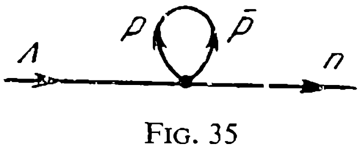


FIG. 35

We shall return to this later on. As a result of the strong interaction, the virtual neutron can emit a π-meson. If a neutron emits a π^0-meson it remains a neutron, while if it emits a π^--meson it becomes a proton (Fig. 36).

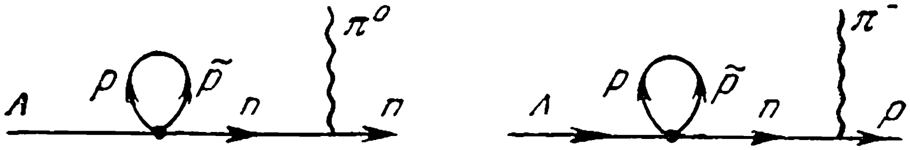


FIG. 36

The diagrams we have considered are the simplest ones. As a matter of fact, all particles drawn in these diagrams are strongly interacting and undergo many interactions with one another. In this case the simplest diagram will get swamped by an infinity of lines of virtual particles. Since we do not have a theory of strong interactions, we cannot calculate the result of this superposition of an infinite number of diagrams.

Proceeding from the simplest matrix element describing the transformation of the Λ-particle into the proton, neutron and antiproton, we cannot find an exact matrix element describing the real processes of the decay of K-mesons and hyperons. Therefore we take on a more limited task. We shall try to determine in our basic underlying interaction those symmetry properties that still remain conserved in the matrix elements, and shall see what properties of the matrix elements are determined by these symmetries.

Thus, the interaction we study leads to Λ-decay, but we cannot use it to obtain quantitative results.

Σ- AND Ξ-HYPERON DECAYS

In an analogous way it can be seen that there must arise nonleptonic decays of other particles. As a matter of fact, the Σ-hyperon as a result of the strong interaction may transform into a π-meson

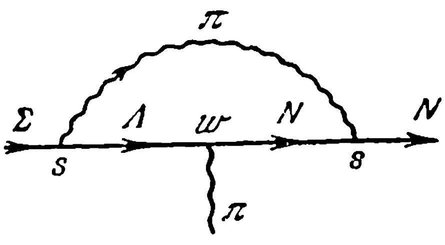


FIG. 37

and a Λ. As for the Λ, we already know that it may transform into a π-meson and a nucleon. If now one of the π-mesons gets absorbed, for example, in the way shown in the diagram (Fig. 37), then a π-meson and a nucleon will remain.

The weak interaction (w) "operated" only at the spot where the Λ decayed, while all other virtual interactions are strong (s), hence the decay of Σ-hyperons must have about the same probability as that of Λ-hyperons.

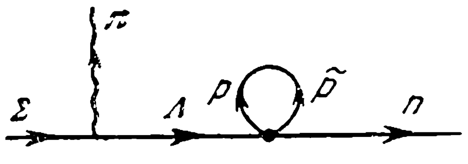


FIG. 38

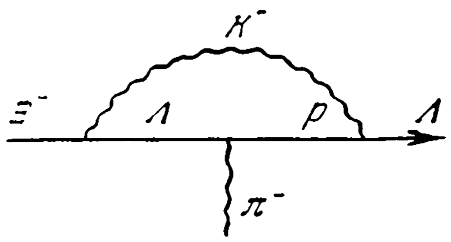


FIG. 39

Another possible diagram for the Σ-hyperon decay is shown in Fig. 38.

Again, as in the case of the Λ-hyperon decay, the number of different diagrams corresponding to the virtual strong interaction is infinite, and we do not know which of these are essential and which are not.

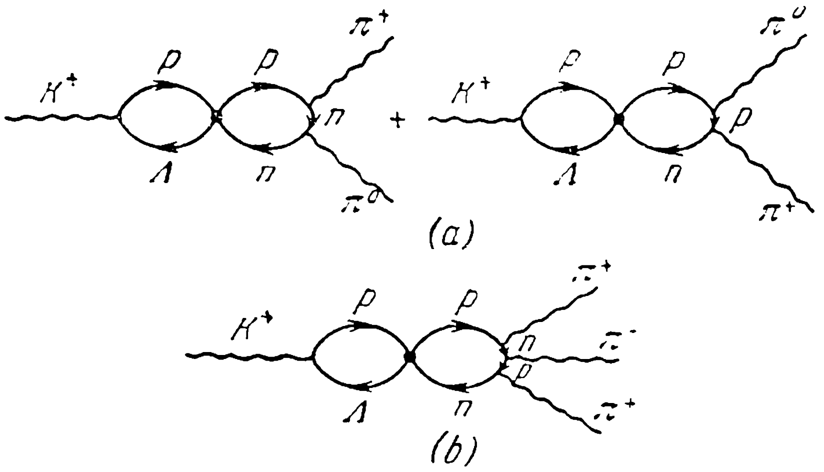
The decay of the cascade hyperon can easily be obtained in an analogous way (Fig. 39).

K-DECAYS

The four-fermion interaction $(\tilde{n}p)\,(\tilde{p}\Lambda)$ is sufficient to give rise also to non-leptonic decays of K-mesons. For example, as a result of a strong interaction, K^+ may go over into a proton and a $\tilde{\Lambda}$; thereupon in consequence of the weak interaction the proton and $\tilde{\Lambda}$ may go over into a proton and an antineutron. The proton and antineutron may annihilate into π-mesons.

The annihilation into one π-meson is forbidden by the energy-momentum conservation law, which also forbids the decay into four or more π-mesons. The transition into two and three-pion states is allowed and leads to the observable $K_{\pi 2}$- and $K_{\pi 3}$-decays respectively (Fig. 40).

These decays are called, respectively, θ and τ decays. Their analysis led by the middle of the fifties to posing the problem of parity non-conservation in weak interactions (famous "$\theta - \tau$ puzzle"). In addition to the ordinary τ-decay there is also the $K^+ \to \pi^+ + 2\pi^0$ decay, called τ'.



(a)

(b)

FIG. 40

Decays of neutral K-mesons into two or three pions can be obtained in an analogous way (Figs. 41 and 42). However, in these decays there is a special feature connected with the combined parity conservation. Shortly we shall return to these decays and consider them in more detail.

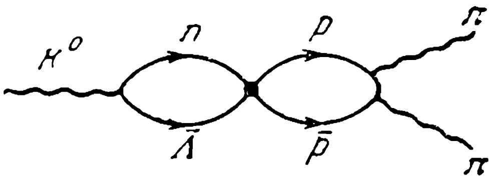


FIG. 41

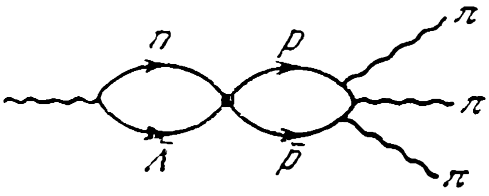


FIG. 42

Thus, on the basis of the universal four-fermion weak interaction we can obtain all known non-leptonic decays. But only qualitatively! The trouble is that the matrix element of any of the decays is very different from the interaction of currents from which it arose and is "dressed" by strong interactions, so that the connection between it and the interaction can be analysed only in the most general way. In particular, we cannot establish from experiment whether the baryon currents are actually of the vector and axial type, and whether the basic interaction is actually the product of two currents.

Even if the four-fermion interaction involved all of the five possible types—scalar, tensor, pseudoscalar, vector and axial—we would be unable to solve this problem, because we have no theory by means of which to pass from the basic interaction to the final matrix element. We cannot point to experiments which would enable us to determine whether the basic interaction has the form *V–A*. However, we can with certainty say that some symmetry properties of the underlying interaction must also be possessed by the matrix elements.

We now study the structure of the matrix elements of different decays which follows from the Lorentz invariance requirement.

$K_{\pi 2}$-DECAY

The decay $K \rightarrow 2\pi$ has a matrix element of the form:

$$f_0 \varphi_K \varphi_{\pi 1}^* \varphi_{\pi 2}^* .$$

Indeed, we have available only the meson wave functions and four-dimensional momenta. From these quantities we have to form a scalar. But the scalar products of the 4-momenta for a two-body decay can be expressed in terms of the masses of the mesons and, consequently, there remain only their wave functions. Such a matrix element corresponds to the fact that the two π-mesons, arising from the decay of a K-meson with spin zero, are in an S-state. Note that the parity of such a system is even. The unknown constant f_θ has the dimensions of energy:

$$[f_\theta] = [m].$$

A theory of strong interactions, if we had it, would allow us to calculate the constant f_θ. We can now estimate theoretically from a consideration of dimensions only its order of magnitude:

$$f_\theta \sim G(x_\theta m_K)^3,$$

where x_θ is a quantity of the order of unity. The θ-decay rate is expressed in terms of f_θ in the following way:

$$W_\theta = \frac{(2\pi)^4}{(2\pi)^6} f_\theta^2 \cdot \int \frac{dp_1\, dp_2}{2E_1\, 2E_2\, 2m_K} \delta^4(p_1 + p_2 - p_K)$$
$$= \frac{f_\theta^2 \sqrt{(m_K^2 - 4m_\pi^2)}}{16\pi m_K^2} .$$

Writing f_θ in the form of $G(x_\theta m_K)^3$ and making use of the experimental probability of the neutral K-meson θ-decay ($\sim 10^{10}$ sec^{-1}), we find

$$x_\theta \approx 0{\cdot}7.$$

The $K_{\pi 2}$-decay of the K^+-meson is 500 times less probable than that of the neutral K-meson. The cause of this is for the present not understood. One usually relates this suppression with the $\Delta T = \frac{1}{2}$ rule (see below). In this connection it should be noted that the total contribution of the two diagrams (*a*) in Fig. 40 is equal to zero within the approximation that the isotopic invariance of strong interactions holds exactly. Hence the basic contribution to the $K^+_{\pi 2}$-decay should be given by diagrams of the type shown in Fig. 43 and by more complex diagrams.

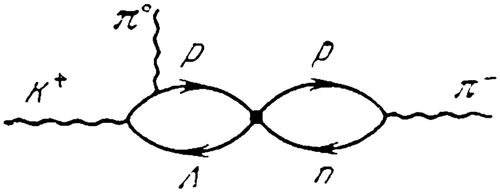


FIG. 43

$K_{\pi 3}$-DECAY

In the case of the $K \to 3\pi$ decay the matrix element has such a form:

$$M = f_\tau \varphi_K \varphi^*_{\pi 1} \varphi^*_{\pi 2} \varphi^*_{\pi 3}.$$

The dimensionless quantity f_τ (as distinct from f_0) is not a constant but a certain function of the scalar products of the 4-momenta of the mesons taking part in the decay, i.e. a function of the energy of the π-mesons. However, because the kinetic energy of the π-mesons arising from the decay is low (the mean kinetic energy of the π-mesons is ~ 25 MeV), one can, in the zeroth approximation, disregard the dependence of the function f_τ on the energy. From a consideration of dimensions it follows that $f_\tau = G(x_\tau m_K)^2$, where x_τ is an unknown coefficient of the order of unity.

We calculate the τ-decay rate, assuming $f_\tau = \text{const}$:

$$W_\tau = \frac{(2\pi)^4}{(2\pi)^9} f_\tau^2 \int \frac{dp_1\, dp_2\, dp_3}{2E_1 \,.\, 2E_2 \,.\, 2E_3 \,.\, 2m_K} \delta^4(p_1 + p_2 + p_3 - p_K).$$

Here the indices 1, 2, 3 refer to the π-mesons. We carry out the integration, disregarding the kinetic energy of the π-mesons in comparison with their mass. Integrating with respect to $\mathbf{p}_2$ and $\mathbf{p}_3$, we obtain easily

$$W_\tau = \frac{f_\tau^2}{(2\pi)^5} \int \frac{dp_1}{16 m_K m_\pi} \cdot \frac{4\pi q}{2m_\pi},$$

where q is the absolute value of the 3-momentum of the mesons 2 or 3 in their centre-of-mass system. We shall express q by means of p_1; for this we introduce the following variables:

$$p_2 + p_3 = -p_1, \qquad p_2 - p_3 = 2q,$$
$$p_2 = q - \frac{p_1}{2}, \qquad p_3 = -q - \frac{p_1}{2}.$$

The total kinetic energy of the mesons 2 and 3 is equal to

$$T_2 + T_3 = \frac{p_2^2}{2m_\pi} + \frac{p_3^2}{2m_\pi}$$
$$= \frac{1}{2m_\pi}\left(\frac{p_1^2}{4} - p_1 q + q^2 + \frac{p_1^2}{4} + p_1 q + q^2\right) = \frac{1}{2m_\pi}\left(\frac{p_1^2}{2} + 2q^2\right).$$

Taking into account that $T_2 + T_3 = Q - T_1$, where $Q = m_K - 3m_\pi$, we obtain

$$q = \sqrt{[m_\pi(Q - \tfrac{3}{2}T_1)]},$$

whence

$$W_\tau = \frac{f_\tau^2}{(2\pi)^5}\cdot\frac{(4\pi)^2}{2^5}\int\limits_0^{T_1^{\max}} \frac{dT_1\,.\,p_1\sqrt{[m_\pi(Q - \frac{3}{2}T_1)]}}{m_\pi m_K}$$
$$= \frac{f_\tau^2}{2^6\pi^3}\int\limits_0^{T_1^{\max}} \frac{dT_1\sqrt{[T_1(2Q - 3T_1)]}}{m_K}$$
$$= \frac{f_\tau^2\sqrt{3}}{2^6\pi^3 m_K}\left(\frac{2Q}{3}\right)^2\int\limits_0^1 \sqrt{[y(1-y)]}\,dy.$$

This last integral is easily calculated and equals $\pi/8$. Finally we have

$$W_\tau = \frac{f_\tau^2 Q^2}{2^7\pi^2 3\sqrt{3}\,m_K}.$$

Writing f_τ in the form $G(x_\tau m_K)^2$ and making use of the experimental τ-decay rate ($W_\tau \approx 6 \times 10^6\,\text{sec}^{-1}$), we find $x \approx 1$. It is seen that the coefficients of the matrix elements of the $K_{\pi3}^+$- and $K_{\pi2}^0$-decays are approximately the same (if a value of the order of $1/m_K$ is taken as the unit length). This means that the matrix element of the $K_{\pi3}^+$-dccay, as distinct from that of the $K_{\pi2}^+$-decay, is not suppressed. That is why the probabilities of these decays differ so little. If the matrix elements of the two decays were equally allowed (or suppressed), the $K^+_{\pi 3}$-decay because of the smallness of phase space would be about three orders of magnitude less probable than the $K^+_{\pi 2}$-decay.

Taking into account the fact that the spin of the K-meson and π-mesons is equal to zero, it is easy to see that the total parity of the system of three π-mesons arising from τ decay is odd. As we have already mentioned, the parity of two π-mesons in the $K^+_{\pi 2}$-decay is even. The discovery of two decay channels with different parities for K^+-mesons ("$\theta - \tau$ puzzle") incited Lee and Yang to raise the famous question: "Does the weak interaction conserve parity?"

DALITZ PLOT

As already mentioned in connection with the K_{e3}-decay (p. 138), the energy distribution of a three-body decay is conveniently presented in a diagram representing an equilateral triangle. Dalitz was the first to point this out, when analysing, by means of this diagram, experimental data on the τ-decay.

Let us consider the Dalitz plot for the general case when a particle with a mass m decays into three particles with masses m_1, m_2 and m_3. We denote the corresponding 4-momenta by p, p_1, p_2, p_3; $p_i = (E_i, k_i)$, $E_i = T_i + m_i$. The region of the variables T_1, T_2 and T_3, satisfying the energy conservation law,

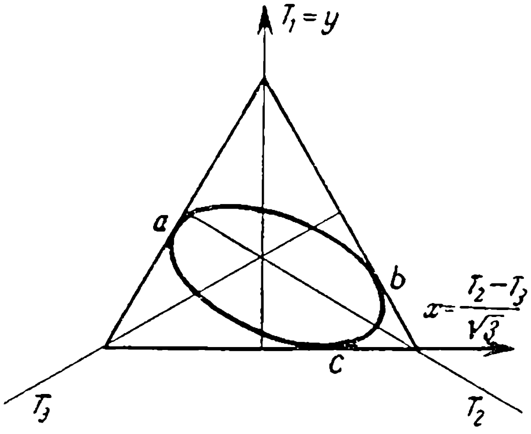


FIG. 44

$$T_1 + T_2 + T_3 = m - m_1 - m_2 - m_3 = \Delta,$$

can be represented by an equilateral triangle with a height equal to Δ (Fig. 44).

However, not all of the points inside the triangle correspond to actual decays. A certain number of these points are forbidden by the 3-momentum conservation law

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0.$$

It can easily be seen that the boundaries of the allowed region are determined by the equations

$$k_1 = k_2 + k_3 \qquad \text{(line } a\,b\text{)},$$
$$k_2 = k_3 + k_1 \qquad \text{(line } b\,c\text{)},$$
$$k_3 = k_1 + k_2 \qquad \text{(line } c\,a\text{)},$$

where $k_i = \sqrt{(E_i^2 - m_i^2)}$.

If one takes into account the energy conservation law, transforms to the variables x and y, and gets rid of the roots, then these equations can be written in the form of one equation:

$$(\sqrt{3}\,x + m_2 - m_3)^2\,[(m - m_1)^2 - 2my] - 2(\sqrt{3}\,x + m_2 - m_3) \times \\ \times (m - m_1 - y)\,(m_2^2 - m_3^2) + (m_2^2 - m_3)^2 - (y^2 + 2m_1 y) \times \\ \times\, [(m - m_1)^2 - 2m_2^2 - 2m_3^2 - 2my] = 0.$$

If $m_1 = \mu_1$, $m_2 = m_3 = 0$, as in the case of the K_{e3}-decay, then

$$[(m - \mu - y)^2 - 2\mu y - y^2]\,[3x^2 - 2\mu y - y^2] = 0,$$

and we obtain

$$y = \frac{(m - \mu)^2}{2m} \qquad \text{for the line } a\,b,$$

$$x = \sqrt{\left(\frac{y^2 + 2\mu y}{3}\right)} \qquad \text{for the line } b\,c,$$

$$x = -\sqrt{\left(\frac{y^2 + 2\mu y}{3}\right)} \qquad \text{for the line } c\,a.$$

In the case of τ-decay $m_1 = m_2 = m_3 = \mu$ and the equation for the boundary assumes the form

$$3x^2\,[(m - \mu)^2 - 2my] - (2\mu y + y^2)\,[(m - \mu)^2 - 4\mu^2 - 2my] = 0.$$

It is easy to see that

$$y_{\max} = \frac{(m - \mu)^2 - 4\mu^2}{2m}.$$

If we denote $m - 3\mu = \Delta$ and assume $\Delta \ll \mu$, then

$$y_{\max} = \frac{(\Delta + 2\mu)^2 - 4\mu^2}{2m} \approx \frac{2}{3}\Delta.$$

Disregarding small terms, we get

$$x^2 - y(\tfrac{2}{3}\Delta - y) = 0.$$

Introducing the variable $y' = y - \Delta/3$, we see that the boundary of the allowed region in the case of τ-decay in the non-relativistic approximation represents a circle of a radius $\Delta/3$, drawn inside the triangle:

$$x^2 + y'^2 = \left(\frac{\Delta}{3}\right)^2.$$

A particular convenience of the Dalitz plot is connected with the fact that an area element $dx\, dy$ of this diagram is proportional to an element of the phase space. This can easily be seen if it is recalled that the expression for the decay rate is proportional to

$$|f_\tau|^2 \int \frac{d\boldsymbol{k}_1\, d\boldsymbol{k}_2\, d\boldsymbol{k}_3}{E_1 E_2 E_3} \delta^4(p_1 + p_2 + p_3 - p).$$

Integrating with respect to $\boldsymbol{k}_3$, we obtain

$$\begin{aligned} dW &\sim |f_\tau|^2 \int \frac{d\boldsymbol{k}_1\, d\boldsymbol{k}_2}{E_1 E_2 E_3} \delta(E_1 + E_2 + E_3 - E) \\ &\sim |f_\tau|^2 \int \frac{k_1 k_2\, dE_1\, dE_2\, d\cos\varphi}{E_3} \times \\ &\qquad \times \delta[E_1 + E_2 + \sqrt{(k_1^2 + k_2^2 - 2k_1 k_2 \cos\varphi + m_3^2)} - E] \\ &= |f_\tau|^2\, dE_1\, dE_2 \sim |f_\tau|^2\, dx\, dy. \end{aligned}$$

It should be stressed that this derivation is connected neither with the assumption of the equality of the masses m_1, m_2 and m_3 nor with that of the validity of the non-relativistic approximation.

From this property of the Dalitz plot it follows that, if $f_\tau = \text{const}$, then the distribution of the points in the diagram must be uniform.

For the decay $K^+ \to 2\pi^+ + \pi^-$ it was experimentally established that

$$|f_\tau|^2 \sim 1 + 0{\cdot}2\left(\frac{2y}{y_{\max}} - 1\right).$$

This energy dependence of $|f_\tau|^2$ seems to be due mainly to the interaction between π-mesons which are produced in the τ-decay.

K_1^0- AND K_2^0-MESONS

Let us see where combined parity conservation in non-leptonic decays of K-mesons leads. First of all, it gives rise to the occurrence of two distinct types of neutral K-mesons: K_1^0 and K_2^0. Where does the basic distinction between leptonic and non-leptonic decays of neutral K-mesons lie?

Take, for example, the $K^0 \to \pi^- + e^+ + \nu$ decay and perform a CP-transformation. We obtain $\tilde{K}^0 \to \pi^+ + e^- + \tilde{\nu}$. It is seen that the disintegrating particles as well as the decay products under CP-transformation go over not to themselves but to their antiparticles: π^- goes over to π^+, e^+ to e^-, ν to $\tilde{\nu}$. Consequently, neither the disintegrating particles nor the decay products have a definite combined parity.

Take the $K^0 \to 2\pi^0$ decay. The combined parity of the system of two π^0-mesons is even. Thus, particles produced in the decay have a definite combined parity, whereas the disintegrating K^0-meson does not: $CPK^0 = \tilde{K}^0$. Consequently, only a combined-parity non-conserving interaction could lead to the $K^0 \to 2\pi^0$ decay. But we are assuming that our interaction does conserve the combined parity. Is there any way out of this? There is, and one can easily see it if one considers the state representing a linear combination of K^0 and $\tilde{K}^0$.

Indeed, consider the state K_1^0 which is described by a linear superposition of the wave functions K^0 and $\tilde{K}^0$: $K_1^0 = \dfrac{K^0 - \tilde{K}^0}{\sqrt{2}}$. Perform the CP-operation on this state. Under the CP-transformation K^0 will go over to $-\tilde{K}^0$, while $\tilde{K}^0$ will go over to $-K^0$, and as a result the state K_1^0 will go over to itself:

$$CPK_1^0 = CP\frac{K^0 - \tilde{K}^0}{\sqrt{2}} = \frac{K^0 - \tilde{K}^0}{\sqrt{2}} = K_1^0.$$

Now perform the CP-transformation on the function $K_2^0 = \dfrac{1}{\sqrt{2}}(K^0 + \tilde{K}^0)$:

$$CPK_2^0 = CP\frac{K^0 + \tilde{K}^0}{\sqrt{2}} = -\frac{K^0 + \tilde{K}^0}{\sqrt{2}} = -K_2^0.$$

Thus, we find that $CPK_1^0 = K_1^0$, while $CPK_2^0 = -K_2^0$. This means that K_1^0 has an even combined parity, while K_2^0 has an odd one.

It is easy to see that the K_1^0 may decay into $2\pi^0$ as well as into $\pi^+ + \pi^-$, which in an S-state have an even combined parity. K_2^0 cannot decay into two π-mesons. We now find the combined parity of a neutral system consisting of three π-mesons. This system can be in two different charge states: either $3\pi^0$, or $\pi^+ + \pi^- + \pi^0$.

Consider first the system $\pi^+ + \pi^- + \pi^0$.

The total angular momentum, I, of this system must be equal to zero, because the spin of the K-meson is zero.

Let us see how a wave function of the three π-mesons having $I = 0$ can be constructed.

We denote the relative angular momentum of the π^+ and π^- by l, and the angular momentum of the π^0 with respect to the centre of mass of $\pi^+ + \pi^-$ by L. As $I = l + L$, in order that I may equal zero it is necessary that L should equal l.

The combined parity of the π^0-meson is always odd, while that of the $\pi^+ + \pi^-$ system is always even (this follows from the fact that for such a system $C = (-1)^l$, $P = (-1)^l$ and, so, $CP = (-1)^{2l} = 1$. Consequently, everything is determined by the parity of the orbital motion associated with the angular momentum L. If L is even ($L = l = 0, 2, 4$), then the combined parity of the $\pi^+ + \pi^- + \pi^0$ system is odd, while if L is odd ($L = l = 1, 3, 5$) then it is even. Consider now the system $3\pi^0$. Since the π^0-mesons are identical, L must be even, and, consequently, the combined parity of the system $3\pi^0$ is odd.

We now sum up and enumerate the allowed decays:

$$K_1^0 \to \left\{\begin{array}{l} \pi^+ + \pi^-, \\ 2\pi^0, \\ \pi^+ + \pi^- + \pi^0 \quad (l = L = 1, 3, 5, \ldots), \end{array}\right\} CP = 1;$$

$$K_2^0 \to \left\{\begin{array}{l} 3\pi^0, \\ \pi^+ + \pi^- + \pi^0 \quad (l = L = 0, 2, 4, \ldots) \end{array}\right\} CP = -1.$$

If the leptonic decays K_{e3}^0- and $K_{\mu 3}^0$ are for a while set aside, then it is clear that the K_1^0-meson should live substantially less than the K_2^0-meson, because the phase space in the τ^0-decay ($K_{\pi 3}^0$) is considerably less than that in the θ^0-decay ($K_{\pi 2}^0$). The experimental values of these decay rates differ by three orders. The probability of the $K_1^0 \to \pi^+ + \pi^- + \pi^0$ decay should be at least two orders of magnitude lower than $W_{K_2^0 \to 3\pi}$, because emission of π-mesons in the state $l = L = 1$ is hindered by a centrifugal barrier.

The leptonic decays K_{e3}^0 and $K_{\mu 3}^0$, which can occur for the K_1^0 as well as for the K_2^0, do not substantially change the ratio between the lifetimes of the K_1^0- and K_2^0-meson. The probabilities of these decays are of about the same order as the $K_2^0 \rightarrow 3\pi$ decay rate. The properties of the K_1^0- and K_2^0-mesons are considered in more detail on pp. 182–200.

Other consequences following from the combined parity conservation will be considered in the next chapter which is devoted to hyperon decay.

CHAPTER 13

NON-LEPTONIC DECAYS OF HYPERONS

AMPLITUDES OF HYPERON DECAYS

We now proceed to hyperon decays. A hyperon decay is described by the diagram shown in Fig. 45. 1 denotes the disintegrating hyperon, 2 denotes the created baryon. If particles 1 and 2 have a spin of $\frac{1}{2}$, then the most general form of the amplitude describing such a decay is

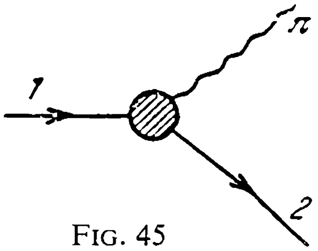


FIG. 45

$$M = \bar{u}_2(\alpha + \beta\gamma_5)\,u_1 \,.\, \varphi_\pi,$$

where α and β are nunkown constants, while $\bar{u}_2$ and u_1 are 4-spinors.

At first sight it seems that we have not made use of all the possibilities. For example, from the spinors we constructed a scalar and a pseudoscalar but not a vector and an axial vector:

$$\bar{u}_2\gamma_\mu u_1 \quad \text{and} \quad \bar{u}_2\gamma_\mu\gamma_5 u_1 .$$

It is, however, easy to see that addition of terms of the type

$$M' = \bar{u}_2(\alpha'\gamma_\mu + \beta'\gamma_\mu\gamma_5)\,u_1 k_\mu \varphi_\pi,$$

where k is the 4-momentum of the π-meson, does not give anything new. Because the decay is a two-body, the momentum of the π-meson is expressed in terms of the momenta k_1 and k_2:

$$k = k_1 - k_2 .$$

Taking advantage of this, it is easy to obtain

$$\begin{aligned} M' &= -\bar{u}_2[\alpha'(\hat{k}_2 - \hat{k}_1) + \beta'(\hat{k}_2 - \hat{k}_1)\gamma_5]\,u_1\varphi_\pi \\ &= -\bar{u}_2[\alpha'(m_2 - m_1) + \beta'(m_2 + m_1)\gamma_5]\,u_1\varphi_\pi . \end{aligned}$$

We have now brought M' into the form of M (recall that we cannot calculate theoretically the constants $\alpha(\alpha')$ and $\beta(\beta')$!). Thus, the amplitudes M and M' are completely equivalent.

Even having found, experimentally, the constants α and β and, consequently, fixed the form of the matrix element, we are unable to find out, whether or not, this matrix element actually results from a four-fermion weak interaction of the type *V–A*. No experiments with hyperons can establish (in so far as we do not have a strong interaction theory) the form of the fundamental weak interaction responsible for hyperon decays. Even if it contains only *S* and *P* and no *V* and *A*, it can in principle give the same type of matrix element. We cannot even verify whether or not this interaction actually involves four fermions.

Still, experiments with hyperons allow one to state that the interaction does not conserve strangeness and charge parity, but conserves combined parity. Parity non-conservation in hyperon decays manifests itself by the fact that the matrix element M contains, along with the scalar $\beta\gamma_5$, also the pseudoscalar α. Under P-inversion the first of these does not change sign (is even), while the second does change sign (is odd), if the intrinsic parities of both baryons are even, whereas that of the π-meson is odd. It is easy to see that the term α describes the formation of a meson in an S-state, while the term β describes the formation of a meson in a P-state. Indeed, waves with $l = 0, 2, 4 \ldots$ possess even parity, while waves with $l = 1, 3, 5 \ldots$ possess odd parity. But because the spins of the disintegrating hyperons and the created baryon are equal to $\frac{1}{2}$, only waves with $l = 0$ and $l = 1$ can give a contribution. Higher values of l are forbidden by the angular momentum conservation law.

Let us find the restrictions which are imposed by combined parity conservation on the coefficients α and β contained in the hyperon decay amplitude:

$$M = \bar{u}_2(\alpha + \beta\gamma_5)u_1\varphi_\pi.$$

For definiteness we shall speak about the $\Xi^- \rightarrow \Lambda^0 + \pi^-$ decay, assuming that the spin of the Ξ-hyperon equals $\frac{1}{2}$. The substantial difference between non-leptonic decays and the processes that we have considered earlier is that the particles produced in these decays interact strongly with each other.

SWITCH OFF THE SCATTERING

Let us forget, for the moment, that the decay of the hyperon results from both the weak and strong interactions, and assume that the π-meson and Λ-hyperon arising from the decay do not interact with each other. In this case we may disregard the π-meson scattering from the Λ-hyperon and consider the decay matrix element as a certain "effective weak interaction". Because we have included by this assumption (completely illegally!) the π-meson and Λ-hyperon in the class of weakly interacting particles, our decay is, in principle, no longer different from the processes considered before, such as the leptonic decays of K-mesons or the neutron β-decay. In order to determine the phase of the coefficients α and β, we can apply a standard procedure to our matrix element (see p. 75). We perform the Hermitian conjugation transformation (H) and the combined inversion (CP), and compare the results of these two transformations.

Under Hermitian conjugation

$$M = \bar{u}_2(\alpha + \beta\gamma_5)\, u_1\varphi_\pi \overset{H}{\rightarrow} M^H = \bar{u}_1(\alpha^+ - \beta^*\gamma_5)\, u_2\varphi_\pi^*.$$

(The minus sign in front of β arises from the anticommutation of the matrices γ_4 and γ_5; these γ's are chosen as Hermitian.)

Under charge conjugation

$$M \overset{C}{\rightarrow} M^C = -\tilde{u}_2 C^+(\alpha + \beta\gamma_5)\, C\tilde{\bar{u}}_1\varphi_\pi^* = -\bar{u}_1(\alpha + \beta\gamma_5)\, u_2\varphi_\pi^*.$$

Under space inversion

$$M \overset{P}{\rightarrow} M^P = -\bar{u}_2(\alpha - \beta\gamma_5)\, u_1\varphi_\pi.$$

(We assume the intrinsic parities of the particles 1 and 2 to be the same, and the π-meson to be a pseudoscalar.) From the definition of M^P and M^C it is obvious that under combined inversion

$$M \overset{CP}{\rightarrow} M^{CP} = \bar{u}_1(\alpha - \beta\gamma_5)\, u_2\varphi_\pi^{+}.$$

REALITY OF α AND β

By virtue of the combined parity conservation, $M^H = M^{CP}$. Whence it follows that

$$\alpha = \alpha^*, \qquad \beta = \beta^*,$$

i.e. the coefficients α and β are real. It should be noted that the common phase of the coefficients α and β is a physically unobservable quantity, because the decay probability is proportional to MM^*. In particular, if we assumed the particles 1 and 2 to have different intrinsic parities, then we would obtain

$$\alpha = -\alpha^* \quad \text{and} \quad \beta = -\beta^*,$$

i.e. the coefficients α and β would be purely imaginary.

The relative phase of the coefficients α and β is a physically observable quantity. It is seen that by virtue of the combined parity conservation it is equal to zero (more precisely, to $n\pi$) independently of what the intrinsic parities of the particles are.

It is easy to see that if we required the conservation of charge parity (C) and not that of combined parity (CP), then the result would be different.

As a matter of fact, from the equality

$$M^H = M^C$$

it follows that

$$\alpha^* = -\alpha, \qquad \beta^* = \beta.$$

This means that the relative phase of the coefficients α and β is equal to $\pi/2$ $\left(\text{more precisely, to } n\pi + \frac{\pi}{2}\right)$.

SWITCH ON THE SCATTERING. S-MATRIX

Let us find out now what will be changed in our inferences when we take into account the strong interaction between the π-meson and Λ-hyperon. For this we have to study more closely the properties of the S-matrix. We illustrate these properties with the same decay $\Xi^- \rightarrow \Lambda^0 + \pi^-$. In this decay we are dealing with three states.

We shall denote the initial state by Ξ^-, and the two possible final states of the $\Lambda^0 + \pi^-$ system with $l = 0$ and $l = 1$ respectively

TABLE 5

Initial state / Final state	Ξ	s	p
Ξ	S_{11}	S_{12}	S_{13}
s	S_{21}	S_{22}	S_{23}
p	S_{31}	S_{32}	S_{33}

by s and p. If also the inverse reaction $\Lambda^0 + \pi^- \rightarrow \Xi^-$ is considered, then, in total, three initial and three final states are possible. The set of matrix elements $S_{\alpha\beta}(\alpha, \beta = 1, 2, 3)$ connecting each initial state to each final state can be presented in the form of Table 5 (S-matrix).

If the Ξ^--hyperon decay and $\pi\Lambda$-scattering are "switched off", the S-matrix will have the form

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

which means that each of the initial states goes over into itself without any change.

UNITARITY AND SYMMETRY

It is obvious that, in the absence of interaction, the S-matrix possesses the unitarity property $SS^+ = I$, where I is the unit matrix, and the symmetry property

$$S_{\alpha\beta} = S_{\beta\alpha}.$$

A remarkable property of the S-matrix is that it preserves these properties even after we "switch on" the interaction.

The unitarity condition on the S-matrix expresses the requirement of total probability conservation (the sum of the probabilities of different final states must be equal to unity) and the conservation of the orthogonality of the states. The condition of the unitarity of the S-matrix is equivalent to that of the hermiticity of the interaction Lagrangian.

The symmetry condition on the S-matrix is a consequence of the interaction invariance with respect to time reversal (or, by virtue of the CPT-theorem, with respect to combined inversion): the matrix elements for the direct and inverse process must be the same.

If, as before, the strong interaction is switched on without switching on the weak decay interaction, then π-meson scattering by the Λ-hyperon occurs, and the S-matrix assumes the form

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\varphi_2} & 0 \\ 0 & 0 & e^{i2\varphi_3} \end{pmatrix},$$

where φ_2 is the scattering phase in the s-state, while φ_3 is the phase in the p-state.

If now the weak interaction is switched on, non-diagonal matrix elements will appear in the matrix:[†]

$$S = \begin{pmatrix} 1 & S_{21} & S_{31} \\ S_{12} & e^{2i\varphi_2} & S_{32} \\ S_{13} & S_{23} & e^{2i\varphi_3} \end{pmatrix}.$$

S_{12} describes the transformation of Ξ into $\Lambda + \pi$ with $l = 0$, while S_{13} describes that with $l = 1$. S_{23} describes the transition of the $\Lambda + \pi$ system from a state with $l = 0$ to a state with $l = 1$. This transition does not conserve parity and can take place only on account of the weak interaction. All the non-diagonal matrix elements are proportional to the weak interaction constant G, and are about 5–7 orders of magnitude smaller than the diagonal matrix elements. Based on this, and making use of the unitarity and symmetry conditions of the S-matrix, we shall express the phases of the non-diagonal matrix elements in terms of those of the diagonal matrix elements.

CONNECTION BETWEEN THE DECAY AND SCATTERING

We write the unitarity condition $SS^+ = I$ in the explicit form

$$S_{\alpha\gamma} S^*_{\beta\gamma} = \delta_{\alpha\beta}.$$

Now we take advantage of the fact that, by virtue of symmetry of the S-matrix,

$$S^*_{\beta\gamma} = S^*_{\gamma\beta}.$$

We obtain

$$S_{\alpha\gamma} S^*_{\gamma\beta} = \delta_{\alpha\beta}. \qquad (*)$$

In the sum of the products $S_{\alpha\gamma} S^*_{\gamma\beta}$ there are terms of zero order, first order and second order in the weak interaction constant. We disregard those of second order in G ($\sim 10^{-10}$–10^{-14}). For zero order terms the relation (*) gives

$$S_{\alpha\alpha} S^*_{\alpha x} = \delta_{\alpha\alpha}.$$

[†] As already mentioned on p. 37, the non-diagonal matrix elements of the S-matrix $S_{\alpha\beta}$ are connected in the following way with corresponding transition amplitudes $M_{\alpha\beta}$:

$$S_{\alpha\beta} = i(2\pi)^4\, \delta^4(p_f - p_i)\, M_{\alpha\beta}.$$

while for those of the first order it gives

$$S_{\alpha\alpha}S^*_{\alpha\beta} + S_{\alpha\beta}S^*_{\beta\beta} = 0 \qquad (\alpha \neq \beta),$$

whence it follows that

$$\frac{S_{\alpha\beta}}{S^*_{\alpha\beta}} = -\frac{S_{\alpha\alpha}}{S^*_{\beta\beta}}.$$

Writting $S_{\alpha\beta}$ in the form $i\varrho_{\alpha\beta}e^{i\varphi_{\alpha\beta}}$ and taking into account that $S_{\alpha\alpha} = e^{i2\varphi_\alpha}$, $S_{\beta\beta} = e^{i2\varphi_\beta}$, we obtain

$$e^{i2\varphi_{\alpha\beta}} = e^{i2(\varphi_\alpha+\varphi_\beta)}$$

or

$$\varphi_{\alpha\beta} = \varphi_\alpha + \varphi_\beta + n\pi.$$

In the $\Xi^- \to \Lambda^0 + \pi^-$ decay $\varphi_1 = 0$, while φ_2 and φ_3 are equal, respectively, to the phases of π-meson scattering by Λ in the s- and p-state.

The result we have obtained has an obvious interpretation. In an ordinary π-meson scattering on the Λ-hyperon these particles come from infinity to the origin and go out again to infinity. In this case the phase 2φ is "accumulated". In the Ξ-hyperon decay Λ and π are created at the origin and go out to infinity. In this case, naturally, the phase φ is "accumulated".

Thus, in the absence of interaction in the final state, combined parity conservation leads to the fact that the coefficients α and β for the s- and p-wave in the hyperon decay amplitude are real. When interaction in the final state is taken into account, one gets

$$\alpha = \pm|\alpha|\, e^{i\varphi_s}, \qquad \beta = \pm|\beta|\, e^{i\varphi_p},$$

where φ_s and φ_p are the corresponding scattering phases (it should be stressed once more that only the relative phase of the coefficients α and β, but not their common phase, has a physical meaning).

WHAT IS THE VALUE OF SCATTERING PHASES?

The phases of the Λ-scattering by π are, for the present, unknown. As will be seen in what follows, these phases (more precisely, their difference) can be determined by analysing the cascade hyperon decay.

In the decays of the Λ-hyperon and Σ-hyperon there arise π-mesons and nucleons. The phases of pion-nucleon scattering are known from corresponding scattering experiments (see Table 6).

TABLE 6. πN-SCATTERING PHASES AT MOMENTA CORRESPONDING TO THE DECAY OF Λ- AND Σ-HYPERONS

momentum	$\varphi_1(T=\frac{1}{2}, l=0)$	$\varphi_3(T=\frac{3}{2}, l=0)$	$\varphi_{11}(T=\frac{1}{2}, l=1)$	$\varphi_{31}(T=\frac{3}{2}, l=1)$
$99{\cdot}5\,\frac{\text{MeV}}{c}\,(\Lambda)$	$\sim +7°$	$\sim -4°$	$\sim 0°$	$\sim 0°$
$189{\cdot}0\,\frac{\text{MeV}}{c}\,(\Sigma)$	$\sim +14°$	$\sim -8°$	$\sim +5°$	$\sim -3°$

The decay $\Xi^- \to \Lambda^0 + \pi^-$ is characterized by the fact that the $\Lambda^0 + \pi^-$ system has a definite isotopic spin ($T = 1$). The system $\pi + N$, produced in the Λ- or Σ-decay, can, in general, be in two isotopic states ($T = \frac{1}{2}$ and $T = \frac{3}{2}$). In this case the S-matrix will contain not 9 (3×3) but 25 (5×5) matrix elements.

NON-RELATIVISTIC AMPLITUDE

Making use of the properties of the amplitude derived above, we calculate angular and spin correlations in non-leptonic decays of hyperons. Since the energy released in hyperon decay is small in comparison to the mass of the nucleon, it is convenient to use a two-component representation of the amplitude.

We write the expression

$$M = \bar{u}_2(\alpha + \beta\gamma_5)\, u_1 \varphi_\pi.$$

in terms of two-component spinors

$$u_1 = \begin{pmatrix}\varphi_1 \\ \chi_1\end{pmatrix}, \qquad \bar{u}_2 = (\varphi_2^+,\ -\chi_2^+),$$

$$\chi_1 = \frac{\sigma p_1}{E_1 + m_1}\varphi_1, \qquad \chi_2^+ = \varphi_2^+ \frac{\sigma p_2}{E_2 + m_2}.$$

Here φ and χ are two-component spinors, while the σ are the Pauli two-component matrices. p, E and m are the momentum, total energy and mass of the particle. Let us consider the decay in the rest system of the disintegrating hyperon ($p_1 = 0$). Then

$$M = \varphi_2^+\left(1,\ -\frac{\sigma p_2}{E_2 + m_2}\right)\left[\alpha\begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} - \beta\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\right]\begin{pmatrix}1 \\ 0\end{pmatrix}\varphi_1\varphi_\pi$$

$$= \varphi_2^+\left[\alpha + \beta\frac{\sigma p_2}{E_2 + m_2}\right]\varphi_1\varphi_\pi.$$

Introducing the notation

$$n = \frac{p_2}{|p_2|}, \qquad \alpha = a, \qquad \beta\frac{|p_2|}{E_2 + m_2} = b,$$

we obtain

$$M = \varphi_2^+ [a + b\sigma n]\, \varphi_1 \varphi_\pi .$$

Up to this point we have not made any approximations—we simply passed from the four-component representation to the two-component one. The transition to a non-relativistic approximation consists in our assuming that both two-component spinors (φ_1, φ_2) are non-relativistic and independent of the corresponding momenta.

The decay rate will be proportional to MM^*:

$$W \sim \varphi_2^+ [a + b\sigma n]\, \varphi_1 \varphi_1^+ [a^* + b^* \sigma n]\, \varphi_2 .$$

Since we are not interested in the total decay rate, we do not write the factor of proportionality.

NON-RELATIVISTIC POLARIZATION OPERATOR

Let us introduce spin projection operators for two-component spinors

$$\lambda^s_{\alpha\beta} = \varphi_\alpha \varphi_\beta^+ = \left(\frac{1 + \sigma\zeta}{2}\right)_{\alpha\beta},$$

where ζ is the unit vector in the direction of the particle's polarization. This expression can be obtained in the non-relativistic region from the expression for the operator

$$\Lambda^s_{\alpha\beta} = u_\alpha \bar{u}_\beta = \tfrac{1}{2}[(\hat{p} + m)(1 - \gamma_5 \hat{s})]_{\alpha\beta},$$

which we considered on p. 59. We recall that $\hat{s} = s_4\gamma_4 - s\gamma$, where

$$s_4 = \frac{(\zeta p)}{m}, \qquad s = \zeta + \frac{(\zeta p) p}{(E + m) m}.$$

In non-relativistic region $s_4 \to 0$, $s \to \zeta$. Taking into account that

$$\gamma_5 = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}, \qquad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

we obtain

$$\Lambda^s \to m\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \sigma\zeta & 0 \\ 0 & -\sigma\zeta \end{pmatrix}\right] \to m[1 + \sigma\zeta] = 2m\lambda^s.$$

By means of the projection operator λ^s we can write the hyperon decay rate in the form

$$W \sim \mathrm{Sp}\, \lambda_2^s [a + b\boldsymbol{\sigma n}]\, \lambda_1^s\, [a^* + b^* \boldsymbol{\sigma n}],$$

where Sp denotes the trace of the 2 × 2-matrix. We denote the polarization vector of the particle 1 by ζ_1, and that of the particle 2 by ζ_2. Then

$$W \sim \mathrm{Sp}[1 + \sigma\zeta_2]\, [a + b\boldsymbol{\sigma n}]\, [1 + \sigma\zeta_1]\, [a^* + b^* \boldsymbol{\sigma n}].$$

(The trace of the unit 2 × 2-matrix equals 2, while that of any of the σ-matrices equals zero.)

CORRELATIONS IN THE s-WAVE

We calculate first the term proportional to

$$\frac{aa^*}{2} \mathrm{Sp}(1 + \sigma\zeta_2)(1 + \sigma\zeta_1) = aa^*(1 + \zeta_1\zeta_2).$$

Because the term proportional to a corresponds to an s-wave, this result is quite natural. The final polarization must coincide with the initial one—the spins of the particles 1 and 2 are parallel. If ζ_2 is antiparallel to ζ_1, the probability reduces to zero. (Note that in calculating the trace we made use of the relation $\sigma_i\sigma_k = \delta_{ik} + i\varepsilon_{ikl}\sigma_l$.)

CORRELATIONS IN THE p-WAVE

We now calculate the term proportional to $b\, b^*$:

$$\frac{bb^*}{2} \mathrm{Sp}(1 + \sigma\zeta_2)\boldsymbol{\sigma n}(1 + \sigma\zeta_1)\boldsymbol{\sigma n} = bb^*[1 + 2(\zeta_2\boldsymbol{n})(\zeta_1\boldsymbol{n}) - (\zeta_2\zeta_1)].$$

In calculating this trace it is convenient to make use of the relation

$$\tfrac{1}{2}\mathrm{Sp}\, \sigma_i\sigma_k\sigma_l\sigma_m = \delta_{ik}\delta_{lm} + \delta_{lm}\delta_{kl} - \delta_{il}\delta_{km}.$$

Note that the contribution of the s-wave as well as that of the p-wave under space inversion does not change sign. The polarization of particle 2 for the p-wave does not coincide any more with that of particle 1, as in the case of s-wave. The probability is a maximum when the vector ζ_2 is parallel to the vector

$$2\boldsymbol{n}(\zeta_1\boldsymbol{n}) - \zeta_1.$$

sp-INTERFERENCE

Finally, we calculate the interference terms proportional to $a\,b^*$ and $a^*\,b$:

$$\frac{ab^*}{2}\,\mathrm{Sp}(1+\sigma\zeta_2)(1+\sigma\zeta_1)\,\sigma\mathbf{n} = ab^*(\zeta_2\mathbf{n} + \zeta_1\mathbf{n} + i\zeta_2[\zeta_1\mathbf{n}]),$$

$$\frac{a^*b}{2}\,\mathrm{Sp}(1+\sigma\zeta_2)\,\sigma\mathbf{n}(1+\sigma\zeta_1) = a^*b(\zeta_2\mathbf{n} + \zeta_1\mathbf{n} + i\zeta_2[\mathbf{n}\zeta_1]).$$

Summing the contribution of interference terms, we obtain

$$(ab^* + a^*b)(\zeta_2\mathbf{n} + \zeta_1\mathbf{n}) + i(ab^* - a^*b)\,\zeta_2[\zeta_1\mathbf{n}].$$

As is easy to see, these terms under space inversion change sign (under space inversion $\zeta \to \zeta$, $\mathbf{n} \to -\mathbf{n}$).

Finally we obtain

$$\begin{aligned} W(\mathbf{n},\zeta_1,\zeta_2) \sim{} & a^*a(1+\zeta_1\zeta_2) + bb^*[1 + 2(\zeta_1\mathbf{n})(\zeta_2\mathbf{n}) - \zeta_1\zeta_2] \\ & + (a^*b + b^*a)(\zeta_1\mathbf{n} + \zeta_2\mathbf{n}) + i(ab^* - a^*b)\,\zeta_2[\zeta_1\mathbf{n}]. \end{aligned}$$

This expression describes angular and spin correlations in the hyperon decays:

$$\begin{array}{lll} \Lambda^0 \to p + \pi^-, & \Lambda^0 \to n + \pi^0, & \\ \Sigma^+ \to n + \pi^+, & \Sigma^+ \to p + \pi^0, & \Sigma^- \to n + \pi^-. \\ \Xi^- \to \Lambda^0 + \pi^-, & \Xi^0 \to \Lambda^0 + \pi^0, & \end{array}$$

Different decays are characterized by different values of the coefficients a and b. If any of the quantities $\mathbf{n}$, ζ_1 and ζ_2 are not measured experimentally, then in our formula the corresponding vector should be assumed to be zero.

DECAY ASYMMETRY

Consider, for example, an experiment in which a polarized Λ-hyperon disintegrates ($\Lambda \to p + \pi^-$), while the polarization of the nucleon is not measured. In this case the angular distribution of the nucleons is of the form:

$$W(\mathbf{n},\zeta_1) \sim aa^* + bb^* + (a^*b + b^*a)\,\zeta_1\mathbf{n}$$

or, in other words,

$$W(\mathbf{n},\zeta_1) \sim 1 + \alpha(\mathbf{n}\zeta_1), \quad \text{where} \quad \alpha = \frac{a^*b + b^*a}{a^*a + b^*b}.$$

A typical scheme for this experiment is as follows. A π-meson beam hits a hydrogen target, and there takes place the reaction

$$\pi^- + p \to \Lambda^0 + K^0.$$

The Λ-hyperons are produced polarized, and their polarization vector ζ_1 is directed along the normal to the production plane. Measurement of the up-down asymmetry with respect to this plane for π-mesons produced in the $\Lambda^0 \to p + \pi^-$ decay allows one to establish the value of αP, where α is the asymmetry coefficient introduced earlier, while P is the degree of polarization of the Λ-hyperon. (Note that $P = \dfrac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$, where $N_\uparrow(N_\downarrow)$ denotes the number of Λ-hyperons polarized along (counter) the normal to the production plane; it is easy to see that $|P| \leqq 1$.)

Experiments similar to the one described, at incident pion energies of about 1—2 BeV, have given the following values of $\alpha\bar{P}$ ($\bar{P}$ denotes P averaged over the hyperon angular distribution):

$$\begin{array}{ll}
\Lambda^0 \to p + \pi^- & \sim 0{\cdot}6, \\
\Lambda^0 \to n + \pi^0 & \sim 0{\cdot}6, \\
\Sigma^- \to n + \pi^- & \sim 0\ , \\
\Sigma^+ \to p + \pi^0 & \sim 0{\cdot}8, \\
\Sigma^+ \to n + \pi^+ & \sim 0\ .
\end{array}$$

Because the sign of the hyperon polarization is not known, these experiments do not give the sign of the coefficient α. The asymmetry coefficient α at given values of $|a|$ and $|b|$ is maximum, if the phases a and b are the same, and becomes zero if these phases differ by $\pi/2$.

As we saw above, the first possibility corresponds to combined parity conservation, while the second corresponds to charge parity conservation. Because of the smallness of the scattering phases, scattering in the final state will affect only slightly this conclusion. The very fact that a large asymmetry in the decay of the Λ-hyperon and Σ^+-hyperon is observed means that charge parity in these decays is not conserved.

NUCLEON POLARIZATION

If $W(\boldsymbol{n}, \zeta_1, \zeta_2)$ is averaged over the direction of ζ_1, we obtain

$$W(\boldsymbol{n}, \zeta_2) \sim aa^* + b^*b + (a^*b + b^*a)(\boldsymbol{n}\zeta_2).$$

It is seen that the degree of nucleon longitudinal polarization is determined by the same coefficient $\alpha = \frac{a^*b + b^*a}{aa^* + bb^*}$. Note that measurements on the nucleon longitudinal polarization provide not only the absolute value of the coefficient but also its sign. In this respect the measurement of the polarization of nucleons has an advantage over that of the asymmetry of the nucleons.

Consider, finally, the nucleon polarization in the direction perpendicular to the plane which is formed by the vectors $\boldsymbol{n}$ and ζ_1. This polarization is determined by the term

$$i(ab^* - a^*b)\,\zeta_2\,[\zeta_1 \boldsymbol{n}].$$

If the relative phase of a and b were equal to zero, then this term would reduce to zero. Whence it follows that, when the combined parity is conserved, such a correlation may arise only on account of the interaction in the final state. Measuring this correlation in the $\Xi^- \to \Lambda^0 + \pi^-$ decay, one can obtain information about $\pi\Lambda$-scattering.

CHAPTER 14

ISOTOPIC SELECTION RULES IN NON-LEPTONIC DECAYS

THE weak interaction $(\bar{\Lambda}p)(\bar{p}n) + (\bar{p}\Lambda)(\bar{n}p)$, leading to non-leptonic decays, violates not only isotopic spin conservation but also strangeness conservation. However, this violation is governed by very strict selection rules: in the decays only certain changes in the isotopic spin (T) and in its third component (T_3) and, consequently, in the strangeness (S), are allowed.

SELECTION ACCORDING TO STRANGENESS

The non-leptonic interaction transforms the Λ-hyperon into the proton, antiproton and neutron; it transforms also the anti-Λ into the proton, antiproton and antineutron. In the first of these transformations $\Delta S = +1$, while in the second one $\Delta S = -1$. Because the strong interaction conserves strangeness, in all non-leptonic decays resulting from a weak interaction and virtual strong interactions the change in the strangeness satisfies the rule

$$\Delta S = \pm 1.$$

This selection rule leads at once to two rather strong restrictions.

The first of these is the absence of the direct decay of cascade hyperons into nucleons and π-mesons. The decays $\Xi^- \to n + \pi^-$, $\Xi^0 \to n + \pi^0$ and $\Xi^0 \to p + \pi^-$ are forbidden, because in them the change in strangeness amounts to $\Delta S = +2$.

In the Sakata model this forbiddenness is accounted for particularly simply: the Ξ-hyperon "consists" of an antinucleon and two Λ-hyperons, and the weak interaction breaks up only one Λ-hyperon at a time.

Not a single $\Xi \to N + \pi$ decay has been observed experimentally, although many $\Xi^- \to \Lambda^0 + \pi^-$ decays have been observed. However, this in itself is no evidence that the $\Xi \to N + \pi$ decays are actually absent. It is by the characteristic Ξ-hyperon decay into Λ^0-hyperon that the Ξ-hyperon is usually detected (Fig. 46).

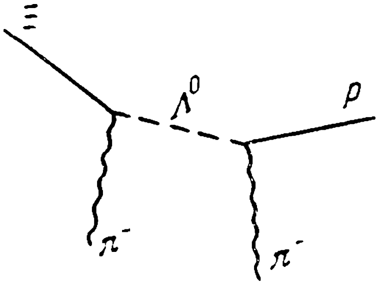


FIG. 46

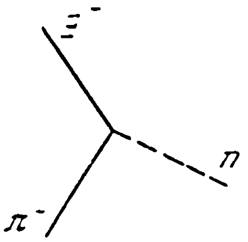


FIG. 47

The $\Xi^- \rightarrow n + \pi^-$ decay, even if it existed, would look in an experiment simply as a break in the track (Fig. 47).

In this respect the $\Xi^0 \rightarrow p + \pi^-$ decay would be much more effective; unfortunately, so far, the number of Ξ^0-hyperons observed experimentally is small. †

$K^0 \nleftrightarrow \tilde{K}^0$ FORBIDDENNESS

We now consider the other consequence of the $\Delta S = \pm 1$ selection rule. Because the strangeness of the K^0- and $\tilde{K}^0$-mesons is (respectively) equal to $+1$ and -1, the matrix element of transition between them must be proportional to the square of the weak interaction constant, G^2. One of the diagrams giving such a transition is shown in Fig. 48. Here s denotes the strong interaction, while w denotes the weak interaction. If the weak interaction were

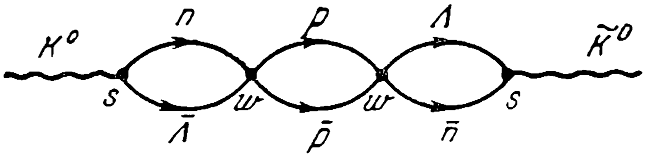


FIG. 48

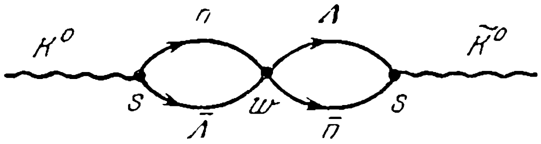


FIG. 49

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† Recent experiments give $\dfrac{\Gamma(\Xi^- \rightarrow n\pi^-)}{\Gamma(\Xi^- \rightarrow \Lambda^0\pi^-)} < 5 \times 10^{-3}$ (note added in 1964).

able to change the strangeness by two units, then the matrix element of the $K^0 \rightleftarrows \tilde{K}^0$ transition would be proportional to G. For example, the interaction $(\tilde{\Lambda} n)(\tilde{\Lambda} n)$, for which $\Delta S = 2$ and which is forbidden in our scheme, could transform K^0 into $\tilde{K}^0$ in the way shown in Fig. 49.

As will be seen in the next chapter, the value of the matrix element of the $K^0 \rightleftarrows \tilde{K}^0$ transition can be determined by analysing the properties of the neutral K-meson beam.

$\Delta T = \frac{1}{2}$ RULE

It is easy to see that the interaction $(\tilde{\Lambda} p)(\tilde{p} n) + (\tilde{p} \Lambda)(\tilde{n} p)$ also puts definite restrictions on the change in isotopic spin in non-leptonic decays. The isotopic spin of the Λ-hyperon is zero, while that of the nucleon is $\frac{1}{2}$.

Consequently, the change in isotopic spin must be equal to $\Delta T = \frac{1}{2}, \frac{3}{2}$. Larger values of ΔT, in particular $\Delta T = \frac{5}{2}$, are forbidden.

We shall not consider restrictions resulting from this selection rule—they are rather weak. Instead we shall find out what would take place if of the two allowed transition amplitudes ($\Delta T = \frac{1}{2}$ and $\Delta T = \frac{3}{2}$) one ($\Delta T = \frac{1}{2}$) were much larger than the other ($\Delta T = \frac{3}{2}$). In this case we would have the approximate selection rule $\Delta T = \frac{1}{2}$ for non-leptonic decays. It should be stressed that no conclusive theoretical explanation of such a selection rule is as yet available. Yet, as will be seen in what follows, all experimental data are in good agreement with it.

In attempts to account for the $\Delta T = \frac{1}{2}$ rule, theoreticians are divided into two schools. Adherents of one school assume that the Lagrangian of the weak non-leptonic interaction has the ordinary form of the product of two charged currents ($\tilde{n} p$ and $\tilde{\Lambda} p$), but

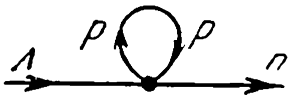

FIG. 50

that virtual strong interactions lead to the fact that transitions with $\Delta T = \frac{3}{2}$, allowed by this Lagrangian, are suppressed. In this case the $\Delta T = \frac{1}{2}$ rule appears to be only approximate. Such a situation would take place if, for example, in non-leptonic decays the major role were played by diagrams of the type shown in Fig. 50, giving the effective transition $\Lambda \rightarrow n$ (see pp. 146, 147). But certain theoreticians prefer to account for the $\Delta T = \frac{1}{2}$ rule by other types of diagrams. Unfortunately, at the present stage of the theory of strong interactions the validity of such explanations cannot be checked.

Adherents of the other school assume that the $\Delta T = \frac{1}{2}$ rule is strict and inherent in the weak interaction Lagrangian. In this case, of course, the Lagrangian must be modified by introducing in it new currents. Within the framework of the Sakata model the $\Delta T = \frac{1}{2}$ rule can be obtained if, in addition to the charged currents $\bar{\Lambda}p$ and $\bar{n}p$, the neutral currents $\bar{\Lambda}n$, $\bar{n}n$, $\bar{p}p$ or at least some of them are introduced. (Neutral currents are understood to be those containing, in the initial *and* final state, particles of the same charge.) It is easy to see, for example, that an interaction of the form

$$\bar{\Lambda}n(\bar{p}p + \bar{n}n)$$

satisfies the condition $\Delta T = \frac{1}{2}$. Indeed, the first factor in it is an isotopic spinor, while the second one is an isotopic scalar.

The question as to whether or not the weak interaction contains the product of neutral currents appears to be, at present, one of the most important problems in the physics of weak interactions. Experimental investigation of this problem is of great interest (see pp. 18 and 209).

Any real explanation to come of the $\Delta T = \frac{1}{2}$ rule will, undoubtedly, enhance our understanding of the properties, not only of the weak interactions, but also those of the strong. In view of the importance of this problem we shall consider, in detail, which experiments point to the existence of the $\Delta T = \frac{1}{2}$ rule and in which experiments it could be checked.

HYPERON DECAYS

Let us first find what the $\Delta T = \frac{1}{2}$ rule leads to in hyperon decays. We note that in the decay of Λ- and Σ-hyperons there arise nucleons and π-mesons, and we may express different charge states of the nucleon + π-meson system in terms of the corresponding isotopic states. It is easy to see that there are two such isotopic states: one with $T = \frac{1}{2}$ and the other one with $T = \frac{3}{2}$. In hyperon decays we have to do with the following states of the nucleon + pion system:

$\pi^- + p,\quad \pi^0 + n$ (in the Λ-hyperon decay),
$\pi^0 + p,\quad \pi^+ + n$ (in the Σ^+-hyperon decay),
$\pi^- + n$ (in the Σ^--hyperon decay).

Each of these states represents a linear superposition of the states with $T = \frac{1}{2}$ and $T = \frac{3}{2}$. The coefficients of this superposition are quoted in the table.

TABLE 7

Charge function	T_3	Coefficient of the function with $T = \frac{1}{2}$	Coefficient of the function with $T = \frac{3}{2}$
$\pi^- + p$	$-\frac{1}{2}$	$+\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
$\pi^0 + n$	$-\frac{1}{2}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$
$\pi^0 + p$	$+\frac{1}{2}$	$+\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$
$\pi^+ + n$	$+\frac{1}{2}$	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
$\pi^- + n$	$-\frac{3}{2}$	0	1

The coefficients contained in this table are the Clebsch–Gordan vector-addition coefficients.

Charge-exchange reactions transform the states $\pi^- + p$ and $\pi^0 + n$ into each other. Therefore these states are not diagonal. Their linear superpositions

$$\psi_{T=\frac{1}{2}} = \sqrt{(\tfrac{2}{3})}(\pi^- + p) - \sqrt{(\tfrac{1}{3})}(\pi^0 + n)$$

and

$$\psi_{T=\frac{3}{2}} = \sqrt{(\tfrac{1}{3})}(\pi^- + p) + \sqrt{(\tfrac{2}{3})}(\pi^0 + n)$$

are diagonal. By means of the table, analogous superpositions can easily be obtained for the states $\pi^0 + p$ and $\pi^+ + n$. The state $\pi^- + n$ is a pure state with $T = \frac{3}{2}$.

Λ-HYPERON DECAYS

If $\Delta T = \frac{1}{2}$, then the system $N + \pi$ that arises in the decay of Λ-hyperon is in a state with $T = \frac{1}{2}$. From the above it follows that the amplitudes a^- and a^0 of the decays

$$\Lambda \rightarrow p + \pi^- \qquad (a^-),$$
$$\Lambda \rightarrow n + \pi^0 \qquad (a^0)$$

should be in the ratio $\sqrt{(2)} : 1$, while the corresponding probabilities should be in the ratio $2 : 1$:

$$B_\Lambda = \frac{W(\Lambda^0 \to p + \pi^-)}{W(\Lambda^0 \to n + \pi^0) + W(\Lambda^0 \to p + \pi^-)} = \frac{2}{3}.$$

From the fact the amplitudes of the two decays are similar it follows that all of the correlations in these decays must be the same. In particular, the polarization of protons must be the same as that of neutrons, while the asymmetry parameter α^- of the π^--meson in the decay of polarized Λ-hyperons must be the same as the asymmetry parameter α^0 of π^0-mesons. Experimentally,

$$B_\Lambda \approx 0{\cdot}65 \pm 0{\cdot}05,$$

$$\alpha^0/\alpha^- = 1{\cdot}10 \pm 0{\cdot}27,$$

which is in good agreement with the theoretical prediction.

Σ-HYPERON DECAYS

We now establish relations between the decays

$$\Sigma^+ \to \pi^0 + p,$$

$$\Sigma^+ \to \pi^+ + n,$$

$$\Sigma^- \to \pi^- + n.$$

We denote the amplitude of these reactions by A^0, A^+ and A^- (according to the charge of the π-meson). (We shall not consider the Σ^0-hyperon decay, because the Σ^0-hyperon disintegrates through the action of the electromagnetic (and not the weak!) interaction into $\Lambda^0 + \gamma$ in about 10^{-19} sec, and has nothing in common with the decays of charged Σ-hyperons.) The fact that in these decays the $\Delta T = \frac{1}{2}$ rule holds can be expressed as isotopic spin conservation ($\Delta T = 0$), if it is postulated that each disintegrating hyperon absorbs a certain unobservable particle with $T = \frac{1}{2}$, which we shall call the "spurion" and denote by S. In essence, we have already dealt with this "particle" on p. 120, when we discussed the $\Delta T = \frac{1}{2}$ rule for leptonic decays. It should be stressed, once more, that the spurion is not a physical object but only a convenient mathematical construction. Introducing the spurion we can write Σ-hyperon decays in the form of the following isotopically invariant reactions:

$$S^0 + \Sigma^+ \rightarrow \pi^0 + p, \qquad A^0,$$

$$S^0 + \Sigma^+ \rightarrow \pi^+ + n, \qquad A^+,$$

$$S^0 + \Sigma^- \rightarrow \pi^- + n, \qquad A^-.$$

According to the definition, the spurion S^0 has $T = \frac{1}{2}$, $T_3 = -\frac{1}{2}$, $S = +1$ and is analogous in its isotropic properties to the K^0-meson.

It is easy to see that the state $S^0 + \Sigma^+$ is analogous to the state $n + \pi^+$, while the state $S^0 + \Sigma^-$ is analogous to the state $n + \pi^-$. In terms of isotopic states we have (see the table of the Clebsch–Gordan coefficients on p. 176):

$$\langle S^0 + \Sigma^+| = -\sqrt{(\tfrac{2}{3})}\varphi_{\frac{1}{2}} + \sqrt{(\tfrac{1}{3})}\varphi_{\frac{3}{2}},$$

$$\langle S^0 + \Sigma^-| = \varphi_{\frac{3}{2}},$$

$$|\pi^0 + p\rangle = \sqrt{(\tfrac{1}{3})}\psi_{\frac{1}{2}} + \sqrt{(\tfrac{2}{3})}\psi_{\frac{3}{2}},$$

$$|\pi^+ + n\rangle = -\sqrt{(\tfrac{2}{3})}\psi_{\frac{1}{2}} + \sqrt{(\tfrac{1}{3})}\psi_{\frac{3}{2}},$$

$$|\pi^- + n\rangle = \psi_{\frac{3}{2}}.$$

We know now the initial as well as final isotopic states. In reactions with spurions the isotopic spin is conserved. This means that the interaction will give matrix elements differing from zero only between states with the same isotopic spin. Taking this into account we obtain

$$A^0 = \frac{\sqrt{2}}{3}(A_3 - A_1),$$

$$A^+ = \tfrac{1}{3}(A_3 + 2A_1),$$

$$A^- = A_3.$$

As a result we may express the three charge amplitudes A^0, A^+, A^- in terms of two isotopic amplitudes $A_3(T = \frac{3}{2})$ and $A_1(T = \frac{1}{2})$. This allows one to establish a relation between A^0, A^+, A^-:

$$A^- = \sqrt{(2)}A^0 + A^+.$$

We now take into account the fact that each of the amplitudes A^0. A^+, A^- is actually a set of two amplitudes s and p, corresponding to $l = 0$ and $l = 1$:

$$A^0 = s^0 + p^0, \quad A^+ = s^+ + p^+, \quad A^- = s^- + p^-.$$

Therefore the relation between A^-, A^+ and A^0 is to be understood not as a relation between numbers but as a relation between the vectors in sp-space. Consequently, we have

$$s^- = \sqrt{(2)}s^0 + s^+,$$
$$p^- = \sqrt{(2)}p^0 + p^+.$$

For further considerations the following experimental facts are essential:

(1) The probabilities of all the three decays agree to within ~ 20 per cent.

(2) In the decays $\Sigma^- \to n + \pi^-$ and $\Sigma^+ \to n + \pi^+$ the up–down asymmetry is almost absent.

(3) In the $\Sigma^+ \to p + \pi^0$ decay the asymmetry is maximum.

From (3) it follows that $s^0 \approx p^0$. From (2) and the relations between the vectors A^+, A^- and A^0 it follows that

$$\text{either} \quad s^+ = p^- = 0, \tag{a}$$
$$\text{or} \quad s^- = p^+ = 0. \tag{b}$$

For the present we cannot choose between these two possibilities. It can easily be seen that both possibilities are consistent with (1). Indeed, consider, for example, (a). From the condition $s^- = \sqrt{(2)}s^0 + s^+$ and $p^- = \sqrt{(2)}p^0 + p^+$, taking into account the equality $s_0 = p_0$, we obtain

$$s^{-2} = 2s^{02} = 2p^{02} = p^{+2}.$$

Taking into account that

$$W^- = A^{-2} = s^{-2},$$
$$W^+ = A^{+2} = p^{+2},$$
$$W^0 = A^{02} = s^{02} + p^{02},$$

we obtain

$$W^- = W^+ = W^0.$$

As is easy to see, the same result is obtained also for possibility (b).

If a coordinate system is constructed such that the amplitude of the s-wave is plotted on one axis while that of the p-wave is plotted on the other axis, then the vector equality $A^- = \sqrt{(2)}A^0 + A^+$ can be shown in this diagram as a rectangular equilateral triangle, the legs of which are directed along the coordinate axes (Fig. 51).

It should be noted that the fact that the triangle representing the vector equality is a rectangular, isosceles triangle along the coordinate axis follows in no way from theory and is in itself rather enigmatic.

In placing the triangle with respect to the coordinate axes in the way shown in the drawing, we have made use of only one of the existing possibilities, namely of (a). Note, also, that the position of the triangle in the first quadrant is also arbitrary, because we do not know the sign of the corresponding s- and p-amplitudes.

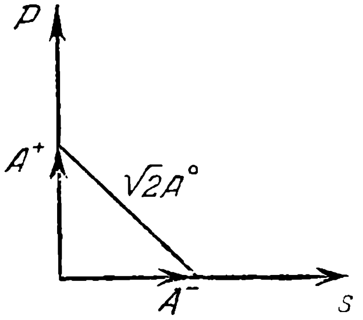

FIG. 51

Thus, the $\Delta T = \frac{1}{2}$ rule predicts that, if in one of the decays, for example in $\Sigma^+ \to n + \pi^+$, there is pure s-wave, then in the second one ($\Sigma^- \to n + \pi^-$) there must be pure p-wave, and vice versa. This prediction can be checked by measuring the neutron polarization in the decay of polarized hyperons. As we have seen on p. 168,

$$\text{for } s\text{-wave} \quad \boldsymbol{\zeta} = \boldsymbol{\eta},$$

$$\text{for } p\text{-wave} \quad \boldsymbol{\zeta} = -\boldsymbol{\eta} + 2(\boldsymbol{n}\,\boldsymbol{\eta})\,\boldsymbol{n}.$$

Here $\boldsymbol{\zeta}$ is the neutron polarization, $\boldsymbol{\eta}$ is the hyperon polarization, while $\boldsymbol{n}$ is a unit vector in the direction of the neutron emission.

If it turns out that in the two decays the correlation of the neutron and Σ-hyperon polarization is of the same form, then this will mean that the $\Delta T = \frac{1}{2}$ rule is violated.

Ξ-HYPERON DECAYS

For decays

$$\Xi^- \to \Lambda^0 + \pi^- \qquad (a^-),$$

$$\Xi^0 \to \Lambda^0 + \pi^0 \qquad (a^0),$$

making use of the concept of spurion, it is easy to obtain the ratio between the amplitudes:

$$a^- : a^0 = \sqrt{(2)} : 1\,.$$

This in turn means that

$$\frac{W(\Xi^- \to \Lambda^0\pi^-)}{W(\Xi^0 \to \Lambda^0\pi^0)} = 2$$

and that all correlations in these decays must be the same.

Let us see now what the $\Delta T = \frac{1}{2}$ rule gives for decays of K-mesons.

$K^+ \to \pi^+ + \pi^0(\theta^+)$ DECAY

Let us consider the possible isotopic states of two π-mesons arising from this decay. The system of two π-mesons, in general, can be in states with $T = 0, 1, 2$. However, in our case the state with $T = 0$ is excluded, because one of the π-mesons is charged while the other one is neutral and, consequently, $T_3 = +1$. The state with $T = 1$ is also forbidden. This is connected with the fact that the spin of the K-meson is equal to zero and that the two π-mesons arise in an s-state. But, being in a symmetric orbital state, the π-mesons, as Bose particles, must also be in a symmetric isotopic state, whereas their state with $T = 1$ is antisymmetric. (If $\boldsymbol{a}$ and $\boldsymbol{b}$ are the isotopic vectors of the first and second meson, respectively, then their state with $T = 1$ is described by the vector product of these vectors, $\boldsymbol{a} \times \boldsymbol{b}$.) Consequently, there remains only the state with $T = 2$. But if the $\Delta T = \frac{1}{2}$ rule holds, then, since the isotopic spin of the K-meson is $\frac{1}{2}$, the transition to this state is forbidden.

Thus, if the $\Delta T = \frac{1}{2}$ rule holds, the $K^+ \to \pi^+ + \pi^0$ decay should be forbidden.

It is known that in fact this decay is not forbidden, and its rate W_{θ^+} amounts to about $2 \times 10^7\ \text{sec}^{-1}$. However, if we compare this decay rate with the probability of decay of the neutral K-meson into two π-mesons, we find that, although there is no strict forbiddenness, none the less there is a very strong suppression. Indeed, $W_{\theta^0} \approx 1 \times 10^{10}\ \text{sec}^{-1}$. We see that the ratio of the θ^+-decay rate to the θ^0-decay rate amounts to $\sim 2 \times 10^{-3}$.

Thus, from the consideration of the θ^+-decay we can conclude that, although the $\Delta T = \frac{1}{2}$ rule is not absolutely strict, it is nevertheless fulfilled with very high accuracy.

$K_1^0 \to \pi^+ + \pi^-$ AND $K_1^0 \to \pi^0 + \pi^0$ DECAYS

The π-meson state with $T = 2$ is forbidden in these decays by the $\Delta T = \frac{1}{2}$ rule. The state with $T = 1$ is forbidden by virtue of the symmetry of the π-meson wave function. The state with $T = 0$ is allowed.

We shall describe the isotopic wave functions of each of the π-mesons by the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. In order to make out of these vectors a state with isotopic spin zero, one must form their scalar product

$$\boldsymbol{ab} = a_1 b_1 + a_2 b_2 + a_3 b_3 .$$

Taking into account that the π^+-meson is described by the wave function $a_+ = \dfrac{a_1 + ia_2}{\sqrt{2}}$, the π^--meson by $a_- = \dfrac{a_1 - ia_2}{\sqrt{2}}$, and the π^0-meson by $a_0 = a_3$ (and similarly for the vector $\boldsymbol{b}$), we rewrite the scalar product in new variables:

$$\boldsymbol{ab} = a_+ b_- + a_- b_+ + a_0 b_0 .$$

Because the probability of formation of the charged mesons is proportional to $|a_+ b_-|^2 + |a_- b_+|^2$, while that of neutral mesons is proportional to $|a_0 b_0|^2$, from the $\Delta T = \frac{1}{2}$ rule it follows that

$$\frac{W(K_1^0 \to \pi^+ + \pi^-)}{W(K_1^0 \to 2\pi^0)} = 2,$$

or

$$B(K) = \frac{W(K_1^0 \to 2\pi^0)}{W(K_1^0 \to \pi^+ + \pi^-) + W(K_1^0 \to 2\pi^0)} = \frac{1}{3}.$$

Taking into account a small admixture of amplitude with $\Delta T = \frac{3}{2}$ the value of B may lie within a range 0·29 to 0·37. $B(K)$ is found experimentally to be of the order of 0·30 with an error of the order of 0·04–0·06, which is in excellent agreement with theory.

$K_{\pi 3}$-DECAYS

We shall now determine those relations which, because of the $\Delta T = \frac{1}{2}$ rule, hold between the probabilities of the four different τ-decays.

1. $K^+ \to \pi^+ + \pi^- + \pi^+ (\tau)$,
2. $K^+ \to \pi^0 + \pi^0 + \pi^+ (\tau')$,
3. $K_2^0 \to \pi^0 + \pi^0 + \pi^0$,
4. $K_2^0 \to \pi^0 + \pi^+ + \pi^-$.

π-mesons arising from these decays are in an s-state (there is also a small admixture of symmetric states with $l = L = 2, 4$; see p. 157). We denote the isotopic functions of the three π-mesons by $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. Then this symmetric state will be described by the isotopic function

$$A = \boldsymbol{a}(\boldsymbol{bc}) + \boldsymbol{b}(\boldsymbol{ca}) + \boldsymbol{c}(\boldsymbol{ab}).$$

The component A^+ corresponds to the decays τ and τ':

$$A^+ = a_+(\boldsymbol{bc}) + b_+(\boldsymbol{ca}) + c_+(\boldsymbol{ab}).$$

The component A^0 corresponds to the neutral $K_{\pi 3}$-decays:

$$A^0 = a_0(\boldsymbol{bc}) + b_0(\boldsymbol{ca}) + c_0(\boldsymbol{ab}).$$

We write A^+ and A^0 in the explicit form:

$$\begin{aligned} A^+ &= a_+b_+c_- + a_+b_-c_+ + a_+b_0c_0 + b_+c_+a_- + b_+c_-a_+ + b_+c_0a_0 \\ &\quad + c_+a_+b_- + c_+a_-b_+ + c_+a_0b_0, \\ A^0 &= a_0b_+c_- + a_0b_-c_+ + a_0b_0c_0 + b_0c_+a_- + b_0c_-a_+ + b_0c_0a_0 \\ &\quad + c_0a_+b_- + c_0a_-b_+ + c_0a_0b_0. \end{aligned}$$

Between the corresponding decay rates there exist the following relations:

$$\frac{W(K^+ \to 2\pi^+ + \pi^-)}{W(K^+ \to 2\pi^0 + \pi^+)} = \frac{|2a_+b_+c_-|^2 + |2b_+c_+a_-|^2 + |2a_+c_+b_-|^2}{|a_+b_0c_0|^2 + |b_+c_0a_0|^2 + |c_+a_0b_0|^2} = \frac{12}{3} = 4,$$

$$\frac{W(K_2^0 \to 3\pi^0)}{W(K_2^0 \to \pi^+ + \pi^- + \pi^0)}$$

$$= \frac{|3a_0b_0c_0|^2}{|a_0b_+c_-|^2 + |a_0b_-c_+|^2 + |b_0c_+a_-|^2 + |b_0c_-a_+|^2 + |c_0a_+b_-|^2 + |c_0a_-b_+|^2}$$

$$= \frac{9}{6} = \frac{3}{2}.$$

Making use of the idea of the spurion, it is easy to establish a relation between the amplitudes of the transitions

$$K^0 \to A^0 + S^0 \quad \text{and} \quad K^+ \to A^+ + S^0.$$

By means of our tabel of Clebsch–Gordan coefficients we find that these amplitudes are in the ratio of $1 : \sqrt{2}$. Taking into account that $K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0)$ we find that the amplitudes of the

transitions
$$K_2^0 \to A^0 \quad \text{and} \quad K^+ \to A^+$$
should be the same. Consequently, the total probabilities of the $K_{\pi 3}$-decay of the K^+ and K_2^0 mesons should also be equal:

$$\frac{W(K_{\pi 3}^+)}{W(K_{2\pi 3}^0)} = \frac{W(K^+ \to 2\pi^+ + \pi^-) + W(K^+ \to 2\pi_0 + \pi^+)}{W(K_2^0 \to 3\pi^0) + W(K_2^0 \to \pi^+ + \pi^- + \pi^0)} = 1 .$$

The ratios 4 : 1, 3 : 2 and 1 : 1 will be somewhat changed if we take into account that, because of the mass difference $\pi^\pm - \pi^0$, the phase space for emitting π^0-mesons is larger than that for emitting charged π-mesons. The replacement of $\pi^\pm$ by π^0 increases the phase space by about 13 per cent. Furthermore, because of the mass difference $K_2^0 - K^+$ the phase space for the decay of the K_2^0 would be 10 per cent larger than that for the decay of the K^+, even if the masses of $\pi^\pm$ and π^0 were the same. Hence

$$W(K^+ \to 2\pi^0 + \pi^+) : W(K^+ \to 2\pi^+ + \pi^-) \approx 1{\cdot}26 : 4 \approx 0{\cdot}32,$$
$$W(K_2^0 \to 3\pi^0) : W(K_2^0 \to \pi^+ + \pi^- + \pi^0) \approx 3 \times 1{\cdot}49 : 2 \times 1{\cdot}23 \approx 1{\cdot}8,$$
$$W(K_2^0 \to 3\pi) : W(K^+ \to 3\pi) \approx (3 \times 1{\cdot}49 + 2 \times 1{\cdot}23) : (4 + 1{\cdot}26) \approx 1{\cdot}3.$$

The first of these three ratios is in good agreement with experiment. The two remaining ones do not contradict experiment, but the experimental errors are extremely large. Verification of these ratios is necessary for deciding the question of the applicability of the $\Delta T = \frac{1}{2}$ rule.

It should be noted that because the π-mesons in the $K_{\pi 3}$-decay are in a symmetric s-state even the presence of transitions with $\Delta T = \frac{3}{2}$ would not change the first two ratios. This is connected with the fact that there only exist two completely symmetric states, which can be made from the three vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$; these are the state with $T = 1$ considered above and a state with $T = 3$. But a transition to the latter is possible only in the case when $\Delta T = \frac{5}{2}$. As distinct from the first two ratios, the third one is sensitive to the admixture of transitions with $\Delta T = \frac{3}{2}$.

LIFETIME OF THE K_2^0-MESON

The set of $\Delta T = \frac{1}{2}$ rules for leptonic and non-leptonic decays makes it possible, knowing the probabilities of K^+-decays, to predict those of individual K_2^0-decays and the lifetime of the K_2^0-meson.

The probabilities of different decays of the K^+-meson are equal to

$$\begin{array}{lll}
K_{\mu 2} & K^+ \to \mu^+ + \nu & 48 \times 10^6 \text{ sec}^{-1} \\
K_{\pi 2} & K^+ \to \pi^+ + \pi^0 & 21 \times 10^6 \text{ sec}^{-1} \\
K_{\pi 3} & K^+ \to 2\pi^+ + \pi^- & 4{\cdot}6 \times 10^6 \text{ sec}^{-1} \\
 & K^+ \to 2\pi^0 + \pi^+ & 1{\cdot}4 \times 10^6 \text{ sec}^{-1} \\
K_{e3} & K^+ \to e^+ + \nu + \pi^0 & 3{\cdot}4 \times 10^6 \text{ sec}^{-1} \\
K_{\mu 3} & K^+ \to \mu^+ + \nu + \pi^0 & 3{\cdot}3 \times 10^6 \text{ sec}^{-1} \\
\hline
 & & \dfrac{1}{\tau_{K^+}} = 81{\cdot}7 \times 10^6 \text{ sec}^{-1}
\end{array}$$

Making use of these data, we obtain for the K_2^0-meson

$$\begin{array}{ll}
K_{\pi 3} \begin{cases} K_2^0 \to 3\pi^0 \\ K_2^0 \to \pi^+ + \pi^- + \pi^0 \end{cases} & \left.\begin{array}{l} 5{\cdot}0 \times 10^6 \text{ sec}^{-1} \\ 2{\cdot}8 \times 10^6 \text{ sec}^{-1} \end{array}\right\} \\
K_{e3} \begin{cases} K_2^0 \to e^+ + \nu + \pi^- \\ K_2^0 \to e^- + \tilde{\nu} + \pi^+ \end{cases} & \left.\begin{array}{l} 3{\cdot}4 \times 10^6 \text{ sec}^{-1} \\ 3{\cdot}4 \times 10^6 \text{ sec}^{-1} \end{array}\right\} \\
K_{\mu 3} \begin{cases} K_2^0 \to \mu^+ + \nu + \pi^- \\ K_2^0 \to \mu^- + \tilde{\nu} + \pi^+ \end{cases} & \left.\begin{array}{l} 3{\cdot}3 \times 10^6 \text{ sec}^{-1} \\ 3{\cdot}3 \times 10^6 \text{ sec}^{-1} \end{array}\right\} \\
\hline
 & \dfrac{1}{\tau_{K_2^0}} = 21{\cdot}2 \times 10^6 \text{ sec}^{-1}
\end{array}$$

Thus it follows that the lifetime of the K_2^0-meson should be about 5×10^{-8} sec. Experiment gives a value close to 6×10^{-8} sec.

From all this it follows that an accurate measurement of the lifetime and branching ratios of the individual decays of the K_2^0-meson would be of great interest.†

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† The numerical values of decay rates given in this paragraph are outdated. For a comparison of experimental decay rates for K^+ and K_2^0 mesons see Tables 9 and 10 on pp. 221 and 222 (note added in 1964).

CHAPTER 15

DUAL PROPERTIES OF NEUTRAL *K*-MESONS

WE NOW consider in more detail the properties of neutral *K*-mesons and the $K_1 - K_2$ complex. In strong interactions neutral *K*-mesons appear in the form of the particles K^0 and $\tilde{K}^0$, having a definite strangeness. The strangeness of K^0 is $+1$, while that of $\tilde{K}^0$ is -1.

In the decays of neutral *K*-mesons into π-mesons there do not appear K^0 and $\tilde{K}^0$ but K_1^0 and K_2^0, which have a definite *CP*-parity. The combined parity of K_1^0 equals $+1$, while that of K_2^0 equals -1 (see p. 156).

We ask ourselves the questions: what is the combined parity of K^0 or $\tilde{K}^0$, and what is the strangeness of K_1^0 or K_2^0? The answer to these questions is the following: K^0 and $\tilde{K}^0$ have no definite value of the combined parity, and K_1^0 and K_2^0 have no definite strangeness. This is connected with the fact that under combined inversion the strangeness changes sign, like electric charge. A particle having a non-zero value of strangeness under *CP*-inversion goes over to its antiparticle (different from itself!) and hence cannot have a definite value of the combined parity.

ANALOGY WITH ANGULAR MOMENTUM

It should be noted that the situation of two mutually incompatible descriptions of the same system is not something new in physics. A system which has an angular momentum *I* and is placed in a magnetic field is in this sense very similar to neutral *K*-mesons. It is well known that the total angular momentum of the system commutes with its projections on any of the axes x, y, z. However, these projections do not commute with one another. Therefore, if, for example, a projection on the x-axis is given, projections on the y- and z-axis will have no definite values. This is similar to the fact that, if the strangeness of the *K*-meson is fixed, its combined parity has no definite value. In the absence of strong as well as weak interactions the K-meson is equivalent to a system which has an angular momentum and is placed in a space where there are no magnetic fields. The switching on of the strong interaction of K-mesons with nuclei, which depends on the value of the strangeness, can be compared, for instance, with that of a magnetic field along the x-axis. Then the switching on of the weak decay interaction of K-mesons with π-mesons, which depends on the value of the combined parity, can be compared with that of a magnetic field along the y- or z-axis.

WHY NOT n_1 AND n_2?

What gives rise to the possibility of analysing K-mesons on the basis of two different quantum characteristics—combined parity and strangeness? Why, for example, cannot the same be done with neutrons? The neutron, like the K^0, is a neutral particle and, one might say that one could consider the linear superpositions of the neutron and antineutron

$$n_1 = \frac{n + \tilde{n}}{\sqrt{2}} \quad \text{and} \quad n_2 = \frac{n - \tilde{n}}{\sqrt{2}}.$$

True, as distinct from K^0-mesons, the strangeness of neutrons is equal to zero. But, to make up for it, neutrons have a baryonic charge, which equals $+1$ for the neutron, and -1 for the antineutron. Therefore the neutron and antineutron, like K^0 and $\tilde{K}^0$, have no definite combined parity. On the other hand, n_1 and n_2 have, like K_1^0 and K_2^0, a definite combined parity and no definite baryonic charge.

Why do we always speak about the neutron and antineutron, and never about n_1 and n_2? This is connected with the fact that there are no processes in which the states n_1 and n_2 would be diagonal; there is no interaction which could transform the neutron into the antineutron. The baryon charge conservation law appears to be strict.

As distinct from baryonic charge, strangeness is not a strictly conserved quantum number. It is this that accounts for the peculiarity of neutral K-mesons.

In general, all particles can be classified into two groups. One of these contains such particles as the proton, neutron, electron and Λ-hyperon, the antiparticles of which differ from the particles by some strictly conserved quantum numbers (for example, by the electric or baryonic charge). The other group contains particles which are identical with their antiparticles (photon, π^0-meson). Such particles are absolutely neutral. Neutral K-mesons behave as if they are just at the boundary of these two groups. K^0 differs from $\tilde{K}^0$ by the value of the strangeness quantum number. However, this difference is not absolute, because strangeness conservation is violated by the weak interaction.

Can particles or systems of particles analogous to neutral K-mesons exist?

MUONIUM

A very good example of such a system is muonium (an atom consisting of μ^+ and e^-). As pointed out by Pontecorvo, muonium could go over to antimuonium, if the transition of the muon into the electron

$$\mu^+ e^- \leftrightarrow \mu^- e^+.$$

were not forbidden. Then the diagonal states of muonium in vacuum are

$$\text{muon one} = \frac{\mu^+ e^- + \mu^- e^+}{\sqrt{2}}$$

and

$$\text{muon two} = \frac{\mu^+ e^- - \mu^- e^+}{\sqrt{2}}.$$

The first of these states has an even combined parity, while the second one has an odd combined parity. In strong electric fields the diagonal states would be the muonium and antimuonium. In solids the fields are so large that the transition of muonium into antimuonium should be absent. In rarefied gases such transitions might take place (if there were no muonic charge conservation law), but, because of the loose structure of the muonium, their characteristic time would be substantially larger than the muon lifetime.

We now return to K-mesons and study the consequences of the fact that neutral K-mesons represent such a peculiar superposition of four states. We consider first phenomena in the neutral K-meson beam, which result from the difference in the lifetime of K_1^0- and K_2^0-mesons.

PAIS–PICCIONI GEDANKEN EXPERIMENT

The properties of neutral K-mesons are manifested very clearly in the classical experiment proposed by Pais and Piccioni in 1955. Speaking literally, this experiment has so far not been performed. However, its ideas in a modified form have been utilized in a number of experiments.

Imagine a chamber on which a π-meson beam is incident (Fig. 52). In plate A there occurs the $\pi^- + p \to K^0 + \Lambda^0$ reaction.

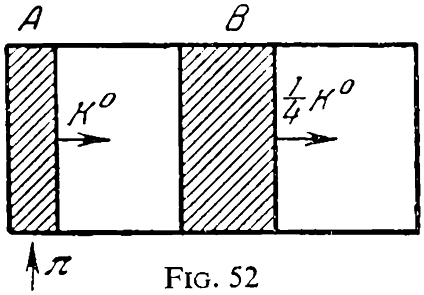

FIG. 52

We shall follow the fate of K^0-mesons emitted to the right of the plate A. These K^0-mesons represent the superposition of K_1^0 and K_2^0:

$$K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0).$$

The K_1^0-particle disintegrates in a time of the order of 10^{-10} sec ($K_{\pi 2}$-decay).

The K_2^0-particle decays in a time of the order of 5×10^{-8} sec ($K_{\pi 3}$-, K_{e3}-, $K_{\mu 3}$-decays). Therefore the decay of K_2^0-mesons can be disregarded in comparison with that of K_1^0-mesons. Within ten centimetres or so practically all of K_1^0-mesons contained in the K^0-beam will decay, and a pure K_2^0-meson beam will remain:

$$K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0) \xrightarrow[K_1^0]{\text{decay}} \frac{1}{\sqrt{2}}K_2^0.$$

Recall now that

$$K_2^0 = \frac{1}{\sqrt{2}}(K^0 + \tilde{K}^0).$$

Thus we find that if the initial beam had a strangeness equal to $+1$ (K^0-mesons), then in the final beam there are also particles with strangeness equal to -1 ($\tilde{K}^0$-mesons). If a plate B is put in the path of K_2^0-mesons, then in the plate there will arise nuclear reactions of the type

$$\tilde{K}^0 + p \rightarrow \Lambda^0 + \pi^+,$$

which cannot be caused by initial K^0-mesons.

As distinct from K^0-mesons, $\tilde{K}^0$-mesons are intensely absorbed in colliding with nuclei. Assume that plate B absorbs all $\tilde{K}^0$-mesons and lets through all K^0-mesons. Then we shall have such a sequence of transformations:

$$K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0) \overset{\text{decay}}{\xrightarrow{K_1^0}} \frac{1}{\sqrt{2}} K_2^0 = \frac{1}{2}(K^0 + \tilde{K}^0) \xrightarrow{\tilde{K}^0\text{-absorption}} \frac{1}{2} K^0.$$

Taking into account the fact that $K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0)$, we expect that behind plate B there will again be K_1^0-decays, their intensity being four times less than that of K_1^0-decays near plate A. We have here to do with regeneration of K_1^0-mesons.

$K_1^0 - K_2^0$ MASS DIFFERENCE

In considering the $K^0 \rightarrow \tilde{K}^0$ transition we have so far taken into account only the lifetime difference between K_1^0- and K_2^0-mesons. If the states K_1^0 and K_2^0 are written in the form of vectors (Fig. 53), then from the above it is seen that we only used the fact that the length of one of the vectors decreases with time more rapidly than that of the other one. However, along with this mechanism of the transition of K^0 to $\tilde{K}^0$ another mechanism of this transition, due to the mass difference between K_1^0- and K_2^0-mesons, is very essential. The $K_1^0 - K_2^0$ mass difference is the cause of the fact that the angle between the vectors K_1^0 and K_2^0 in our diagram (Fig. 55) varies with time ($\varphi = \Delta m t$).

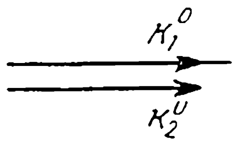

FIG. 53

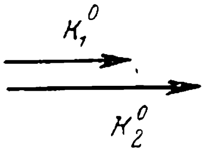

FIG. 54

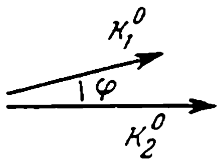

FIG. 55

To what is the $K_1^0 - K_2^0$ mass difference due? Because the combined parities of the K_1^0- and K_2^0-meson are different, not only real states into which these mesons can transform but also virtual ones are different. Thus, transitions to $2\pi^0$, $4\pi^0$, $6\pi^0$ and so on are allowed for K_1^0, while those to $1\pi^0$, $3\pi^0$, $5\pi^0$ and so on are allowed for K_2^0. If states in which K_1^0- and K_2^0-mesons spend a part of their time are different, then it is natural that the masses of these particles should also be different. The $K_1^0-K_2^0$ mass difference is determined by the mass difference between virtual states and by the time interval during which K_1^0- and K_2^0-mesons are in these virtual states. In essence, to estimate Δm we have to estimate the contribution of matrix elements of the type shown in Fig. 56.

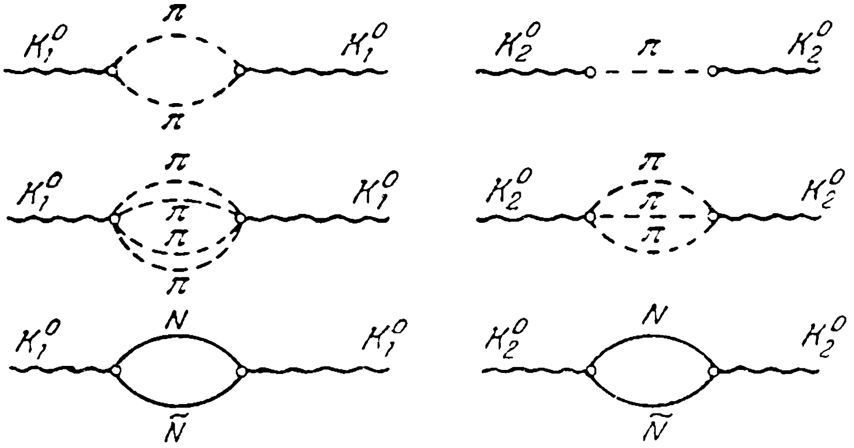

FIG. 56

Since every transformation with a change of strangeness is proportional to the weak interaction constant G, the contribution of these matrix elements must also be proportional to G^2. Consequently, the $K_1^0-K_2^0$ mass difference must be proportional to G^2: $\Delta m = \alpha G^2$. The coefficient α must have the dimensions of $[m]^5$. If $\alpha = m_\pi^5$ is chosen, then we obtain

$$\Delta m = 10^{-10}\frac{m_\pi^5}{m_p^4} \approx 10^{-14} m_K \approx 10^{-5}\,\mathrm{eV} \approx 10^{10}\,\mathrm{sec}^{-1}.$$

For $\alpha = m_K^5$ we would obtain $\Delta m \approx 10^{-11}\ m_K \approx 10^{13}\,\mathrm{sec}^{-1}$. The value of α depends on which effective energies of virtual particles play a part. If these energies were very large (of the order of a hundred BeV), then the value of Δm would be many orders of magnitude larger than $10^{10}\,\mathrm{sec}^{-1}$. However, experiment yields $\Delta m \sim 10^{10}\,\mathrm{sec}^{-1}$. Therefore it can be assumed that the effective energies of the virtual particles are in reality not large (of the order of 1 BeV). Unfortunately, because we cannot treat virtual strong interactions, we cannot as yet calculate the $K_1^0-K_2^0$ mass difference, and we have to be content with qualitative estimates. We cannot even predict which of these mesons is heavier.

It should be stressed that the $K_1^0 - K_2^0$ mass difference would exist even if the decays of these particles and their lifetimes were absolutely the same. Imagine for the moment a fantastic universe in which π-mesons are heavier than K-mesons. Then the real decay

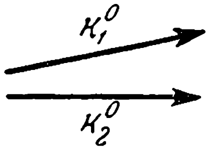

FIG. 57

of K-mesons into π-mesons would be forbidden and, because leptonic decays of K_1^0- and K_2^0-mesons have the same probabilities, these mesons would have the same lifetimes. But, even in this case, because of the difference in the virtual states, their masses would be different. In the vector diagram above the length of the vectors K_1^0 and K_2^0 would not be changed, and only the angle between them would alter (Fig. 57).

The $K_1^0 - K_2^0$ mass difference could be equal to zero only if the strangeness were a strictly conserved quantum number and if the $K^0 \leftrightarrow \tilde{K}^0$ transition were forbidden. This is seen particularly clearly from the equations describing the variation of the wave functions K and $\tilde{K}$ with time:

$$-i\dot{K} = mK + \delta\tilde{K},$$
$$-i\dot{\tilde{K}} = m\tilde{K} + \delta K.$$

The dot denotes a derivative with respect to the time, while δ denotes the matrix element of the $K \leftrightarrow \tilde{K}$ transition. We disregard decays. Adding and subtracting these equations, we obtain

$$-i\dot{K}_1 = (m - \delta)\, K_1,$$
$$-i\dot{K}_2 = (m + \delta)\, K_2,$$

It follows that $\Delta m = 2\delta$. Note that if there were an interaction with $\Delta S = 2$, then the amplitude of the $K^0 \to \tilde{K}^0$ transition, as mentioned before (see p. 173), would be proportional to G and not to G^2. In this case the $K_1^0 - K_2^0$ mass difference would be equal to

$$\delta m \sim G\, m_\pi^3 \sim 10^{-5} \cdot \frac{m_\pi^3}{m_p^2} \sim 10^{-7}\, m_K \sim 10^{17}\ \mathrm{sec}^{-1}.$$

An example of an interaction which might give transitions with $\Delta S = 2$ is an interaction of the type $(\bar{\Lambda} n)(\bar{\Lambda} n)$ (Fig. 58). As pointed out by Glashow, only the parity conserving part of this interaction would give the $K^0 \rightarrow \tilde{K}^0$ transition.

The experimental fact that $\Delta m_{K_1 K_2} \approx 10^{10}\ \text{sec}^{-1}$ means that the interaction with $\Delta S = 2$, even if it existed, does not contain any parity conserving terms (contrary to the universal V–A-interaction).

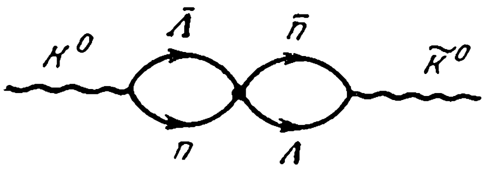

FIG. 58

Let us examine once again the Pais–Piccioni experiment and derive the formulae corresponding to the vector diagrams drawn above. At the time $t = 0$, the beam would represent a pure K^0-meson state:

$$K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0) \quad \text{at} \quad t = 0.$$

After a time t we have

$$\frac{1}{\sqrt{2}}\left(K_1^0 e^{im_1 t - \frac{\gamma_1 t}{2}} + K_1^0 e^{im_2 t - \frac{\gamma_2 t}{2}}\right),$$

where γ_1 and γ_2 are the decay rates, while m_1 and m_2 are the masses of the K_1^0-meson and K_2^0-meson, respectively. The time t is measured in the rest system of the K-meson. Making use of the definition of K_1^0 and K_2^0, we rewrite the preceding expression in the form

$$\tfrac{1}{2}\left[K_0\left(e^{im_1 t - \frac{\gamma_1 t}{2}} + e^{im_2 t - \frac{\gamma_2 t}{2}}\right) - \tilde{K}^0\left(e^{im_1 t - \frac{\gamma_1 t}{2}} - e^{im_2 t - \frac{\gamma_2 t}{2}}\right)\right].$$

Knowing the beam wave function, it is easy to find how many K^0- and $\tilde{K}^0$-mesons will be in the beam after a time t:

$$N(K^0) = \tfrac{1}{4}\left[e^{-\gamma_1 t} + e^{-\gamma_2 t} + 2e^{-\frac{\gamma_1+\gamma_2}{2}t}\cos(\Delta m\,.\,t)\right],$$

$$N(\tilde{K}^0) = \tfrac{1}{4}\left[e^{-\gamma_1 t} + e^{-\gamma_2 t} - 2e^{-\frac{\gamma_1+\gamma_2}{2}t}\cos(\Delta m t)\right],$$

$$\text{for} \quad t = 0, \quad N(K^0) = 1, \quad N(\tilde{K}^0) = 0,$$

$$\text{for} \quad \frac{1}{\gamma_2} \gg t \gg \frac{1}{\gamma_1}, \quad N(K^0) = N(\tilde{K}^0) = \frac{1}{4}.$$

It is seen that, measuring the number of $\tilde{K}^0$-mesons in a beam, which initially consisted of K^0-mesons, as a function of the "age" of the beam, one can determine Δm. The appearance of $\tilde{K}^0$-mesons as a result of the "ageing" of the K^0-mesons will show up in the formation of Λ-hyperons in the matter and their subsequent decay.

OSCILLATIONS OF LEPTONIC DECAYS

The leptonic decays K_{e3} and $K_{\mu 3}$ may serve as a good analyser of K^0- and $\tilde{K}^0$-states. If the $\Delta Q = \Delta S$ rule is valid, then the decays

$$K^0 \to e^+ + \nu + \pi^-, \qquad K^0 \to \mu^+ + \nu + \pi^-,$$
$$\tilde{K}^0 \to e^- + \tilde{\nu} + \pi^+, \qquad \tilde{K}^0 \to \mu^- + \tilde{\nu} + \pi^+$$

are allowed, while the decays

$$K^0 \to e^- + \tilde{\nu} + \pi^+, \qquad K^0 \to \mu^- + \tilde{\nu} + \pi^+,$$
$$\tilde{K}^0 \to e^+ + \nu + \pi^-, \qquad \tilde{K}^0 \to \mu^+ + \nu + \pi^-$$

are forbidden. Thus, the number of decays which give rise to e^+ (or μ^+) (N^+) is proportional to $N(K^0)$, while the number of decays which give rise to e^- (or μ^-) (N^-) is proportional to $N(\tilde{K}^0)$. Whence it follows that

$$R = \frac{N^+ - N^-}{N^+ + N^-} = \frac{2e^{-\frac{\gamma_1+\gamma_2}{2}t}\cos(\Delta m t)}{e^{-\gamma_1 t} + e^{-\gamma_2 t}}.$$

From this formula it is seen that the ratio R changes its sign with a period $T = 2\pi/\Delta m$. The value of R differs substantially from zero, in so far as both K_2^0- and K_1^0-mesons are present in the beam ($t \lesssim 1/\gamma_1$). Comparison of the formula for R with experiment would enable one to find the absolute value of Δm and to check the $\Delta Q = \Delta S$ rule.†

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† Experimental data on the K_{e3}-decays of neutral K-mesons are in agreement with the $\Delta Q = \Delta S$ rule; see p. 221 (note added in 1964).

CHAPTER 16

DUAL PROPERTIES OF NEUTRAL K-MESONS (*continued*)

THREE TYPES OF REGENERATION

As mentioned above, when a beam of K_2^0-mesons passes through matter an admixture of K_1^0-mesons will arise in the beam, because K and $\tilde{K}$ possess different nuclear properties.

As a result of interaction with nucleons the K wave will be multiplied by a certain number a, and the $\tilde{K}$ wave by a certain number b; a and b are, in general, complex and contain the phase shifts of the waves as well as their damping:

$$K_2 = \frac{K + \tilde{K}}{\sqrt{2}} \rightarrow \frac{aK + b\tilde{K}}{\sqrt{2}} = \frac{a(K_1 + K_2)}{2} - \frac{b(K_1 - K_2)}{2}$$
$$= \frac{a - b}{2} K_1 + \frac{a + b}{2} K_2 .$$

If the nuclear properties of K and $\tilde{K}$ were the same, then a would be equal to b and no K_1^0-mesons would appear. We can call the quantity $(a - b)/2$ the amplitude of the $K_2^0 \rightarrow K_1^0$ transition. The regeneration of K_1^0-mesons, occurring when a beam of K_2^0-mesons passes through matter, can proceed in three ways: (1) on account of the interaction of the K-meson wave with individual nucleons of the nucleus; (2) on account of the K-meson wave scattering on nuclei (basically, diffraction scattering); and (3) on account of the coherent interaction of the K-meson wave with groups of nuclei. The angular distribution for each of these mechanisms is different. The basic contribution to the forward regeneration is given by the two last mechanisms. As shown by Good, the ratio of the intensity of K_1^0-mesons regenerated by the coherent interaction to that of K_1^0-mesons regenerated by the diffraction scattering depends strongly on the $K_1^0 - K_2^0$ mass difference Δm. This fact was used by Piccioni and collaborators for the measurement of Δm. Let us look at the idea of their experiment.

PICCIONI'S EXPERIMENT

Let f_{21} be the amplitude for regeneration of K_1^0's on an individual nucleus. We calculate the amplitude of the coherent K_1^0-wave produced as a beam of K_2^0-particles passes through a thin plate of thickness dx (Fig. 59). The wave coming to a point A from an area with its centre at point B is equal to

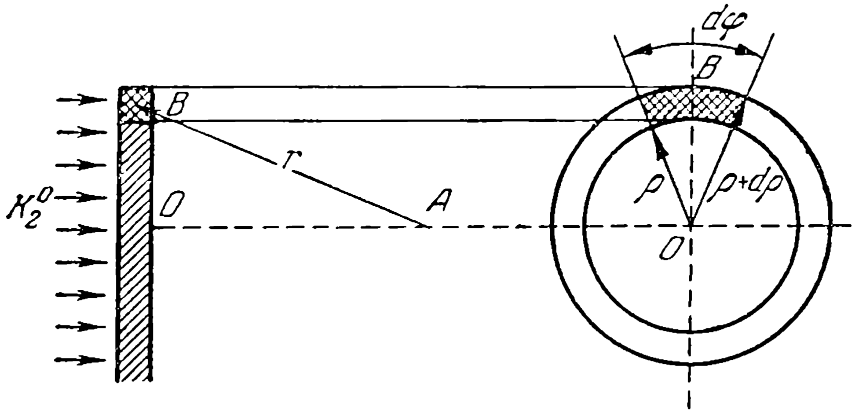

FIG. 59

$$f_{21} N\, dx \frac{e^{-ik_1 r}}{r} \varrho\, d\varrho\, d\varphi,$$

where N is the number of atoms in 1 cm^3. For the sake of simplicity we shall assume that f_{21} does not depend on the angle. Taking advantage of the fact that $r\,dr = \varrho\, d\varrho$, and integrating with respect to φ and r, we obtain

$$\frac{2\pi i}{k_1} f_{21} N\, dx e^{-ik_1 x} = i\lambda f_{21} N\, dx e^{-ik_1 x},$$

where $\lambda = 2/k_1$.

If we take into account that f_{21} is small, and if transitions $K_1 \to K_2$ are disregarded, then it is easy to obtain an analogous formula for a plate of finite thickness L (Fig. 60).

The K_1^0-wave, generated in a layer dx at a depth x, will come to the right edge of the plate with the amplitude

$$da_1 = iN\lambda f_{21}\, dx \exp\left\{-ik_2 x - ik_1(L-x) - \frac{L-x}{2v\gamma\tau_1} - \frac{L}{2u}\right\}.$$

Here u is the collision length, which is the same for K_1^0 and K_2^0, v is the velocity of the particles, while $\gamma = (1 - v^2)^{-\frac{1}{2}}$. The first term in the exponent corresponds to the propagation of the K_2^0-

wave to a depth x, the second one corresponds to the propagation of the K_1^0-wave, the third one describes the damping of the K_1^0-wave on account of the decay of the K_1^0-mesons and, finally, the last one describes the absorption of K_2^0- and K_1^0-mesons, due to nuclear collisions. We denote the decay length of the K_1^0-mesons by $\Lambda (\Lambda = v\gamma\tau_1)$, and introduce the dimensionless quantities

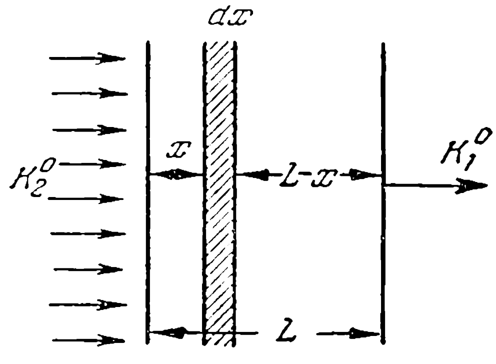

FIG. 60

$$l = \frac{L}{\Lambda} \quad \text{and} \quad \delta = |m_2 - m_1|\tau_1.$$

Let us relate the difference between the momenta k_2 and k_1 to that between the masses m_2 and m_1 of K_2^0- and K_1^0-mesons. The energy of the K-meson in small-angle scattering on a heavy nucleus practically does not change, hence

$$k_1^2 + m_1^2 = k_2^2 + m_2^2.$$

It follows at once that

$$k\Delta k + m\Delta m = 0,$$

$$\Delta k = -\frac{m}{k}\Delta m.$$

Integrating da_1 with respect to x from 0 to L, and taking the square of the absolute value of the expression obtained, we find the intensity of the coherent K_1^0-wave:

$$|a_1|^2 = \frac{4|f_{21}|^2 N^2 \Lambda^2 \lambda^2}{1 + 4\delta^2} |e^{-i\delta l} - e^{-l/2}|^2 \, e^{-L/u}.$$

Consider the diffraction production of K_1^0-mesons on individual nuclei. The differential production cross section is equal to

$$\frac{d\sigma_{21}}{d\omega} = |f_{21}|^2.$$

The number of K_1^0-mesons generated in a layer dx and traversing the layer $L - x$ in the direction of the incident beam is equal to

$$d\left(\frac{dn_1}{d\omega}\right)^0 = |f_{21}|^2 N \,.\, \exp\left(-\frac{L-x}{v\gamma\tau_1} - \frac{L}{u}\right) dx.$$

On integrating this expression with respect to x, we obtain

$$\left(\frac{dn_1}{d\omega}\right)^0 = |f_{21}|^2 N\Lambda(1 - e^{-l})\, e^{-L/u}.$$

The ratio of the coherent to diffraction intensity is equal to

$$R = \frac{|a_1|^2}{\left(\frac{dn_1}{d\omega}\right)^0} = \frac{4N\Lambda\lambda^2|e^{-i\delta l} - e^{-l/2}|^2}{(1 - e^{-l})(1 + 4\delta^2)}$$

and does not depend on the value of f_{21}. If the mass difference Δm is large in comparison with the inverse lifetime $1/\tau_1$ of the K_1^0-meson ($\delta = |m_2 - m_1|\,\tau \gg 1$), then the value of R, as seen from the formula, will be small. If $\delta \sim 1$, then R can be substantially larger than unity. This result has a simple physical interpretation: if $\Delta m \gg 1/\tau_1$, the coherent regeneration takes place over a length of the order of $v\gamma/\Delta m$, whereas if $\Delta m \ll 1/\tau_1$ it takes place over a length of the order of $v\gamma\tau_1$. Hence the coherent amplitude in the second case is $\delta = \Delta m\tau$ times larger than in the first.

Passing a beam of K_2^0-mesons through iron plates, Piccioni and collaborators measured the value of R and established that the value of δ does not much exceed one. Consequently, $\Delta m \approx 1/\tau_1$.

WHICH IS HEAVIER, K_1^0 OR K_2^0?

The experiments considered above give no answer to this question. Several experiments that would be able to give the sign of Δm have been proposed.

The idea behind one of them is based on the fact that the amplitude f_{21} of regeneration of K_1^0-mesons in different substances has different phases φ.

Consider a monochromatic beam of K_2^0-mesons passing through two successive thin plates a and b of thickness d_a and d_b, respectively, and set apart at a distance L (Fig. 61). The amplitude of K_1^0-mesons on the right of the plate b to within an irrelevant factor is equal to

$$1 + \alpha e^{i(\varphi_a - \varphi_b)t + i\Delta m t - \frac{t}{2\tau_1}},$$

where α is a real number characterizing the plates:

$$\alpha = \frac{N_a |f_{21}^a| d_a}{N_b |f_{21}^b| d_b},$$

t is the transit time between the plates ($t = L\sqrt{(1 - v^2)}/v$), τ_1 is the lifetime of K_1^0, while φ_a and φ_b are the phases discussed above:

$$f_{21}^a = |f_{21}^a|\, e^{i\varphi_a}, \qquad f_{21}^b = |f_{21}^b|\, e^{-i\varphi_b}.$$

Thus the probability for the decay of K_1^0 to be observed on the right of the plate b is proportional to

$$1 + 2\alpha e^{-t/2\tau_1} \cos(\Delta\varphi + \Delta mt) + \alpha^2 e^{-t/\tau_1},$$

Therefore, knowing the sign of $\Delta\varphi$ and changing l, one can determine the sign of Δm. Detailed calculations of this experiment taking into account the finite thickness of the plates were carried out by Matinyan, who discovered that a combination of two thick plates put close together is optimum. The role of the variable distance between the plates is, in this case, played by the variable thickness of the plate b.

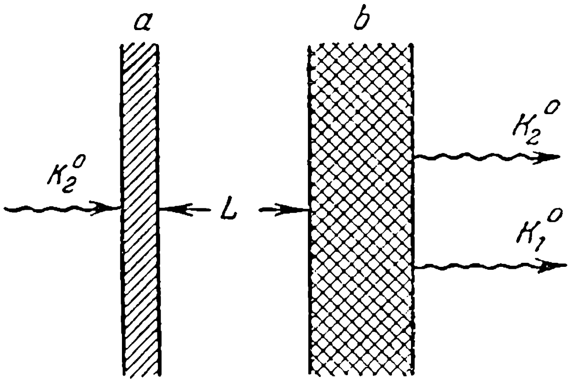

FIG. 61

Another possibility for measuring the sign of Δm consists in the determination of the number of $\tilde{K}^0$-mesons after the beam of K_2^0-mesons has traversed the plate a. This number, as is easy to see, must be proportional to

$$1 + 2\beta e^{-t/2\tau_1} \sin(\varphi_a + \Delta mt),$$

where β is determined by the properties of the plate. The number of $\tilde{K}^0$-mesons can be measured either by observing hyperons produced by them in the plate b, or by observing their leptonic decays. (If the $\Delta Q = \Delta S$ rule is valid, then $\tilde{K}^0$ decays into $\pi^+ + e^-(\mu^-) + \tilde{\nu}$, whereas K^0 decays into $\pi^- + e^+(\mu^+) + \nu$.)

The sign of Δm can, in principle, be determined also by observing the interference of the decays of K_1^0 and K_2^0 into $\pi^+ + \pi^- + \pi^0$. In this case the corresponding phase shift is due to the different final state interaction of π-mesons produced in these two decays, because their orbital states are different.

CHAPTER 17

PARITY NON-CONSERVATION IN NUCLEAR FORCES

CONTRIBUTION OF WEAK INTERACTION

Of all the weak processes enumerated in the table of weak interactions on p. 15, there remain only the two processes to be considered: neutron–proton scattering and Λ-hyperon–proton scattering, and the strangeness-conserving weak interaction of all strongly interacting particles, which is due to these processes and virtual strong processes. In contrast to the other examples of the weak interaction, these last two are very difficult to investigate, because the corresponding processes can also occur by strong interactions, which are, of course, much stronger than the weak interaction contributions. However, the weak interaction part can be separated if advantage is taken of the fact that it does not conserve parity.

Consequently, it is necessary to investigate experimentally parity non-conservation in strong interactions, which is due to the universal weak interaction. We expect that the "odd amplitude" admixture will be of the order of 10^{-7}. Since parity violating correlations are due to interference terms, observable effects will be of the same order. Experiments so far done have not detected parity non-conservation in nuclear forces.† Unfortunately, one cannot calculate quantitatively the size of parity non-conserving terms in the nuclear forces, because the theory of strong interactions plays a role.

Virtual strong interactions can change beyond recognition the basic "weak" amplitude, and can even change its sign.

In the literature a statement can be found that, if the sign of the amplitude of the parity non-conserving np-scattering turns out to be negative, this will exclude the hypothesis that the weak interaction is due to intermediate *W*-boson exchange (see Chapter 18). Indeed, the "bare" amplitude of *np*-scattering in the case of the *W*-boson exchange is positive. But this cannot be said for the amplitude when account is taken of virtual strong interactions, and, hence, this prediction, strictly speaking, is incorrect.

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† Parity non-conservation in nuclear forces was observed experimentally in 1964; see p. 219 (note added in 1964).

CAN PARITY NON-CONSERVATION IN STRONG INTERACTIONS BE LARGE?

In spite of the uncertainty in the estimates of virtual strong interactions, it is certain that parity non-conservation effects resulting from the universal weak interaction must always be small, provided that the parity conserving amplitude does not turn out, for some reason or another, to be anomalously small.

However, the question can be put in a different way: do strong interactions themselves conserve parity? For example, Solovyev put forward the hypothesis that strong interactions involving strange particles do not conserve parity. At one time, some experiments seemed to corroborate this hypothesis. However, at present, more accurate experimental data provide evidence that there is no strong parity non-conservation in the strong interactions.

As to our model, the universality of the strong interaction excludes for it the possibility of strong parity non-conservation in the interactions of strange particles, so long as parity is conserved in the interactions of π-mesons and nucleons. But the experimentally observed violation of unitary symmetry indicates that the Λ-hyperon, along with its universal strong interaction, possesses some additional anomalous interaction. We cannot, for the present, exclude the possibility that this anomalous interaction does not conserve parity.

CHAPTER 18

WEAK INTERACTION AT SMALL DISTANCES

STATEMENT OF THE PROBLEM

In all the effects that we have so far considered we have assumed that the weak four-fermion interaction is local. We have assumed that all four fermion fields interact at one point in space-time. Such an assumption, however, only applies to particles whose momenta are sufficiently small. To see this, let us return to the process of neutrino–electron scattering (convenient because it involves no strongly interacting particles).

In considering neutrino–electron scattering (Chapter 4) we found that the cross section for this process increases with energy:

$$\sigma \approx \frac{G^2 E^2}{\pi} \quad \text{when} \quad E \to \infty,$$

where E is the total energy in the centre-of-mass system. This formula was obtained under two assumptions. First, we assumed the weak interaction to be local. Second, we assumed the weak interaction to be so weak that one can apply the first order of perturbation theory. The energy dependence that we obtained points out that at high energies these assumptions are incompatible, because they lead to an absurd result.

Indeed, if the initial interaction is local and if the first order of perturbation theory is applicable, the matrix element of the process is also local, i.e. the effective interaction characterizing a given process is local. If the effective interaction of two fermions is local, only the wave with a definite angular momentum $I \leqq 1$ can take part in the scattering (in the case of νe-scattering, $I = 0$). From the unitarity of the scattering matrix it follows that the upper limit of the cross section in this case is $4\pi \lambda\!\!\!^{-2}/2$, where $\lambda\!\!\!^{-}$ is the wavelength of the incident particle. (The factor $\frac{1}{2}$ is due to the averaging over the polarization states of the initial electron.)

Thus it follows that there must be a critical energy E_{cr} beyond which the formula for neutrino–electron scattering that we obtained violates the unitarity of the matrix element. At this energy

$$\sigma \approx \frac{G^2 E_{cr}^2}{\pi} \approx \frac{4\pi \bar{\lambda}_{cr}^2}{2}.$$

Taking into account that, for an ultra-relativistic particle, $\bar{\lambda} \approx 2/E$, we may determine E_{cr} from the preceding equation:

$$E_{cr} = \sqrt{\left(\frac{2\sqrt{(2)}\pi}{G}\right)} = \sqrt{[2\sqrt{(2)}\pi\, 10^5]}\, M \approx 10^3 \text{ BeV}.$$

Thus, at an energy of the order of 10^3 BeV in the centre-of-mass system the formula which we obtained on the basis of the concept of an effective point interaction is obviously incorrect. Consequently, we have to give up at least one of the starting assumptions. Thus, either (1) the weak interaction is local, but it is incorrect to apply perturbation theory, or (2) perturbation theory can be applied, but the weak interaction is non-local, or, finally, (3) the weak interaction is non-local and perturbation theory cannot be applied.

The last possibility is so hopeless that we shall not consider it in what follows, although it is not excluded that it is just this one that takes place in nature. We shall dwell in more detail on the first and second possibility.

A LOCAL FOUR-FERMION INTERACTION

If the first possibility holds, i.e. if the weak four-fermion interaction is local, then inapplicability of perturbation theory is quite natural. This was pointed out as long ago as 1936 by Heisenberg. Indeed, at $E \sim E_{cr}$ the weak interaction can no longer be considered as weak, because the scattering resulting from it in a channel with given partial wave is close to the maximum possible one. Consequently, speaking in the language of perturbation theory, it is necessary to take into account diagrams of higher order. The simplest examples of these diagrams are shown in Fig. 62. Unfortunately, we are unable to calculate them. To each loop in such a diagram there corresponds an integral, the calculation of which gives an infinite value. In essence, we encounter here again the unsolved problem of strong interactions. We cannot even say, for the present, whether or not such a theory is consistent.

conservation laws. At high energies these laws have not been verified with an accuracy corresponding to their fundamental character.

INTERMEDIATE BOSONS

However, another possible model of the non-locality of the weak interaction, considered in a number of studies, is more likely. This is the model of heavy intermediate particles.

We assume that the weak interaction, which at low energies looks like a contact interaction of two currents, is in fact due to some heavy intermediate quantum (*W*-boson) exchange (Fig. 63).

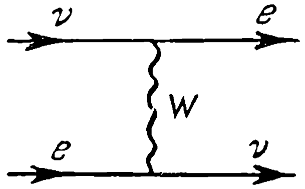

FIG. 63

Because the weak current is a Lorentz vector, these bosons must be vector bosons. Since the weak current changes the electric charge ($\Delta Q = \pm 1$), they must be charged. Thus the weak interaction reduces to the interaction of a weak current with charged vector bosons, and has the form

$$f(\varphi_\alpha j_{w\alpha} + \varphi^+_\alpha j^+_{w\alpha}),$$

where f is the interaction constant, while φ is the wave function of the boson field. Such a picture of weak interaction is particularly attractive because it contains, in a natural way, the product of currents. *W*-bosons interrelate all four terms of the weak current (Fig. 64). In this case there are not only the products of different currents but also quadratic terms of the type $(\bar\nu e)(\bar e \nu)$.

The matrix element for νe-scattering resulting from the *W*-boson exchange is of the form

$$M_{e\nu} = -4\pi f^2 (\bar e O_\alpha \nu) \frac{\delta_{\alpha\beta} - \dfrac{q_\alpha q_\beta}{\mu^2}}{q^2 - \mu^2} (\bar\nu O_\beta e),$$

where q is the 4-momentum transferred by the boson, μ is the boson mass, while $O_\alpha = \gamma_\alpha(1 + \gamma_5)$. At $q^2 \ll \mu^2$ this expression goes over to that for the local four-fermion interaction:

$$M_{e\nu} = \frac{G}{\sqrt{2}} (\bar{e} O_\alpha \nu)(\bar{\nu}_\alpha O_\alpha e),$$

where

$$\frac{G}{\sqrt{2}} = \frac{4\pi f^2}{\mu^2}.$$

The boson connects, by something like a rigid spring, the two pairs of particles. If the momentum transferred to the spring is small in comparison with the rigidity of the spring, then the spring will appear like a rigid body (this is the local approximation); if the momentum is large, the spring gets deformed (this momentum-dependence is a manifestation of the non-locality).

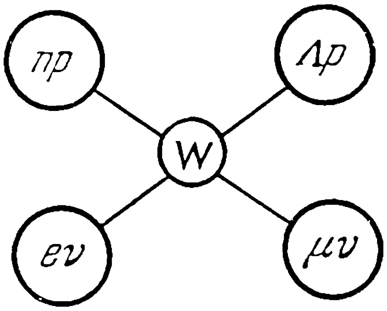

FIG. 64

The departure of the electron spectrum in the muon decay from the theoretical one with $\varrho = \frac{3}{4}$ would be one of the manifestations of non-locality. As shown by Lee and Yang, this departure can be expressed through a change in the parameter ϱ:

$$\varrho - \frac{3}{4} = \frac{1}{3} \frac{m_\mu^2}{\mu^2},$$

where m_μ is the muon mass, while μ is the W-boson mass.

The interaction of W-boson with the weak current is not a weak but a "semi-weak" interaction: $f \sim \sqrt{G}$. Therefore, if the W-boson were sufficiently light, then the probability of a decay of strange particles with emission of the W-boson would exceed, by many orders, the probabilities of decays via other channels. The absence of the $K \to W + \gamma$ decay points out that the mass of the W-boson must be larger than that of the K-meson.

The W-boson must decay in times that are intermediate between "weak" and "strong" times. For a mass of the order of 1 BeV the lifetime of the W-boson must be of the order of 10^{-17} sec.

Hence it would be impossible to observe the track of this particle. Still, one would be able to "see" it quite reliably on the basis of its decay products. The W-boson must decay into leptons ($W \to \mu + \nu$, $W \to e + \nu$) as well as into π-mesons ($W \to 2\pi$, $W \to 3\pi, \ldots$) and, if its mass is sufficiently large, also into K-mesons ($W \to K + \pi$ and so on). In this case, strangeness conserving and strangeness changing decays must have comparable probabilities.

The same "semi-weak" interaction is bound to lead also to the production of W-boson. Thus, the neutrino in the nuclear Coulomb field can "dissociate" into a W-boson and a muon or into a W-boson and an electron:

$$\nu + Z \to \mu(e) + W + Z.$$

This process is described by the diagrams shown in Fig. 65.

With "semi-weak" cross sections, W-bosons should be produced in nuclear collisions at large energies. Such processes will look, because of the short lifetime of the W-boson, like lepton production in nuclear interactions or like strangeness non-conservation (in the case of decays of the type $W \to K + \pi$).

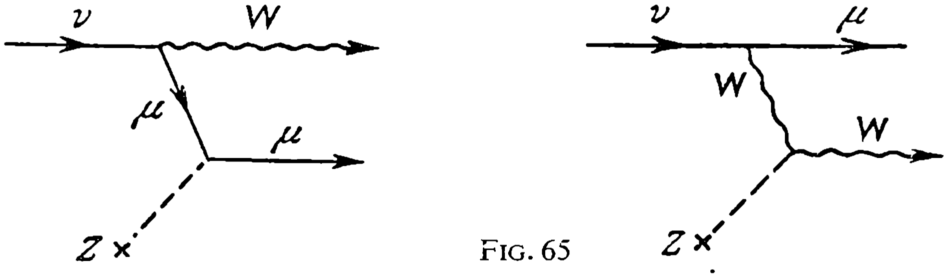

FIG. 65

The search for W-bosons in stars formed by cosmic rays is of great interest. If the mass of the W-boson is large (for example, of the order of 50 BeV), then its decay products will possess large momenta (of the order of 25 BeV) and should be distinguishable from the background of particles with small transverse momenta, which are characteristic for cosmic rays.

W-bosons can also be produced by the electromagnetic interaction, for example the production of the $W^+ + W^-$ pair in the Coulomb field of the nucleus. The cross section for this process on a heavy nucleus and sufficiently far from threshold should be of the order of 10^{-31} cm^2.

Beside charged W-bosons, neutral ones have also been considered. For instance, Lee and Yang presented a scheme in which W^0 and $\tilde{W}^0$ are introduced in addition to W^+ and W^-; these four W-bosons are called schizons. The introduction of neutral as well as charged W-bosons becomes necessary if the weak interaction is assumed to involve neutral currents. Such an hypothesis has been put forward by a number of authors in order to account for the $\Delta T = \frac{1}{2}$ rule in the non-leptonic decays of strange particles (see pp. 175 and 18). While the introduction of neutral baryonic currents encounters no difficulties, the introduction of neutral leptonic currents within the framework of the schizon scheme contradicts the absence of such decays as $K^+ \to \pi^+ + e^+ + e^-$. Therefore the universality of weak interaction disappears in the schizon scheme.

Thus, there are two possibilities.

1. The weak interaction is actually a four-fermion interaction. In this case its "effective constant" should remain constant up to an energy of the order of 1000 BeV. (This refers, of course, only to the weak interaction of leptons; the structure of the strongly interacting particles will manifest itself considerably earlier.) The first possibility means that the range of the weak interaction is small ($\sim 10^{-17}$ cm).

2. The four-fermion form of weak interaction is only a low-energy approximation. In this case, the weak interaction has a structure, and the dependence of its "constant" on energy may manifest itself from an energy only of the order of 1 BeV. An example of such a possibility is the theory with intermediate vector mesons. The second possibility implies that the range of the weak interaction is large ($\lesssim 10^{-14}$ cm).

Since no direct data on the range of the weak interaction are as yet available, we shall consider possible indirect manifestations of the effective "dimensions" of the weak interaction.

Unfortunately, our conclusions will be rather indefinite, because for the local four-fermion interaction as well as for the interaction of charged vector mesons there is no mathematical method for computing virtual processes. Contrary to quantum electrodynamics, these theories are non-renormalizable. Any virtual particle which we wish to take into account is a new infinity in the expression for the matrix element, and the number of these infinities is infinite. (In electrodynamics there are only two infinite quantities: the mass and charge of the electron.)

Nevertheless we shall try to find out about the range of the weak interaction by examining the meaning of the equality of the vector constants in β-decay and μ-decay.

EQUALITY OF THE VECTOR CONSTANTS OF β-DECAY AND μ-DECAY

In the first approximation with respect to the weak interaction this equality (to within electromagnetic corrections) is ensured by the nucleonic vector current conservation. However, the virtual weak interactions are bound to violate this equality. If the range of the weak interaction is assumed to be small, then corrections due to virtual weak interactions will be of the order of unity.

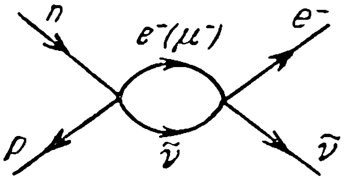

FIG. 66

We estimate, for example, corrections arising in the second order of perturbation theory. In β-decay we have just a diagram of the type of Fig. 66. If the contribution of the latter is taken into account, the β-decay constant reads

$$\left[1+\sqrt{2}\frac{\Lambda^2 G}{(2\pi)^2}\right]G,$$

where Λ is the cut-off constant. (We assume that the contribution of diagrams with a virtual proton and virtual neutron is negligibly small, because these particles have a size of $\sim 10^{-14}$ cm due to the strong interaction.)

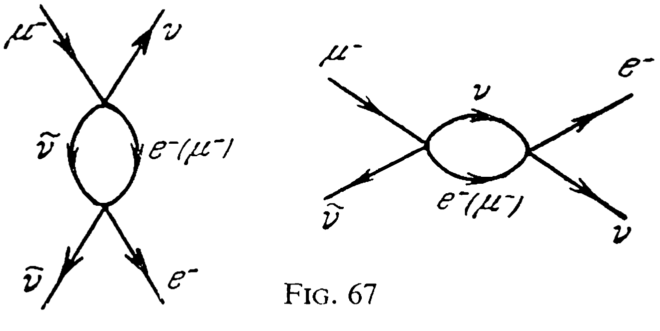

FIG. 67

In μ-decay there are two types of diagrams (Fig. 67). These diagrams differ in the total leptonic charge of the virtual particles; in the first this charge equals zero, while, in the second, it equals two. If the contribution of these diagrams is taken into account the μ-decay constant reads

$$\left[1 + 5\sqrt{2}\frac{\Lambda^2 G}{(2\pi)^2}\right] G.$$

As a result we obtain

$$\frac{G_\beta}{G_\mu} \approx 1 - 4\sqrt{2}\frac{\Lambda^2 G}{(2\pi)^2}.$$

If it is assumed from experiment that $\dfrac{G_\mu - G_\beta}{G_\beta}$ does not exceed 3 per cent, then we obtain $\Lambda \lesssim 140$ BeV.

The cut-off energy Λ is connected with the range of the weak interaction r $\left(r \sim \dfrac{1}{\Lambda}\right)$. Thus, the equality of the vector constants in β-decay and μ-decay seems to show that $r \gtrsim 10^{-16}$ cm. Any further improvement in the accuracy of the ratio of the β-decay constant to the μ-decay constant encounters difficulties in interpretation due to the electromagnetic corrections.

ABSENCE OF THE $\mu^+ \to 2e^+ + e^-$ DECAY

A consideration of the virtual weak interactions led to the question whether the muon and electron neutrino are different particles. The point is that, if $\nu_e \equiv \nu_\mu$, then it is very difficult to explain the absence of the decays $\mu \to 3e$ and $\mu \to e + \gamma$, which we are going to consider. If $\nu_e \equiv \nu_\mu$, virtual weak interactions should give the decay $\mu \to 3e$ in the second order of perturbation theory (see the diagram shown in Fig. 68). The matrix element corresponding to this diagram is equal to

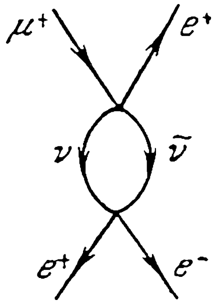

FIG. 68

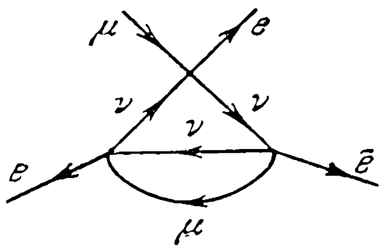

FIG. 69

$$\frac{G^2 \Lambda^2}{2(2\pi)^2}\,[\bar{u}_e \gamma_\alpha (1 + \gamma_5)\, u_\mu]\,[\bar{u}_e \gamma_\alpha (1 + \gamma_5)\, u_e].$$

The ratio of the probability of this decay to that of the ordinary decay $\mu \to e + \nu + \tilde{\nu}$ turns out to be $\frac{G^2\Lambda^4}{(2\pi)^4}$. From experiment this ratio is lower than 10^{-7}, so we obtain $\Lambda \lesssim 35$ BeV. (It should be noted that the spectrum of e^+ in this decay has the form $(3 - 2\varepsilon)\,\varepsilon^2 d\varepsilon$, while the spectrum of e^- has the form $(1 - \varepsilon)\,\varepsilon^2 d\varepsilon$, where $\varepsilon = E/E_{\max}$.)

If the interaction $(\bar{e}\nu)(\bar{\nu}e)$ is not present or if the neutrino loop gives a zero contribution (a possibility considered by B. L. Ioffe), then the $\mu \to 3e$ decay will arise in the third order of perturbation theory with respect to the weak interaction (see the diagram in Fig. 69). In this case the upper limit of Λ amounts to several hundred BeV.

ABSENCE OF THE $\mu \to e + \gamma$ DECAY

If $\nu_e \equiv \nu_\mu$, then this decay should take place on account of the virtual weak and electromagnetic interaction (see, for example, the diagrams of Fig. 70). The ratio of the probability of this decay to that of the decay $\mu \to \nu + \tilde{\nu} + e$, obtained by B. L. Ioffe, is equal to

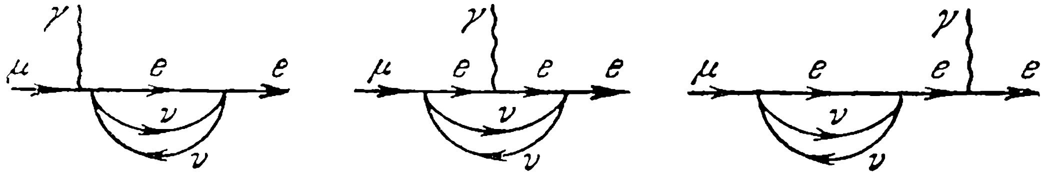

FIG. 70

$$\frac{2\alpha}{3\pi^5}\, G^2\Lambda^4 \left[\ln \frac{\Lambda^2}{\mu^2}\right]^2.$$

This ratio, as found experimentally, is lower than 10^{-7}, so it follows that $\Lambda \lesssim 30$ BeV. It should be noted that the lower the limit on Λ, the more reliable our estimates, because the parameter of expansion is $G\Lambda^2/(2\pi)^2$.

From the above one might get the impression that at a sufficiently large range of weak interaction ($\Lambda \sim 10$ BeV, $r \sim 10^{-15}$ cm) the decays $\mu \to 3e$ and $\mu \to e + \gamma$ should have negligible probabilities so that we get rid of any difficulties. However, such an impression is erroneous. Assuming that the weak interaction has in itself a large effective volume, we see at once that photons (and, consequently, $e^+ + e^-$ pairs) will begin to be emitted directly from the volume in first order perturbation theory with respect to the weak interaction (and not in the second, as was so far the case). This can be displayed symbolically by a diagram (Fig. 71; the circle represents the region of non-locality). In general, such a diagram cannot be calculated. However, if the non-locality is due to an intermediate charged boson (see above), then the calculation can be carried out. And, indeed, it has been carried out by a number of authors for the diagrams shown in Fig. 72, and has given for the ratio R of the decay rate of the $\mu \to e + \gamma$ decay to that of the $\mu \to e + \nu + \tilde{\nu}$ decay

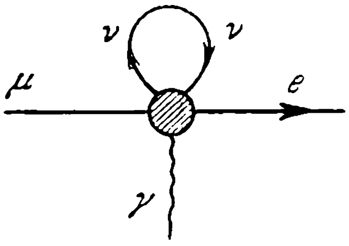

FIG. 71

$$R = \frac{3\alpha}{8\pi} N,$$

where N is a number dependent on the value of the anomalous magnetic moment of the W-boson and on the cut-off limit Λ (again the integrals diverge as the limiting momenta of the virtual particles tend to infinity).

If the anomalous magnetic moment of the W-boson is equal to its normal moment ($\mu_{\text{anom.}} = \mu_0$, $\mu_{\text{total}} = \mu_{\text{anom.}} + \mu_0 = 2\mu_0$), then $N = 1$. If $\mu_{\text{anom.}} = 0{\cdot}7\mu_0$, then $N \ll 1$; if $\mu_{\text{anom.}}$ is close to zero, then $N \sim 1$. Apparently, it is most reasonable to assume $\mu_{\text{anom.}} \approx 0$, because the W-boson does not take part in strong interactions.

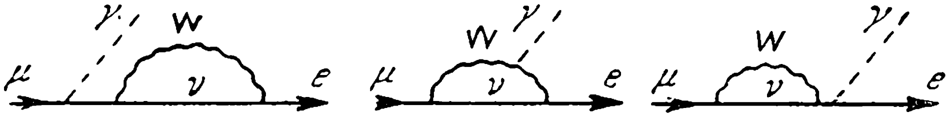

FIG. 72

Then we obtain $R \sim 10^{-3}$ and have, as before, a disagreement with experiment.

Besides the absence of $\mu \to e + \gamma$ and $\mu \to 3e$ decays, the fact that the $\mu \to e$ transitions are absent in the Coulomb field of nuclei is also of considerable interest. (This process might turn out to be more likely than the $\mu \to e + \gamma$ decay, if the matrix element of the $\mu \to e$ transition had basically the character of a mono-field.)

Thus, it is seen that the absence of the $\mu \to 3e$ and $\mu \to e + \gamma$ decays appears to be very strange if the muon and electron neutrino are the same particle. On the contrary, if ν_e and ν_μ are different particles, then the absence of $\mu \to 3e$ and $\mu \to e + \gamma$ decays is quite natural. These decays turn out to be strictly forbidden if the muon neutrino and electron antineutrino, produced in the $\mu \to e + \nu_\mu + \tilde{\nu}_e$ decay, can become neither a photon nor an $e^+ + e^-$ pair by virtue of muon charge conservation.†

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† The problem of the existence of two kinds of neutrino was solved in 1962 by an experiment, which was carried out at the Brookhaven laboratory. In this experiment it was discovered that muon neutrinos cannot cause the reactions $\tilde{\nu}_\mu + p \to n + e^+$, $\nu_\mu + n \to p + e^-$, but give rise to analogous reactions in which muons are emitted instead of electrons (see p. 19).

CHAPTER 19

WHAT IS TO BE MEASURED, AND WHY?

THE physics of weak interactions is in a special position. First, there is a theory which describes in a unified manner a wide range of phenomena. Second, the theory predicts a number of regularities which have so far not been checked. Third, there are experimental results which are not accounted for by the theory.

All this gives grounds for expecting that, in the near future, important results, theoretical as well as experimental, will be obtained in the domain of weak interactions. Some problems in the physics of weak interactions, whose experimental investigation would be of considerable interest, are enumerated below.

A. TEST OF THE GENERAL PROPERTIES OF THE THEORY

1. Check of the universality of the V–A-interaction. (Measurement of the K_{e2}-decay rate, measurement of the spectra, angular distributions and polarization of particles in the K_{e3}- and $K_{\mu 3}$-decays, measurement of leptonic decays of hyperons.)

2. Check of the square current hypothesis. (Search for νe- and $\tilde{\nu} e$-scattering and for the $\mu^+\mu^-$ and e^+e^- leptonic pair production in the interaction of the neutrino with the Coulomb field of nuclei. Investigation of parity non-conservation in strong interactions.)

3. Check of CP-invariance. (Measurement of the $\tilde{\Sigma}^- \rightarrow \tilde{p} + \pi^0$ and $\tilde{\Sigma}^- \rightarrow \tilde{n} + \pi^-$ decays, search for $K_2^0 \rightarrow 2\pi$ decays.)

4. Check of the hypothesis that the muon neutrino and electron neutrino are different particles. (Search for $\mu \rightarrow e + \gamma$ and $\mu \rightarrow 3e$ decays. Search for the process $\nu_\mu + n \rightarrow p + e^-$. A more accurate determination of the upper limit of the muon neutrino mass. Search for the anomalous interaction of the muon neutrino.)

5. Check of the charged current hypothesis. (Search for the processes with neutral currents $K \to \pi + e^+ + e^-$, $\mu \to 3e$, $\mu^- + p \to e^- + p$, $\nu + p \to \nu + p$, and so on.)

6. Search for the intermediate vector W-boson. (Photoproduction of W-bosons, production of W-bosons by leptons, search for strangeness non-conservation in strong interactions.)

7. Search for a possible non-locality of leptonic interactions. (K_{e3}- and $K_{\mu 3}$-spectra. Measurement of the parameter ϱ in μ-decay.)

B. TEST OF THE ISOTOPIC PROPERTIES OF THE THEORY

8. Check of the nucleonic vector current conservation. (Measurement of the vector constant in μ-capture. Measurement of the $\pi^+ \to \pi^0 + e^+ + \nu$ decay rate. Investigation of the decays $\Sigma^+ \to \Lambda^0 + e^+ + \nu$, $\Sigma^- \to \Lambda^0 + e^- + \tilde{\nu}$.)

9. Check of the isotopic properties of the nucleonic current. (Search for the "weak dipole" and "effective scalar" in μ-capture (β-decay), and comparison of the probabilities for the decays $\Sigma^+ \to \Lambda^0 + e^+ + \nu$ and $\Sigma^- \to \Lambda^0 + e^- + \tilde{\nu}$.)

10. Check of the $|\Delta S| \leqq 1$ rule for strangeness changing leptonic decays. (Search for the $\Xi \to N + e + \nu$ decay.)

11. Check of the $\Delta Q = \Delta S$ rule for strangeness changing leptonic decays. (Search for the decays $\Sigma^+ \to n + e^+ + \nu$, $K^+ \to e^- + \tilde{\nu} + 2\pi^+$, $K^0 \to e^- + \tilde{\nu} + \pi^+$.)

12. Check of the $\Delta T = \frac{1}{2}$ rule for strangeness changing leptonic decays. (Measurement of the decay rates, spectra and angular distributions of the decays $K_2^0 \to \pi^+ + e^- + \tilde{\nu}$ and $K^+ \to \pi^0 + e^- + \tilde{\nu}$.)

13. Check on the $|\Delta S| \leqq 1$ rule for non-leptonic decays. (Search for the $\Xi \to N + \pi$ decay.)

[The points 8–13 are consequences of the Sakata model, or, in a more general way, of the minimal model. In any of these, experiment can disprove the model.]

14. Check on the hypothesis of the restricted universality of the weak interaction. (Accurate measurement of hyperon β-decay rates, K_{e3}- and $K_{\mu 3}$-spectra, the spectra and polarization of particles in the $K_{\mu 3}$-decay.)

C. ACCUMULATION OF DATA WHICH CANNOT AS YET BE ACCOUNTED FOR BY THEORY. VARIOUS PROBLEMS

15. Check of the $\Delta T = \frac{1}{2}$ rule in non-leptonic decays. (Measurement of the τ^0-decay rates, measurement of the probabilities of the $\Xi^- \to \Lambda^0 + \pi^-$ and $\Xi^0 \to \Lambda^0 + \pi^0$ decays and determination of angular and spin correlations in these decays. Measurement of the polarization of nucleons in the decay of polarized Σ-hyperons.)

16. Problem of the $K_1^0 - K_2^0$ mass difference. (Observation of the oscillations. Determination of the sign of the mass difference.)

17. Information on strong interactions. (Determination of the $\pi\pi$-interaction from the angular and energy distributions of π-mesons in τ-decay; determination of the $\Lambda\pi$-interaction from the $\Xi \to \Lambda + \pi$ decay.)†

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† Since 1961, when this paragraph was written, some of the problems enumerated in it have been investigated experimentally to some degree or other. However, these problems need further study (see Chapter 20).

CHAPTER 20

WEAK INTERACTION AND UNITARY SYMMETRY

THIS chapter, written in August 1964, has two purposes. The first one is to summarize briefly the results of experimental test of two hypotheses which can be assumed as the basis of the theory of weak interaction, i.e. the hypothesis of universal V–A-interaction, determining the kinematic properties of weak processes, and the hypothesis of the so-called minimal model, determining the isotopic properties of these processes.

According to the hypothesis of universal weak interaction all weak processes result from the interaction between currents (leptonic and hadronic) having the same form (V–A) and the same coupling constant G.

The hypothesis of the minimal model postulates that the strong, electromagnetic and weak interactions of hadrons are due to the strong, electromagnetic and weak interactions of a minimum number of fundamental baryon fields. The number of these fields is determined by the symmetry properties of the strong interaction and equals three, if one remains within the framework of isotopic invariance. In this case two of the fields form an isodoublet, while the third forms an isosinglet.

The two hypotheses lead to a number of consequences, which have been tested thoroughly during the last few years.

The second purpose of this chapter is to acquaint the reader with a new domain of symmetries higher than isotopic symmetry. The chapter deals in detail with unitary symmetry and its manifestations in the weak interactions of hadrons.

We shall begin with an enumeration of basic experimental results obtained during the past few years.

TEST OF THE GENERAL PROPERTIES OF THE THEORY

1. All experiments on the weak interactions of leptons and hadrons, which have up to now been carried out, confirm the V–A-theory of the weak interaction. In particular, data on K_{e3}- and $K_{\mu 3}$-decays are in excellent agreement with the theory. The K_{e2}-decay is still waiting investigation.

2. Experimental physicists at ITEF (Inst. of Theor. and Exper. Phys. in Moscow) and Caltech reported that they have discovered parity non-conservation in nuclear forces, due to the strangeness conserving hadronic weak interaction. At ITEF an "up-down" asymmetry of photons was observed with respect to the vector of polarization of the neutron in the reaction

$$n + \mathrm{Cd}^{113} \rightarrow \mathrm{Cd}^{114*} \rightarrow \mathrm{Cd}^{114} + \gamma .$$

At Caltech, a circular polarization of photons was detected in the decay of non-polarized Ta^{181} nucleus. If the interpretation of these experiments is correct, then this means that the self-interaction of the weak nucleonic current—a phenomenon predicted theoretically—has been detected.

3. A check of CP-parity conservation in the decays of the K_2^0-meson showed that the branching ratio for the $K_2^0 \rightarrow 2\pi$ decay, forbidden by CP-parity conservation, is less than 3×10^{-3}. Recently a Princeton group working at Brookhaven reported that this decay was observed to have a branching ratio of $\sim 2 \times 10^{-3}$. If this result is confirmed, it will be of fundamental importance.

4. Experiments done at Brookhaven and CERN confirmed definitively the existence of two kinds of neutrino: ν_μ and ν_e. The CERN experiments did not lead to a detection of the intermediate W-boson. It follows from them that the intermediate W-boson, if it exists, has a mass larger than 1·8 BeV.

The CERN experiments allowed one also to establish the upper limit for a possible elastic scattering of the muon neutrino on the proton:

$$\sigma(\nu_\mu p \rightarrow \nu_\mu p) \lesssim 3 \times 10^{-2}\, \sigma(\nu_\mu n \rightarrow \mu p).$$

This result is an argument against the existence of neutral leptonic currents.

TEST OF THE ISOTOPIC PROPERTIES OF THE THEORY

5. The conservation of the nucleonic vector current has been confirmed by various experiments. Thus, at Dubna and CERN a value of $(1{\cdot}14 \pm 0{\cdot}09) \times 10^{-8}$ was obtained for the ratio of the probabilities of π_{e3}- and $\pi_{\mu 2}$-decays, which is in good agreement with the theoretical value $(1{\cdot}00 \pm 0{\cdot}02) \times 10^{-8}$.

The hypothesis of vector current conservation is also confirmed by data on the measurement of the weak magnetism in the decays $N^{12} \rightarrow C^{12}e^{+}\nu$ and $B^{12} \rightarrow C^{12}e^{-}\tilde{\nu}$.

The situation with the presumed equality of the μ-decay constant G_μ and the vector constant G_n of neutron β-decay is not yet settled. Experimentally, $G_n = 0{\cdot}99\,G_\mu$ if no electromagnetic corrections are applied, whereas $G_n = 0{\cdot}98\,G_\mu$ if account is taken of these corrections. Among theoreticians there is no accord as to how this 2 per cent departure is to be understood. Some assume that the constants are in fact equal, while the departure is associated with the fact that electromagnetic corrections for the neutron are calculated incorrectly (speaking strictly, for strongly interacting particles the calculation of such corrections is not reliable). Others assume that in calculating the corrections no large error is

TABLE 8. LEPTONIC DECAYS OF HYPERONS

Decay	N	$R \times 10^{-4}$
$\Lambda \rightarrow pe^{-}\tilde{\nu}$	194	$8{\cdot}6 \pm 0{\cdot}9$
$\Lambda \rightarrow p\mu^{-}\tilde{\nu}$	13	$1{\cdot}3 \pm 0{\cdot}6$
$\Sigma^{-} \rightarrow ne^{-}\tilde{\nu}$	69	12 ± 2
$\Sigma^{+} \rightarrow ne^{+}\nu$	1?	$\lesssim 1$
$\Sigma^{-} \rightarrow n\mu^{-}\tilde{\nu}$	22	$7 \pm 1{\cdot}5$
$\Sigma^{+} \rightarrow n\mu^{+}\nu$	1?	$\lesssim 2$
$\Sigma^{-} \rightarrow \Lambda^{0}e^{-}\tilde{\nu}$	20	$0{\cdot}8 \pm 0{\cdot}3$
$\Sigma^{+} \rightarrow \Lambda^{0}e^{+}\nu$	4	$0{\cdot}7 \pm 0{\cdot}4$
$\Xi^{-} \rightarrow \Lambda^{0}e^{-}\tilde{\nu}$	3	24 ± 13
$\Xi^{-} \rightarrow \Lambda^{0}\mu^{-}\tilde{\nu}$	0	?
$\Xi^{-} \rightarrow \Sigma^{0}e^{-}\tilde{\nu}$	0	?
$\Xi^{0} \rightarrow \Sigma^{+}e^{-}\tilde{\nu}$	0	?

The table was compiled on the basis of a report of I. V. Chuvilo (Dubna Conference, August 1964). N is the number of observed events; R is the ratio of the width of given decay to the total width for the hyperon. for instance,

$$R_\Lambda = \frac{\Gamma_{\Lambda \rightarrow pe\nu}}{\Gamma_\Lambda} = (8{\cdot}6 \pm 0{\cdot}9) \times 10^{-4}.$$

involved and that, consequently, G_n and G_μ differ from each other. This small departure would mean that the simple formulation of universality as meaning the equality of weak constants in different processes needs a revision. We shall later return to this problem in connection with unitary symmetry.

Experiments checking whether or not the axial nucleonic current has a definite G-parity have so far been insufficiently accurate (see in Table 8 the probabilities of $\Sigma^+ \rightarrow \Lambda^0 e^+ \nu$ and $\Sigma^- \rightarrow \Lambda^0 e^- \tilde{\nu}$ decays).

6. Experimental check of the $\Delta Q = \Delta S$ rule has abounded in dramatic moments. At the 1962 Conference on the Physics of Elementary Particles in Geneva, various experimental proofs of the violation of this rule were presented. By the end of 1963 the situation changed, and, at the 1964 Conference on the Physics of Elementary Particles in Dubna, the validity of this rule was no longer called in question. At present, the following experimental results speak in favour of the $\Delta Q = \Delta S$ rule:

(i) About 100 cases of the $K^+ \rightarrow e^+ \nu \pi^+ \pi^-$ decay and none of the $K^+ \rightarrow e^- \tilde{\nu} \pi^+ \pi^+$ decay were detected.

TABLE 9. DECAY RATES FOR THE K^+-MESON

Decay	Exper. decay rate ($\times 10^6$ sec^{-1})
$K^+ \rightarrow \mu^+ \nu$	$50{\cdot}9 \pm 0{\cdot}5$
$K^+ \rightarrow \pi^+ \pi^0$	$18{\cdot}6 \pm 0{\cdot}6$
$K^+ \rightarrow \pi^+ \pi^+ \pi^-$	$4{\cdot}36 \pm 0{\cdot}08$
$K^+ \rightarrow \pi^0 \pi^0 \pi^+$	$1{\cdot}41 \pm 0{\cdot}08$
$K^+ \rightarrow \mu \nu \pi^0$	$2{\cdot}49 \pm 0{\cdot}25$
$K^+ \rightarrow e^+ \nu \pi^0$	$3{\cdot}79 \pm 0{\cdot}35$
$K^+ \rightarrow e^+ \nu \pi^+ \pi^-$	$(3{\cdot}4 \pm 0{\cdot}7) \times 10^{-3}$
$\tau_{K^+} = (1{\cdot}224 \pm 0{\cdot}013) \times 10^{-8}$ sec	

The table was compiled on the basis of a report of I. V. Chuvilo (Dubna Conference, August 1964).

(ii) The ratio of the widths Γ_1 of the K_{e3}-decays of the K_1^0-meson and Γ_2 of the K_2^0-meson amounts to

$$\Gamma_1 : \Gamma_2 = 1{\cdot}3 \pm 0{\cdot}7 .$$

If the $\Delta Q = \Delta S$ rule is valid, this ratio must be equal to one. (Note that in 1962 it was close to 12.)

(iii) About 70 cases of the $\Sigma^- \to ne^-\tilde{\nu}$ decay and ~ 20 cases of the $\Sigma^- \to n\mu^-\tilde{\nu}$ decay were observed. Two cases, which can be interpreted, but not very convincingly, as a $\Sigma^+ \to ne^+\nu$ and a $\Sigma^+ \to n\mu^+\nu$ decay, were detected (the latter in 1962).

(iv) Finally, in the neutrino experiment at CERN not a single case of the $\nu + n \to \Sigma^+ + \mu^-$ reaction, forbidden by the $\Delta Q = \Delta S$ rule, was detected.

Thus, the minimal model apparently stood its first experimental test with regard to the $\Delta Q = \Delta S$ rule. It should be stressed that the problem of the $\Delta Q = \Delta S$ rule still cannot be considered as settled, because the theoretical accuracy of this rule is of the order of 10^{-13}, whereas the accuracy reached in experiments is of order 10^{-1}–10^{-2}.

7. Experimental check of the selection rules $|\Delta S| = 1$ and $\Delta T = \frac{1}{2}$ for strangeness changing leptonic decays yielded positive results. The fact that at present five cases of $\Xi^- \to \Lambda^0 e^-\tilde{\nu}$ decay and none of the $\Xi^- \to ne^-\tilde{\nu}$ decay have been detected speaks in favour of the first rule. This result will look more conclusive if it is taken into account that in the latter decay the phase space is about 25 times larger than in the former.

As is known, the $\Delta T = \frac{1}{2}$ rule predicts that the ratio of K_{e3}- and $K_{\mu 3}$-decays for K_2^0- and K^+-mesons must be equal to two. This ratio was found experimentally to be 2·0 ± 0·5 (see Tables 9 and 10). It would be very interesting to increase the accuracy of

TABLE 10. DECAY RATES FOR THE K_2^0-MESON

Decay	Decay rate ($\times 10^6$ sec^{-1})	
	predicted by the $\Delta T = \frac{1}{2}$ rules	experimental
$K_2^0 \to \pi^+\pi^-\pi^0$	2·9 ± 0·2	2·8 ± 0·5
$K_2^0 \to \pi^0\pi^0\pi^0$	4·7 ± 0·4	5·1 ± 1·2
$K_2^0 \to e^+\nu\pi^-$ $K_2^0 \to e^-\tilde{\nu}\pi^+$	7·6 ± 0·7	7·7 ± 1·2
$K_2^0 \to \mu^+\nu\pi^-$ $K_2^0 \to \mu^-\tilde{\nu}\pi^+$	5·0 ± 0·5	5·1 ± 1
$\Gamma_{K_2^0}$	20·2 ± 1·0	21 ± 2

The table was compiled on the basis of a report of I. V. Chuvilo (Dubna Conference, August 1964).

the measurement of this ratio to such a degree that departures from the $\Delta T = \frac{1}{2}$ rule might be detected. Such deviations must, in any case, occur because of virtual electromagnetic interaction.

8. No departures from the $|\Delta S| = 1$ and $\Delta T = \frac{1}{2}$ rules were detected in non-leptonic decays, in spite of a considerable increase in the accuracy. With regard to the first of these rules, the fact that $\Xi^- \to n\pi^-$ decays, forbidden by it, were not found experimentally imposes a new independent restriction on the interaction with $|\Delta S| = 2$. (As is known, the observed mass difference between K_1^0- and K_2^0-mesons restricts only the parity conserving part of this interaction.) According to Ticho's data

$$\frac{\Gamma(\Xi^- \to n\pi^-)}{\Gamma(\Xi^- \to \Lambda^0\pi^-)} < 5 \times 10^{-3}.$$

The $\Delta T = \frac{1}{2}$ rule has been checked practically in all known non-leptonic decays. At present, the following values are in agreement with this rule:

1. $\Gamma(K_1^0 \to 2\pi^0) : \Gamma(K_1^0 \to 2\pi)$ (~ 10 per cent).
2. $\Gamma(K^+ \to 2\pi^+\pi^-) : \Gamma(K^+ \to 2\pi^0\pi^+)$ (~ 6 per cent).
3. $\Gamma(K_2^0 \to \pi^+\pi^-\pi^0) : \Gamma(K^+ \to 2\pi^0\pi^+)$ (~ 20 per cent).
4. $\Gamma(K_2^0 \to 3\pi^0) : \Gamma(K_2^0 \to \pi^+\pi^-\pi^0)$ (~ 40 per cent).
5. $\Gamma(\Lambda^0 \to n\pi^0) : \Gamma(\Lambda^0 \to N\pi)$ (~ 10 per cent).
6. $\Gamma(\Xi^- \to \Lambda^0\pi^-) : \Gamma(\Xi^0 \to \Lambda^0\pi^0)$ (~ 20 per cent).

Uncertainties, i.e. experimental errors and straggling in different experiments, are shown in parentheses. It is not impossible that at least a few of them should, following a prescription of Landau, be multiplied by 3. An accuracy of 10–20 per cent was achieved also in the measurement of the spin spectra from the $K_{\pi 3}$-decays $K^+ \to 2\pi^+\pi^-$, $K^+ \to 2\pi^0\pi^+$ and $K_2^0 \to \pi^+\pi^-\pi^0$, and in the measurement of the correlation parameters in hyperon decays. To elucidate the nature of the $\Delta T = \frac{1}{2}$ rule it would be very important to increase, by an order, the accuracy of the experiments.

FROM THE ISOTOPIC PROPERTIES TO THE UNITARY ONES

Thus, all predictions of the minimal model with regard to isotopic selection rules in weak interactions are apparently confirmed by experiment.

At the same time, new data concerning strong electromagnetic and weak interactions apparently point to the existence of a new type of symmetry—a unitary symmetry, higher than isotopic invariance. On the basis of the octet version of unitary symmetry one can modify the formulation of universality of weak interaction and obtain relations between the weak decays of mesons and baryons belonging to different isotopic multiplets. These relations are in good agreement with experiment.

The minimal model, in the form presented earlier in the book, cannot give the octet version of unitary symmetry. It must be modified in such a way as to preserve all the old (isotopic) properties of the symmetry and include new (unitary) ones. In what follows we shall describe the general properties of unitary symmetry, its manifestations in strong, electromagnetic and weak interactions and possible modifications of the minimal model.

SUPERMULTIPLETS

During the last few years the family of known strongly interacting particles—hadrons—has grown several-fold. The basic result to which the discovery of new particles led was the establishment of the fact that hadrons are grouped into supermultiplets. Every supermultiplet, with the exception of the simplest one, i.e. the scalar supermultiplet, contains several isotopic multiplets of particles possessing a given spin and parity but different isotopic spins T and hypercharges Y.† The existence of supermultiplets means that strong interactions satisfy a symmetry higher than isotopic invariance. However, considerable mass differences between particles belonging to a given supermultiplet point out that this symmetry is violated substantially more than isotopic invariance. The type of higher symmetry that is the most natural, most fully investigated and, so far, the one in best agreement with experiment, is the so-called SU_3-symmetry. Here U means that the matrix of transformations corresponding to the group is a unitary matrix, $UU^+ = 1$, while S denotes that the group is a special one satisfying the requirement of unimodularity: det $U = 1$. The index 3 indicates that the matrix U is 3×3 and that, after the scalar representation, the triplet representation of the group is the simplest one.

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† It should be recalled that the electric charge Q of a particle is expressed in terms of the hypercharge Y and the third isospin component T_3 by the relation $Q = T_3 + \frac{Y}{2}$. It follows that $Y = S + B$, where S is the strangeness, and B is the baryonic charge.

As is well known, isotopic invariance means invariance with respect to the group SU_2. The group of isotopic transformations SU_2 is of first rank and has one additive quantum number T_3, while the group SU_3 is of second rank and has two additive quantum numbers T_3 and Y.

In the literature, SU_3-symmetry is frequently called unitary. From the above it follows that this nomenclature is, strictly, incorrect because the property of unitarity does not exhaust all properties of this symmetry and because other symmetries, for instance SU_2, also possess the property of unitarity. Nevertheless, we shall employ in what follows this term, since it is convenient and widely used.

TY-DIAGRAMS

An isomultiplet is characterized by two numbers: T and Y. Therefore, the isomultiplets belonging to a given supermultiplet are conveniently presented in a plane, the values of Y being plotted

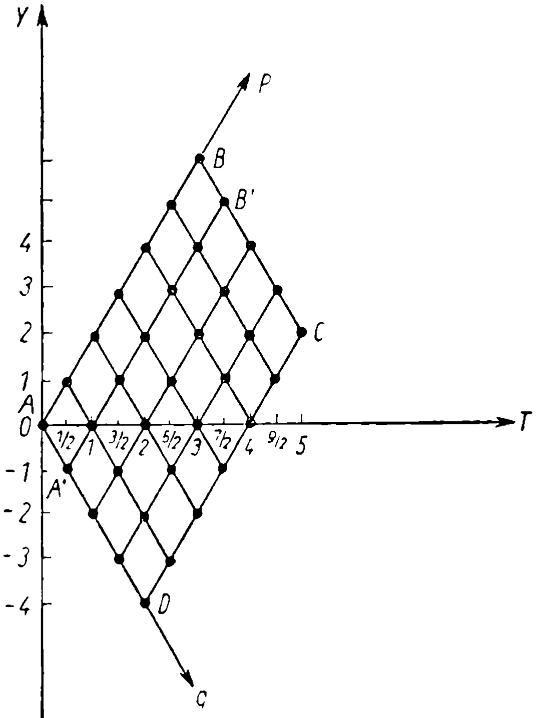


FIG. 73

on the y-axis and those of T on the x-axis. We shall choose the ratio of scales on these axes to be $\sqrt{3}/2$.

As will be shown below, supermultiplets are presented in TY-plane by parallelograms whose sides make an angle of $\pm 60°$ with the x-axis (see Fig. 73). To each point with integer values of $2T$ and Y in the interior of the parallelogram and on its sides there corresponds an isomultiplet.

We specify a supermultiplet by two numbers (p, q). If p shows the number of segments into which the side AB can be divided (see Fig. 73), while q shows the number of segments into which the side AD can be divided, then the total number of isomultiplets in the supermultiplet will prove to be $(p + 1)(q + 1)$. Taking into account that the number of particles in an isomultiplet is $2T + 1$, and inspecting Fig. 73, is is easy to find that the mean number of particles in the isomultiplets of a given supermultiplet is equal to $\frac{1}{2}(p + q + 2)$. It follows that the total number of particles in the supermultiplet (p, q) is

$$n = \tfrac{1}{2}(p + 1)(q + 1)(p + q + 2).$$

One can convince oneself in a simple way of the fact that SU_3-supermultiplets have isotopic content described by Fig. 73, if one proceeds from the explicit form of the tensor representation of the group SU_3. The wave functions of the particles of a given representation (p, q) transform like a tensor $T^{(p)}_{(q)} \equiv T^{\alpha,\dots,\alpha_p}_{\beta,\dots,\beta_q}$ with p contravariant indices α and q covariant indices β and trace zero, which is symmetric in all upper and all lower indices. Each of the indices α and β runs through three values: 1, 2, 3. We shall assume that indices 1 and q correspond to two states of an isodoublet with $Y = +1$ for upper indices and $Y = -1$ for lower indices, while the index 3 corresponds to an isosinglet with $Y = 0$.

Let us first consider states in which all $\beta = 3$. Also let all $\alpha = 3$. This state has $2T = Y = 0$. If one of α's is not equal to three, then $2T = Y = 1$. If now two of α's differ from 3, then $2T = Y = 2$. If all of $p\,\alpha$ indices differ from 3, we obtain $2T = Y = p$. Thus we have reproduced the side AB.

Now let $q - 1$ indices $\beta = 3$, while one of the β's differs from 3. If all the $\alpha = 3$, then $2T = 1$, $Y = -1$; if one of the α's differs from 3, then $2T = 2$, $Y = 0$. If for all p indices $\alpha \neq 3$, then $2T = Y + 2 = p + 1$. We have thus reproduced the next line, $A'B'$, parallel to AB. In an analogous way the entire diagram in

Fig. 73 can be reproduced. At first sight it may seem that we might obtain some states several times. Thus, for example, a state in which one of β's and one of α's differ from 3 gives not only $T = 1$ but also $T = 0$. However, by virtue of the condition of tracelessness of the tensor $T_{(q)}^{(p)}$, we have in essence already taken it into account. We propose to the reader that he prove this point by himself.

TRIALITY

As is known, isomultiplets are divided into two large groups: isobosons (with integer T) and isofermions (with half-integer T). A system of isobosons is an isoboson. An even number of isofermions forms an isoboson, while an odd number of isofermions forms an isofermion. These properties are conveniently described if one introduces the quantity "duality" $d = 2T \bmod 2$ which assumes two values: 0 for isobosons, and 1 for isofermions. It is easy to see that the duality of a system is equal to the sum of the dualities of the components of this system, taken modulo two.

Likewise, for supermultiplets it is convenient to introduce the quantity "triality", t. We define t in the following way:

$$t = (p - q) \bmod 3 .$$

It is easy to see that, according to the value of t, all supermultiplets are divided into three large groups:

$t = +1$, for example: (1,0), (0,2), (2,1) ...

$t = -1$, for example: (0,1), (2,0), (1,2) ...

$t = 0$, for example: (0,0), (1,1), (2,2), (3,0), (0,3) ...

As will be seen below, the triality of a system of particles is equal to the sum of the trialities of these particles, taken modulo 3.

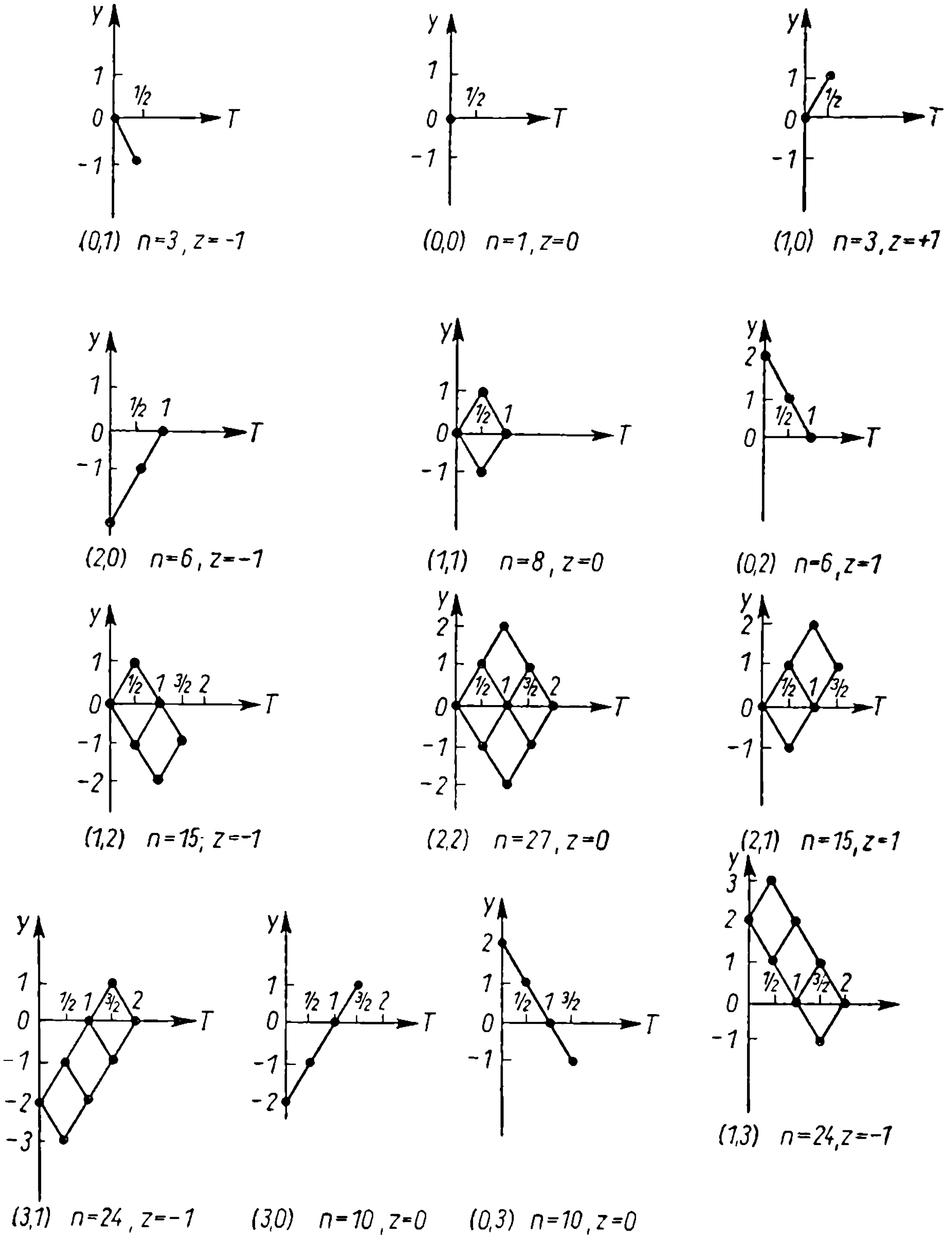
If in a system there are n_+ particles with $t = +1$, n_- particles with $t = -1$ and n_0 particles with $t = 0$, the triality of the system is equal to

$$t = (n_+ - n_-) \bmod 3 .$$

From this it follows immediately, for example, that a system consisting of particles belonging to supermultiplets with $t = 0$ is described by one or several supermultiplets with $t = 0$.

SUPERCHARGE

If the numbers p and q are given, the supermultiplet is still not defined completely, since it can be displaced in the TY-plane in parallel to the y-axis. In order to define completely a supermultiplet, it is necessary to fix the hypercharge of at least one of the constituent isomultiplets. It is convenient to fix Y^0, i.e. the hypercharge of an isosinglet (the vertex A of the parallelogram in Fig. 73). Since to



(0,1) $n=3, z=-1$

(0,0) $n=1, z=0$

(1,0) $n=3, z=+1$

(2,0) $n=6, z=-1$

(1,1) $n=8, z=0$

(0,2) $n=6, z=1$

(1,2) $n=15; z=-1$

(2,2) $n=27, z=0$

(2,1) $n=15, z=1$

(1,3) $n=24, z=-1$

(3,1) $n=24, z=-1$

(3,0) $n=10, z=0$

(0,3) $n=10, z=0$

FIG. 74 (*continued opposite*).

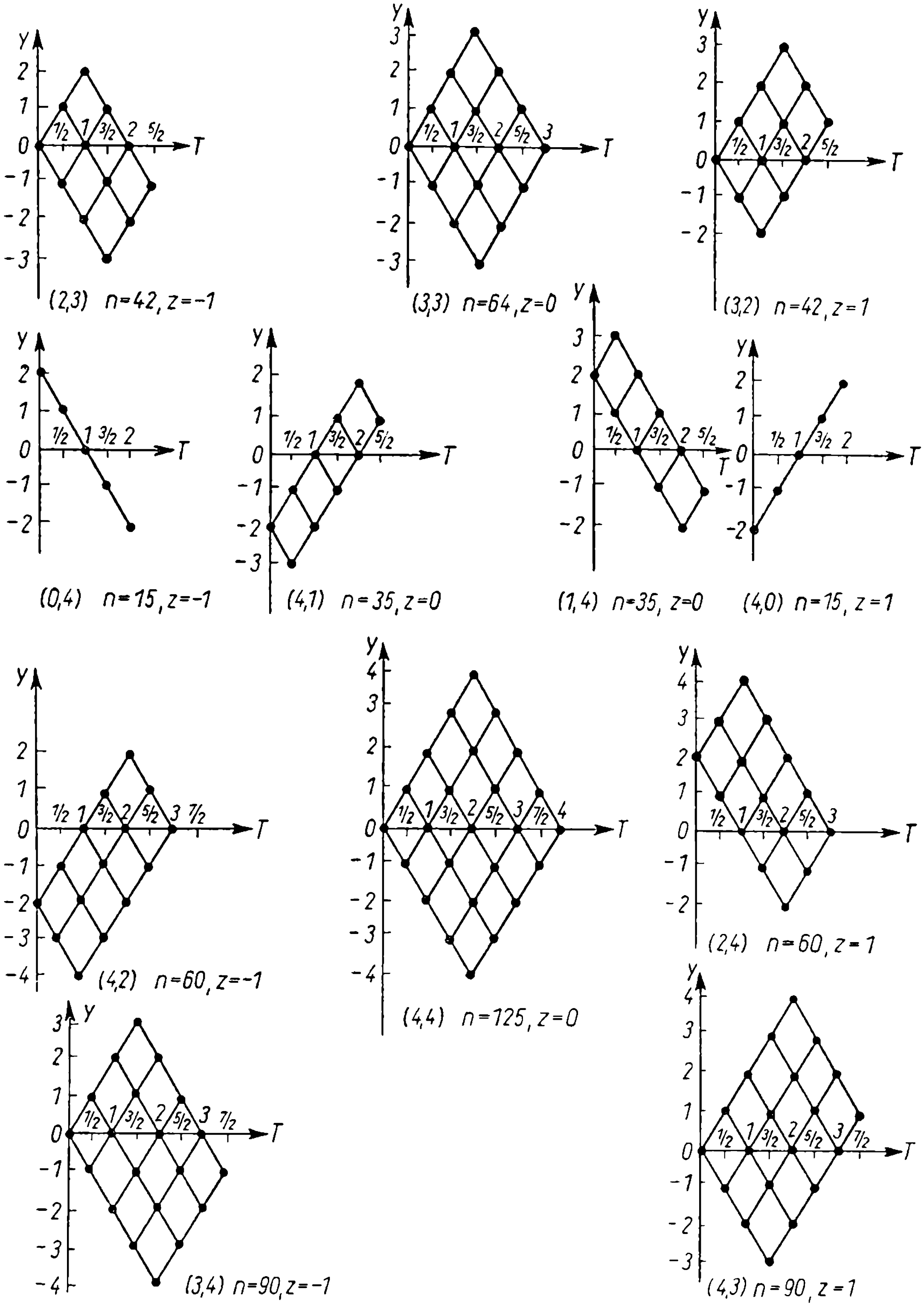


FIG. 74. The drawing gives TY-diagrams of the simplest supermultiplets. The left-hand column gives supermultiplets with $Z = -1$, the central column gives those with $Z = 0$, while the right-hand column gives those with $Z = +1$. The values of Y^0 are chosen in such a way that the supercharge Z, defined by the relation $Z = p - q + \frac{3}{2}Y^0$, be equal to -1 for the left-hand column, 0 for the central column and $+1$ for the right-hand column.

It is easy to see that for fractional values of Q and, consequently, for fractional values of Y and Y^0, the above relations between d and Y and between t and Z do not hold.

ADDITIVITY OF Z AND MULTIPLICATION OF UNITARY REPRESENTATIONS

We have noted above that the supercharge of a system of particles is equal to the sum of their supercharges. This property of supercharge is expressed mathematically in the fact that the product of two unitary representations $(p_1, q_1)_{Z_1}$ and $(p_2, q_2)_{Z_2}$ decomposes into the sum of irreducible representations $(p, q)_Z$, where $Z = Z_1 + Z_2$.

Let us convince ourselves of the additivity of Z by decomposing into irreducible representations the product of two unitary representations $(p_1, q_1)_{Z_1}$ and $(p_2, q_2)_{Z_2}$. In decomposing we can perform the following operations:

1. Symmetrize the indices.
2. Lower the indices by means of the invariant tensor ε_{ijk}.
3. Raise the indices by means of the invariant tensor ε^{ijk}.
4. Contract the indices by means of the invariant tensor δ^i_j.
5. Subtract the trace.

Upon symmetrization, $p = p_1 + p_2$, $q = q_1 + q_2$, $Y^0 = Y^0_1 + Y^0_2$; consequently, $Z = p - q + \frac{3}{2} Y^0 = Z_1 + Z_2$.

Upon multiplication by ε_{ijk}, $p \to p - 2$, $q \to q + 1$, but in this case $Y^0 \to Y^0 + 2$ and, consequently, $Z \to Z$. The fact that $Y^0 \to Y^0 + 2$ is associated with the fact that in the isosinglet two upper indices assume the values 1 and 2 instead of 3. It is easy to see that the multiplication by ε_{ij_k} can be carried out $p_{\min}$ times, where $p_{\min} = \min(p_1, p_2)$.

Upon multiplication by ε^{ijk}, $p \to p + 1$, $q \to q - 2$, $Y^0 \to Y^0 - 2$, $Z \to Z$. This multiplication can be done $q_{\min}$ times, where $q_{\min} = \min(q_1, q_2)$.

Upon multiplication by δ^i_j, $p \to p - 1$, $q \to q - 1$, $Y^0 \to Y^0$, and, consequently, $Z \to Z$. Thus we see that the supercharge of the resulting representations is equal to the sum of the supercharges of the representations being multiplied.

SCHEME FOR MULTIPLICATION OF REPRESENTATIONS

The decomposition of a product into irreducible representations is conveniently done according to a scheme which we shall demonstrate by an example. Let the representations to be multiplied be $(p_1, q_1) = (5,2)$ and $(p_2, q_2) = (4,4)$. We write them in the order

$$(52)\,(44).$$

Above them we write additional pairs of numbers (p_1^i, q_1^i) and (p_2^i, q_2^i), in which p_1^i and p_2^i are obtained by a successive reduction by unity of the numbers p_1 and p_2, until the lowest of these becomes zero, while $q_1^i = q_1$ and $q_2^i = q_2$. Below them we write additional pairs of numbers, in which q_1^i and q_2^i are obtained by a successive reduction by unity of the numbers q_1 and q_2 until the lowest of these becomes zero, while $p_1^i = p_1$ and $p_2^i = p_2$:

$$\begin{array}{c}
(12)\,(04)\\
(22)\,(14)\\
(32)\,(24)\\
(42)\,(34)\\
\hline
(52)\,(44)\\
\hline
(51)\,(43)\\
(50)\,(42)
\end{array}$$

We now write a representation obtained by symmetrization: (9,6), and above (below) it representations obtained as a result of action of the symbols ε_{ijk} (ε^{ijk}). Then we operate in every line by the symbols δ_j^i. The additional pairs of numbers that we wrote earlier exhibit the number of free indices in a given line that can be acted upon by symbols δ_j^i.

In every line we can operate with the δ-symbol $a_i + b_i$ times, where $a_i = \min(p_1^i, q_2^i)$ and $b_i = \min(p_2^i, q_1^i)$. As a result we obtain the following table of irreducible representations into which the sum of representations to be reduced is decomposed:

(12) (04)	$(1,10)_1\,(0,9)_1$
(22) (14)	$(39)_1\,(28)_2\,(17)_2\,(06)_1$
(32) (24)	$(58)_1\,(47)_2\,(36)_3\,(25)_3\,(14)_2\,(03)_1$
(42) (34)	$(77)_1\,(66)_2\,(55)_3\,(44)_3\,(33)_3\,(22)_2\,(11)_1$
(52) (44)	$(96)_1\,(85)_2\,(74)_3\,(63)_3\,(52)_3\,(41)_2\,(30)_1$
(51) (43)	$(10,4)_1\,(93)_2\,(82)_2\,(71)_2\,(60)_1$
(50) (42)	$(11,2)_1\,(10,1)_1\,(90)_1$.

The indices show the multiplicity of the representations. In every line the values of multiplicity form a trapezium. The multiplicity of the first and the last representation in a line is equal to unity. It increases monotonically in steps of unity toward the centre of the line up to a value of $\alpha_i + 1$, where $\alpha_i = \min(a_i, b_i)$. Upon reaching this value it remains constant. Such a behaviour of the multiplicity is easily understood if one takes into consideration all possible ways of closing the free indices of two tensors by means of given number of δ-symbols. For any reader who has already multiplied two such cumbersome representations it will not be difficult to verify the well-known result

$$\begin{array}{ccccccccccccccc}(1,1) & \times & (1,1) & = & (0,0) & + & (1,1) & + & (1,1) & + & (3,0) & + & (0,3) & + & (2,2) \\ 8 & \times & 8 & = & 1 & + & 8 & + & 8 & + & 10 & + & \overline{10} & + & 27.\end{array}$$

TABLES OF SUPERMULTIPLETS

Hence, we have acquainted ourselves with the mathematical description of supermultiplets and with their classification. Now we have to distribute known particles over supermultiplets (see Tables 11 and 12). This distribution, of course, is not definitive, and is to be revised with the discovery of new particles and with progress in investigating the properties of the already known particles.

Unitary symmetry establishes very strict relations between various characteristics of particles belonging to a given supermultiplet. These characteristics include masses, magnetic moments and the amplitudes of strong, electromagnetic and weak interactions. In particular, unitary symmetry leads to the consideration of new group properties of weak currents. However, before proceeding to the description of these consequences of unitary symmetry we shall discuss various "composite" models from the standpoint of unitary symmetry. This discussion will also allow us to make more obvious the interpretation of the above-mentioned relations among the characteristics of particles.

SUPERMULTIPLETS IN THE SAKATA MODEL

We shall denote the unitary triplet underlying the Sakata model by $\boldsymbol{p}, \boldsymbol{n}, \boldsymbol{\Lambda}$ in order to stress the difference between these and the physical particles p, n, Λ. So far we have not considered unitary

TABLE 11. SUPERMULTIPLETS OF BARYONS

Y	$+2$			$+1$			0			-1			-2			Super-multiplet
T	0	1	2	1/2	3/2	5/2	0	1	2	1/2	3/2	5/2	0	1	2	
Symbol	Z_0	Z_1	Z_2	$N_{1/2}$	$N_{3/2}$	$N_{5/2}$	Y_0	Y_1	Y_2	$\Xi_{1/2}$	$\Xi_{3/2}$	$\Xi_{5/2}$	Ω_0	Ω_1	Ω_2	
J^P				N	Δ		Λ	Σ		Ξ						
$1/2^+$				939			1115	1193		1318?						8
				1480					1415??							?
$1/2^-$							1405? 1680??									1 ?
$3/2^+$					1238			1385		1530			1675??			10
$3/2^-$				1512				1660??		1810??						8?
							1520									1
$5/2^+$				1688			1815?									8
$5/2^-$								1765?								?
$7/2^+$					1920											?
$7/2^-$																?
$9/2^+$				2190??												?
$9/2^-$				2700??												?
11/2					2360??											?

In the table all known baryons are classified according to the values of spin J, parity P, hypercharge Y and isotopic spin T. The column on the extreme right gives the multiplicity of the supermultiplet. For the isomultiplets $N_{1/2}$, $N_{3/2}$, Y_0, Y_1, $\Xi_{1/2}$ the symbols $N, \Delta, \Lambda, \Sigma, \Xi$, widely used in the literature, are given. The state $N_{1/2}$ with $J^P = 1/2^+$ and a mass of 1480 MeV is perhaps a representative of one more octet with $J^P = 1/2^+$. If the parity of a particle is unknown, the value of the mass is accompanied by a sign ?. If also the spin of the particle is unknown, then the mass is accompanied by signs ??.

TABLE 12. SUPERMULTIPLETS OF MESONS

Y	0			1			2	Super-multiplet
T	0	1	2	1/2	3/2	5/2	0 1 2	
Symbol	π_0	π_1	π_2	$\varkappa_{1/2}$	$\varkappa_{3/2}$	$\varkappa_{5/2}$	σ_0 σ_1 σ_2	
J^P	η	π		$\varkappa$				
0^+								
0^-	549(η) 959	137(π)		496(K)				8 1
1^+		1215(B)		725($\varkappa$)??	1215(C)??			?
1^-	1020(φ) 783(ω) 1410(E)?	763(ϱ)		891(K^*)				8 1 ?
2^+	1253(f)?	1310(A_2)						1? ?
2^-		1090(A_1)??						?

In the table all known mesons are classified according to the values of spin J, parity P, hypercharge Y and isotopic spin T. The column on the extreme right gives the multiplicity of the supermultiplet. Since there are no well standardized denotations for mesons, we give beside the symbols π_0, π_1 and $\varkappa_{1/2}$ also their widely used synonims η, π, $\varkappa$. Moreover, the table gives the "personal names" of mesons, if they have one. A sign ? means that the spin and (or) parity of the meson are unknown. Signs ?? mean that it is not well established whether the meson actually exists.

symmetry but have remained within the framework of the isotopic group, and there was no special necessity of indicating this difference by means of particular symbols, since the isotopic properties of $\mathbf{p}$, $\mathbf{n}$, $\mathbf{\Lambda}$ and p, n, Λ are the same. In passing over to the unitary group it becomes necessary to distinguish between the two sets, and, thus, $\mathbf{p}$, $\mathbf{n}$, $\mathbf{\Lambda}$ shall stand for the particles in the triplet representation, while the usual p, n, Λ belong, apparently, to another representation (octet).

Remaining within the framework of the isogroup we could, for example, write

$$\Lambda = \mathbf{\Lambda} \quad \text{or} \quad \Sigma^+ = \mathbf{p}\tilde{\mathbf{n}}\mathbf{\Lambda},$$

taking the minimal set of "fundamental" particles which are necessary for constructing physical Λ^0- or Σ^+-hyperons. Since Λ^0 and Σ^+ belong to different isomultiplets, the number of particles in these sets could be different. If now we suppose that Λ^0 and Σ^+ belong to one and the same supermultiplet, then also the minimal number of fundamental particles from which the sets can be formed must be the same. The composite model must now satisfy additional requirements. Therefore, a model which describes correctly the isotopic properties of hadrons may turn out to be incapable of describing their unitary properties.

In the Sakata model the "building material" is the triplet of baryons $\mathbf{p}$, $\mathbf{n}$, $\mathbf{\Lambda}$ with $Z = 1$ and the triplet of antibaryons $\tilde{\mathbf{p}}$, $\tilde{\mathbf{n}}$, $\tilde{\mathbf{\Lambda}}$ with $Z = -1$. Since mesons consist of equal numbers of baryons and antibaryons, for them $Z = 0$ in the Sakata model. Analogous reasoning leads to the conclusion that all baryon supermultiplets in the Sakata model must have $Z = 1$. In general, in the Sakata model

$$Z = B,$$

where B is the baryonic charge of the particle or the system of particles.

If we had chosen the triplet Λ, Ξ^0, Ξ^- with $Z = -1$ to be fundamental, then in such a model we would have

$$Z = -B.$$

Here we have arrived at a basic point, which is crucial for the Sakata model as well as for any minimal model. In a minimal model there are no superneutral supermultiplets. At the same time, as found experimentally (see Table 11), the baryon supermultiplets: singlet (0,0), octet (1,1) and decuplet (3,0) with $Z = 0$ apparently exist. We say "apparently", since experimental investigation of the mass spectrum of baryons and determination of their spins and parities is not only not yet completed but is rather in its initial stage.

In the minimal model the lowest supermultiplet, which comprises 8 known baryons N, Λ, Σ, Ξ, must contain 15 particles: either (2,1) or (1,2). In the first case this multiplet must contain also the resonances $N_{3/2}$ ($T = \frac{3}{2}$, $Y = +1$) and $Z_1(T = 1, Y = 2)$. In the second case it must contain the resonances $\Xi_{3/2}(T = \frac{3}{2}, Y = -1)$ and $\Omega_1(T = 1, Y = -2)$ (see Fig. 74). Such resonances, in the mass range investigated, have not been observed experimentally. True, considering the strong violation of unitary symmetry, one might hope that these resonances will show up at larger masses. One might even hope that the recently discovered Ω^--hyperon has $T = 1, J = \frac{1}{2}$ and $P = 1$. Its quantum numbers are not yet known. However, these hopes are in contradiction with the Gell-Mann–Okubo formula, which establishes definite relations between the masses of isomultiplets contained in a given supermultiplet, in the case that unitary symmetry is violated. We shall discuss this formula later. In the meanwhile we note that, instead of the decuplet (3,0) in the minimal model, one would have to consider either the supermultiplet (3,1) containing besides $(\Delta, \Sigma, \Xi, \Omega)$ four more other isomultiplets (in all 24 particles), or the supermultiplet (4,0) containing only one additional isomultiplet $Z_2(T = 2, Y = +2)$. According to the Gell-Mann–Okubo formula, the mass of this particle would be about 1090 MeV.

THE EIGHTFOLD WAY

The difficulties enumerated above are absent in the Gell-Mann–Ne'eman eightfold way scheme. According to it, the baryon octet N, Λ, Σ, Ξ with $Z = 0$ is taken as the basis of the theory. Hence all supermultiplets in this scheme have $Z = 0$.

In the set-up of the eightfold way the octet representation (1,1) of the group SU_3 plays an important role. (It is sometimes called the vectorial representation.) This representation exists for meson supermultiplets as well as for baryon ones.

In the spirit of the eightfold way one may naturally take the electromagnetic and the weak currents, the non-leptonic weak interaction and the unitary symmetry violating moderately strong interaction to be components of the corresponding 8-vectors. The consequences of these assumptions are in good agreement with experiment.

A shortcoming of the scheme of eightfold way is the fact that the selection rules that arise automatically in a minimal model must be postulated here. Moreover, it is still not clear why, in a theory based on eight initial fields, SU_3-symmetry holds and not SU_8. To avoid these difficulties, Gell-Mann and, independently, Zweig have put forward a theory based on triplets with $Z = 0$; Gell-Mann calls the triplets quarks, and Zweig calls them aces.

QUARKS

Ordinary unitary triplets (see Fig. 74) have $Z = \pm 1$. In order to obtain $Z = 0$, it is necessary to introduce particles with fractional values of Q and, consequently, with fractional values of Y. The baryon octet and decuplet observed experimentally are in this case obtained as a system of three quarks q_1, q_2, q_3, the quantum numbers of which are given in Table 13. The quarks q_1 and q_2, like p and n in the Sakata model, form an isodoublet, whereas the quark q_3, like Λ, forms an isosinglet; altogether they form a unitary triplet. It is easy to see that a system consisting of three quarks, each of which can be in three charge states, has, in all, $3^3 = 27$ states. These states are classified in the following way. Let us consider 9 states of two quarks. They are divided into 6 symmetric and 3 antisymmetric states. Adding to the 3 antisymmetric states the third quark, we obtain 1 completely antisymmetric state and 8 states of mixed symmetry. Adding the third quark to the 6 symmetric states, we get 8 states of mixed symmetry and 10 completely symmetric states. Thus, for baryons in the quark

TABLE 13

Quantum numbers / Quarks	Q	T_3	Y	Z
q_1	2/3	+1/2	+1/3	0
q_2	−1/3	−1/2	+1/3	0
q_3	−1/3	0	−2/3	0

model we have

$$3 \times 3 \times 3 = 1 + 8 + 8 + 10.$$

(In the tensor formalism we have

$$T^{(1)}_{(0)} T^{(1)}_{(0)} T^{(1)}_{(0)} = T^{(0)}_{(0)} + T^{(1)}_{(1)} + T^{(1)}_{(1)} + T^{(3)}_{(0)}.)$$

Mesons in the quark model are obtained as bound states of a quark and an antiquark:

$$3 \times \bar{3} = 1 + 8.$$

It is easy to see that Q and Y are in this case integers.

The quark model possesses all the merits of a minimal model with regard to weak interactions: conservation of the vector current with $\Delta Y = 0$, the $T = 1$ rule for the vector current and the axial current with $\Delta Y = 0$, the rules $\Delta Q = \Delta Y$ and $\Delta T = \frac{1}{2}$ for a current with $|\Delta Y| = 1$, and the absence of currents and hadronic decays with $|\Delta Y| = 2$. Moreover, the quark model, in contrast to a minimal model, gives baryon supermultiplets with $Z = 0$. The only disadvantage of this model is that neither quarks nor diquarks (systems of two quarks) have as yet been observed in nature.

FOUR-BARYON MODEL AND SUPERCHARGED PARTICLES

If there are no particles with fractional charges, then the most economic generalization of the minimal model, preserving all merits of the latter, is a model in which, along with a unitary baryon triplet, a unitary singlet is introduced. In this case, as in the minimal model, any of the "dumb-bells" shown in Fig. 75 can be chosen to be the fundamental triplet. One has to take as the singlet the baryon V with $T = 0$ and $Y = 0$. The introduction of the unitary baryon singlet breaks the relation between baryonic charge

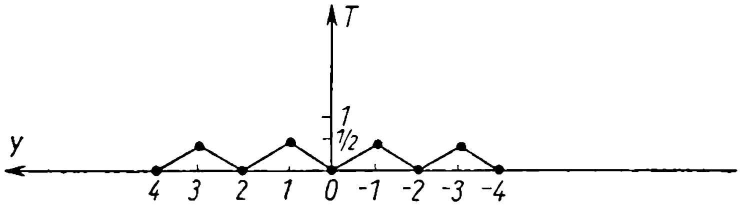


FIG. 75

and supercharge, existing in the minimal model. Thus, for example, adding a V-hyperon to a meson octet, we obtain a baryon octet with $Z = 0$. In this sense, the addition of a V-hyperon to the Sakata model is analogous to the addition of a Λ-hyperon to the Fermi–Yang model, which Sakata did ten years ago. Introducing the Λ-hyperon allowed one to break up the relation between the baryonic charge and hypercharge, which existed in the Fermi–Yang model where all mesons had to have $Y = 0$ and all baryons $Y = 1$. However, there is an essential difference between the introduction of the Λ-hyperon and that of V-hyperon. By 1956, when the Sakata model was proposed, π-mesons and nucleons, as well as strange particles, were already known. But at present only superneutral particles (with $Z = 0$) are known, whereas supercharged ones (with $Z \neq 0$) have not been observed experimentally. If a composite model based on four baryon fields is valid, then one can expect that supercharged mesons and baryons should exist along with superneutral ones. The fact that the former have not as yet been discovered is perhaps associated with the fact that their masses are larger than those of superneutral particles. It is also not excluded that some already discovered resonances belong to such supercharged supermultiplets. Experimental observation of supercharged particles would allow one to answer the very important and interesting question as to which interactions conserve the supercharge Z.

In principle, supercharge may turn out to be a strictly conserved quantum number. Then the lightest particles among the supercharged ones should be stable. Of great interest is the possibility that supercharge is conserved by all known interactions and its conservation is violated only by a new interaction as yet unknown, which might be, for example, weaker than the weak interaction. The supercharge might be not conserved by the weak interaction. Then supercharged weak currents should exist. To discover and investigate effects associated with these currents one might make use of high-energy neutrino experiments.

It may be that the supercharge is conserved only by the unitary symmetric strong interaction and is not conserved even by the moderately strong interaction, which violates unitary symmetry and whose properties will be discussed in detail below. In this case, supercharged particles might be generated in strong interactions individually (not in pairs).

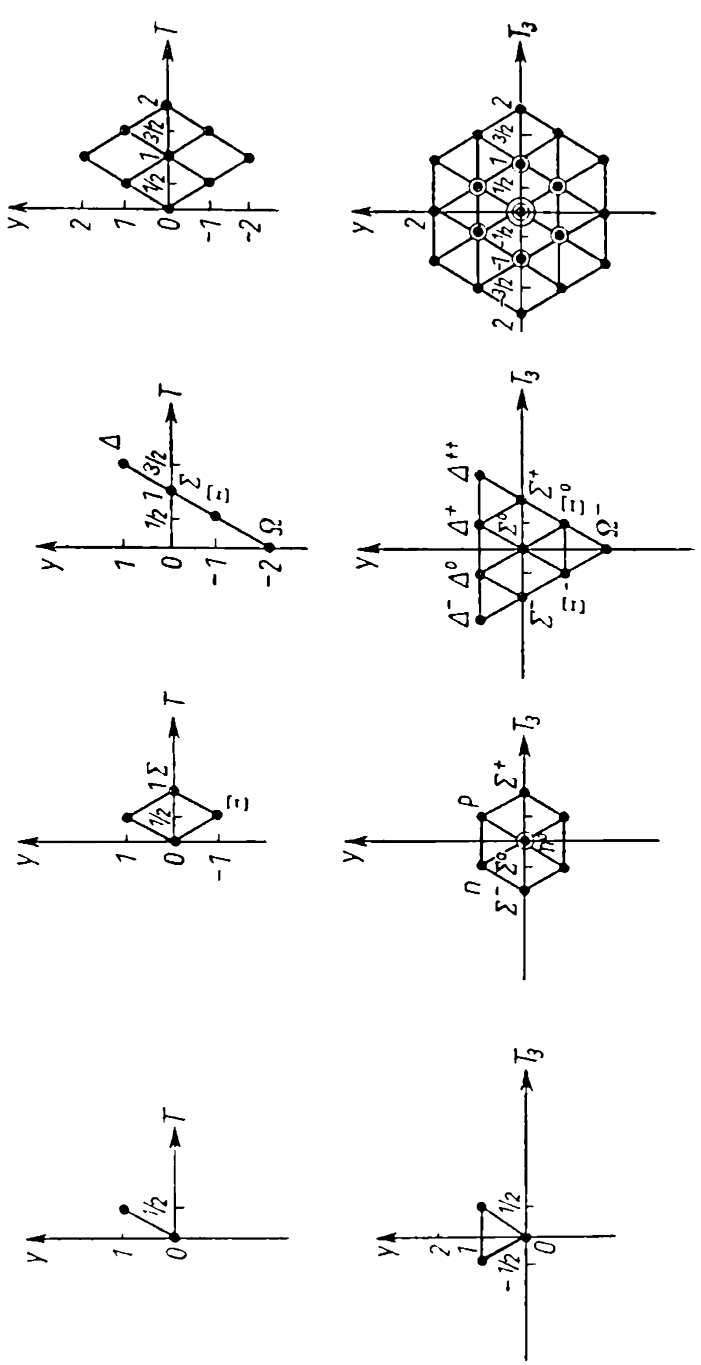

Y
T
T_3
1/2
1
3/2
2
-1/2
-1
-3/2
-2
0
Σ
Ξ
Δ
Ω
n
p
Σ^-
Σ^0
Σ^+
Λ
Δ^-
Δ^0
Δ^+
Δ^{++}
Ξ^-
Ξ^0
Ω^-

Fig. 76

OTHER MODELS

The introduction of the V-hyperon appears to be the simplest but not the only way of breaking the relation between the baryonic number and supercharge, existing in the minimal model. It is obvious that the same result can be achieved by adding to a baryon triplet, for example, a meson triplet, or a triplet of particles with baryonic charge 2.

All such models predict along with the existence of superneutral particles the existence of supercharged particles as well.

MODELS AND MATHEMATICAL FORMALISM

So far we have studied the problem of how supermultiplets are classified and which supermultiplets occur in one or another model.

We shall now discuss which experimentally observable consequences will emerge if one assumes that the known hadrons and weak hadronic currents belong to the representations of the unitary group, and that the strong interaction is SU_3-invariant. In considering the consequences of unitary symmetry we shall widely use the language of composite models. This makes the discussion less formal and more obvious. However, it should be stressed that the results which we shall derive are not a consequence of composite models but of the symmetry properties we postulate, which are manifested most simply in composite models. In what follows

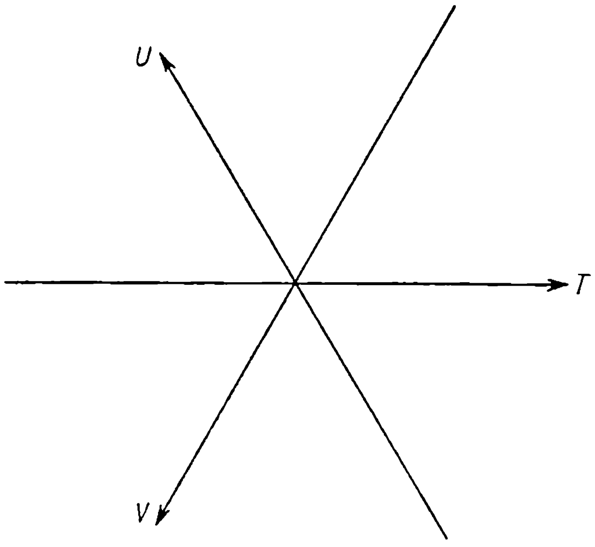


FIG. 77

we shall start the discussion with the example of the Sakata model, using the fact that it is better known and that the transition to other models (e.g. to the quark model or the four-baryon model) is trivial.

The discussion which we give below using the example of the Sakata model is easily "formalized" if advantage is taken of the fact that p, n, Λ ($\tilde{p}$, $\tilde{n}$, $\tilde{\Lambda}$) are three components of the unitary triplet T^1, T^2, T^3 (T_1, T_2, T_3) and that "composite" supermultiplets are irreducible representations into which the products of "fundamental" triplets are decomposed.

T_3Y-DIAGRAMS

We begin with a more detailed consideration of some supermultiplets, for instance, (1,0), (1,1), (3,0) and (2,2). "Turning" the TY-diagrams, from right to left, for these supermultiplets we obtain diagrams in which the values of Y are plotted as before on the y-axis while on the x-axis the values of the third component, T_3, of the isotopic spin of the particles contained in the supermultiplets are plotted (see Fig. 76). Note that in certain cases several particles get into one point in the diagram. These points are marked by concentric circles.

U-SPIN

As is easy to see, all T_3Y-diagrams are symmetric with respect to rotations through 120° (see Fig. 77). This graphical symmetry reflects a symmetry which is most obviously manifested in the Sakata model: when all interactions, except the unitary symmetric, are "switched off", the three fundamental particles p, n, Λ can be interchanged.

TABLE 14

	Singlet	Doublet
T-spin	Λ	p, n
U-spin	p	n, Λ
V-spin	n	Λ, p

In the approximation of a unitary interaction the masses of all particles contained in a supermultiplet are the same. Taking this into account, we can classify the particles in a supermultiplet not only into isotopic multiplets (T-multiplets), containing particles lying on one horizontal line, but also into U-multiplets and V-multiplets (see Fig. 77). Thus, along with T-spin we can consider U-spin and V-spin. This corresponds to the fact that in the unitary approximation the three fundamental fields can be broken up into a doublet and a singlet in three ways (see Table 14).

The concept of U-spin proves to be extremely fruitful in studying the moderately strong and electromagnetic interaction. For the latter, this is easily understood if we take into account that the photon, interacting in the Sakata model only with a proton, represents a mixture of a scalar and a vector in T-spin space and a scalar in U-spin space.

Thus, the electromagnetic interaction conserves U-spin.

MODERATELY STRONG INTERACTION

In the picture that we are describing the strong interaction can be divided into two parts: the very strong interaction possessing unitary symmetry, and the moderately strong interaction violating this symmetry. It is the moderately strong interaction that we shall assume to be responsible for the considerable departures from unitary symmetry that occur in nature. In particular, just this interaction is responsible for the mass difference between particles having different T and Y but belonging to one unitary supermultiplet.

In the Sakata model, for instance, the moderately strong interaction leads to a mass difference between $\boldsymbol{p}$ and $\boldsymbol{n}$ on the one hand, and Λ on the other hand. It would hence be natural to try to interpret it, for example, as a correction $\Delta m\bar{\Lambda}\Lambda$ to the mass of the Λ-hyperon. From the standpoint of T-spin such a term is an isotopic scalar. From the standpoint of U-spin it is a linear superposition of a U-scalar and the neutral component of a U-vector. Such a hypothesis on the transformation properties of the moderately strong interaction has two merits. First, it is simple. Second, it leads to consequences which are in striking agreement with experimental data.

MASS FORMULA FOR DECUPLET

In what follows we shall consider the mass splitting to first order in the moderately strong interaction. From the point of view of U-spin, the negatively charged particles Ω^-, Ξ^-, Σ^-, Δ^-

contained in a decuplet form a U-quadruplet (see Fig. 76). If it is assumed that the moderately strong interaction is a mixture of a U-scalar and a U-vector, then it must change the masses of these particles: the U-scalar part will change them all by the same amount, while the U-vector part will change them proportionally to the values of the third component U_3 of U-spin, which are equal to $+\frac{3}{2}$ for Δ^-, $+\frac{1}{2}$ for Σ^-, $-\frac{1}{2}$ for Ξ^-, and $-\frac{3}{2}$ for Ω^-. From this it follows immediately that the masses of $\Delta, \Sigma, \Xi, \Omega$ must be spaced equally, varying linearly with Y. This equal spacing for Δ, Σ and Ξ was known long ago and allowed Gell-Mann to predict the mass of the Ω-hyperon, the existence of which was at that time still hypothetical. In 1964 this hyperon was discovered, and its mass turned out to be exactly the one predicted (see Table 11).

MASS FORMULA FOR OCTET

The derivation of a formula for the octet is somewhat more complex. This is associated with the fact that in the centre of the T_3Y-diagram for octet there are two states. If they are classified according to the T-spin, then for a baryon octet they are Σ^0 and Λ^0. (We shall denote them by Σ_T and Λ_T.) If they are classified according to the U-spin, then they will be two other states, Σ_U and Λ_U, which are linear combinations of Σ_T and Λ_T. For an octet of pseudoscalar mesons, there are π_T and η_T (π_U and η_U) in the centre. We shall now find by means of the Sakata model a relation between π_U, η_U and π_T, η_T. The same relation will hold between Σ_U, Λ_U and Σ_T, Λ_T. In the Sakata model, charged and (or) hypercharged mesons, lying on the periphery of the hexagon in Fig. 76, have the following structure:

$$\pi^+ = p\tilde{n} \quad \pi^- = \tilde{p}n$$

$$K^+ = p\tilde{\Lambda} \quad K^- = \tilde{p}\Lambda$$

$$K^0 = n\tilde{\Lambda} \quad \tilde{K}^0 = \tilde{n}\Lambda.$$

The two neutral components of the octet in the SU_3-symmetry model are chosen in the following way. One of them, π_T with $T = 1$, must have the form

$$\pi_T = \frac{1}{\sqrt{2}}(p\tilde{p} - n\tilde{n}).$$

(The term $\langle \bar{\Sigma}_T | M | \Lambda_T \rangle$ is equal to zero by virtue of the isotopic invariance of M.) Finally we obtain

$$3m_{\Lambda} + m_{\Sigma} = 2m_{\Xi} + 2m_n.$$

In the meson octet, an analogous relation is written usually for the squared masses of the mesons, taking into account that it is just these that enter the Klein–Gordon equation:

$$3m_{\eta}^2 + m_{\pi}^2 = 4m_K^2.$$

These two relations are in good agreement with experiment. For vector mesons the analogous relation

$$3m_{\varphi}^2 + m_{\varrho}^2 = 4m_{K*}^2$$

does not hold strictly, which can be accounted for by the fact that the unitary singlet ω is near by and that the mixing $\omega \leftrightarrow \varphi$, through which each of these states becomes a mixture of a unitary singlet and an octet, plays an important role.

MASS FORMULA FOR ANY SUPERMULTIPLET

The formula of mass splitting for any supermultiplet was derived by Okubo and has the form

$$m(Y, T) = a + bY + c[4T(T + 1) - Y^2],$$

where a, b, c are certain constants characteristic of given multiplet, while Y and T are the values of hypercharge and isospin for the particles of given isomultiplet entering the supermultiplet.

For lines with given p_i or q_i in the TY-diagrams (see Fig. 73) the relation $2T = \pm Y +$ const holds, hence isomultiplets distributed along such lines are equidistant. The example of decuplet considered above is a particular case of this regularity. It is easy to verify that also the mass relation for the octet is a particular case of the Okubo formula. This formula was derived under the assumption that the mass splitting interaction has the transformation properties of a component of an octet $T_{(1)}^{(1)}$. In this case the "mass amplitudes" of the particles of a supermultiplet must themselves be the components of an 8-vector. Phenomenologically, we have to construct these "mass amplitudes" from the wave functions of the particles. The representation (1,1), in general, enters the product $(p, q)(q, p)$ twice. To this there correspond two splitting parameters (b and c) in the mass formula. For representations of the type $(p, 0)$ the splitting is determined by one parameter and is equidistant, since the representation (1,1) enters a product of the type $(p, 0)\,(0, p)$ once.

It is easy to see that the assumption that the moderately strong interaction is the sum of a U-scalar and a U-vector is equivalent to the assumption that it is a component of an 8-vector with $T = 0$ plus a unitary scalar. Indeed, in the Sakata model,

$$\bar{\Lambda}\Lambda = \frac{1}{\sqrt{3}}\,\frac{p\bar{p} + n\bar{n} + \Lambda\bar{\Lambda}}{\sqrt{3}} - \sqrt{\left(\frac{2}{3}\right)}\,\frac{(p\bar{p} + n\bar{n} - 2\Lambda\bar{\Lambda})}{\sqrt{6}}.$$

The hypothesis according to which the Lagrangian of the moderately strong interaction is a component of the triplet $T^{(1)} + T_{(1)}$, is very attractive. In this case, octet properties arise in the second order of perturbation theory in this interaction: $T^{(1)}T_{(1)} = T^{(0)}_{(0)} + T^{(1)}_{(1)}$. The simplest realization of this idea is the transition $\Lambda \to V$ in the four-baryon model, which in the second order gives $\Lambda \to V \to \Lambda$.

ELECTROMAGNETIC PROPERTIES

The U-invariance of electromagnetic interaction allows one to obtain easily a number of relations between the electromagnetic properties of the particles contained in a supermultiplet. For example, for magnetic moments the following relations must hold:

$$\mu_{\Sigma^+} = \mu_p$$

$$\mu_{\Xi^-} = \mu_{\Sigma^-}$$

$$\mu_n = \mu_{\Xi^0}$$

$$\mu_n = \frac{3}{4}\mu_{\Lambda^0} + \frac{1}{4}\mu_{\Sigma^0} - \frac{\sqrt{3}}{2}\mu_{\Sigma^0\Lambda^0} \qquad (\mu_n = \mu_{\Sigma^0_U})$$

$$\mu_{\Sigma^0\Lambda^0} = \frac{\sqrt{3}}{2}(\mu_{\Sigma^0} - \mu_{\Lambda^0}) \qquad (\mu_{\Sigma^0_U\Lambda^0_U} = 0).$$

(Here $\mu_{\Sigma^0\Lambda^0}$ is the magnetic moment of the transition $\Sigma^0 \to \Lambda^0$, determining the probability of the $\Sigma^0 \to \Lambda^0 + \gamma$ decay.) These five relations follow directly from the fact that the photon is a U-scalar. From the two last relations and the isotopic relation $2\mu_{\Sigma^0} = \mu_{\Sigma^+} + \mu_{\Sigma^-}$ it follows that

$$\mu_n = \tfrac{3}{2}\mu_{\Lambda^0} - \tfrac{1}{2}\mu_{\Sigma^0} = \tfrac{3}{2}\mu_{\Lambda^0} - \tfrac{1}{4}\mu_{\Sigma^+} - \tfrac{1}{4}\mu_{\Sigma^-}.$$

In models (of the type of the four-baryon model) in which, along with superneutral particles, there are supercharged ones, no other relations arise between magnetic moments in a baryon octet. But, if supercharged particles are absent, as, for example, in the eightfold way or in the quark model, then this implies one more relation between the magnetic moments of the baryons. As was noted by Prentki, this additional relation could be written in the form

$$\mu_p + \mu_n + \mu_{\Xi^0} + \mu_{\Xi^-} + \mu_{\Sigma^+} + \mu_{\Sigma^-} + \mu_{\Sigma^0_U} + \mu_{\Lambda^0_U} = 0,$$

from which it follows that

$$\mu_{\Sigma^0} + \mu_{\Lambda^0} = 0.$$

Taking into account the relations obtained above, we have:

$$\mu_n = 2\mu_{\Lambda^0}, \quad \mu_n = -\mu_{\Sigma^+} - \mu_{\Sigma^-}.$$

The existence of the additional relation in the case of superneutral models is connected with the fact that in them the electromagnetic current contains no term that is a unitary scalar (see below).

Proceeding from the fact the photon is a *U*-scalar, it is easy to obtain relations between electromagnetic mass splittings in different isomultiplets of a given supermultiplet. In particular, for electromagnetic mass shifts in a baryon octet, we have

$$\delta m_{\Sigma^+} = \delta m_p, \quad \delta m_n = \delta m_{\Xi^0}, \quad \delta m_{\Xi^-} = \delta m_{\Sigma^-}.$$

In order to compare these relations with experiment, we write them in the form

$$\delta m_{\Sigma^+} - \delta m_{\Sigma^-} = \delta m_p - \delta m_n + \delta m_{\Xi^0} - \delta m_{\Xi^-}$$

or

$$m_{\Sigma^+} - m_{\Sigma^-} = m_p - m_n + m_{\Xi^0} - m_{\Xi^-}.$$

This equality is valid if in calculating electromagnetic mass differences the moderately strong interaction can be disregarded. Experimentally this relation holds to within an accuracy of the order of 20 per cent.

MODIFIED UNIVERSALITY OF THE WEAK INTERACTION

The unitary symmetry of the strong interaction leads to a number of important consequences, concerning not only the form of the matrix elements of weak processes but also that of the weak interaction Lagrangian. This is accounted for by the fact that in the presence of unitary symmetry the constants of the vector currents do not change under the action of virtual strong interactions.

A well-known example of this is the non-renormalization of the constant G_n of the strangeness conserving nucleonic vector current. In the Sakata model this current has the form $\bar{n}\gamma_\alpha p$, and, like the electric current $\bar{p}\gamma_\alpha p$, is conserved if the strong interactions of n and p are the same. In the limit of a unitary symmetric strong interaction, the interactions of all three fundamental particles p, n, Λ are the same. Hence the current $\bar{\Lambda}\gamma_\alpha p$, which changes strangeness by unity, is also conserved, while the corresponding constant G_Λ must remain unchanged. Since experimentally the probabilities of strangeness changing leptonic decays are suppressed by a factor of ~ 20 in comparison with those of strangeness conserving leptonic decays, it follows that $G_\Lambda^2 \sim \frac{1}{20} G_n^2$. Thus, we are led to the hypothesis of restricted universality of the weak interaction: the V–A form of various currents is the same, while the constants are different. Complete universality can be re-established if it is assumed that the neutron and Λ-hyperon, themselves, do not enter the weak current, but rather a linear combination

$$n \cos \vartheta + \Lambda \sin \vartheta .$$

In correspondence with this version of universality we choose the weak interaction constant of the current $(n \cos \vartheta + \bar{\Lambda} \sin \vartheta)\, p$ to be equal to the weak interaction constant of leptonic currents G_μ, determined from the μ-decay. Then we have: $G_n = G_\mu \cos \vartheta$, $G_\Lambda = G_\mu \sin \vartheta$ and, consequently,

$$G_\Lambda^2 + G_n^2 = G_\mu^2 .$$

The universality of the weak interaction constant turns out to be re-established in the sense that now the constants for the hadronic current $(\bar{n} \cos \vartheta + \Lambda \sin \vartheta)\, p$ and for each of the two leptonic currents (muonic current $\bar{\mu}\nu_\mu$ and electronic current $\bar{e}\nu_e$) are the same. (It should be noted that, if there were not two kinds of neutrinos, but only one, the requirements of universality would lead us to the relations

$$G_n = \sqrt{(2)}\, G_\mu \cos \vartheta, \quad G_\Lambda = \sqrt{(2)}\, G_\mu \sin \vartheta, \quad G_n^2 + G_\Lambda^2 = 2 G_\mu^2,$$

which contradict experiment.)

For $G_\Lambda^2 \approx 0{\cdot}05 G_\mu^2$ we obtained $G_n^2 \approx 0{\cdot}95 G_\mu^2$. On taking into account numerous corrections of electromagnetic origin, experiment gives $G_n^2 \approx 0{\cdot}98 G_\mu^2$. Unfortunately, the question of how reliable this result is remains, as yet, open.

MATRIX OF EIGHT CURRENTS

It is easy to see that the weak hadronic current $(\bar{n}\cos\vartheta + \bar{\Lambda}\sin\vartheta)\,p$, with which we are concerned, belongs to the (1,1) representation of the group SU_3. We have already considered this representation, which is called the vectorial or regular representation, in connection with the octets of mesons and baryons. Eight components of this representation play the same role in the group SU_3 as three components of the vector representation in the group SU_2. The eight-current tensor $J^{(1)}_{(1)}$, corresponding to the regular representation, can be written in the form of a traceless matrix, which in the Sakata model has the form

$$J^\alpha_\beta = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{3}(2\bar{p}p - \bar{n}n - \bar{\Lambda}\Lambda) & \bar{p}n & \bar{p}\Lambda \\ \bar{n}p & \frac{1}{3}(2\bar{n}n - \bar{\Lambda}\Lambda - \bar{p}p) & \bar{n}\Lambda \\ \bar{\Lambda}p & \bar{\Lambda}n & \frac{1}{3}(2\bar{\Lambda}\Lambda - \bar{p}p - \bar{n}n) \end{pmatrix}$$

We do not write here spin-space variables. Terms with γ_μ form an octet of vector currents, while those with $\gamma_\mu\gamma_5$ form an octet of axial currents. Note that the generally adopted normalization of currents which we are using differs from the normalization of the wave functions of the composite mesons (see above) by a factor of $1/\sqrt{2}$.

The diagonal elements of the matrix J^α_β are a linear combination of two orthogonal neutral currents which are usually denoted by j_3 and j_8:

$$j_3 = \tfrac{1}{2}(p\bar{p} - \bar{n}n), \qquad T = 1,$$

$$j_8 = \frac{1}{2\sqrt{3}}(\bar{p}p + \bar{n}n - 2\bar{\Lambda}\Lambda), \quad T = 0.$$

Three complex currents $\bar{p}n$, $\bar{p}\Lambda$, $\bar{n}\Lambda$ and three conjugate currents $\bar{n}p$, $\bar{\Lambda}p$, $\bar{\Lambda}n$ can be written as linear combinations of six real currents $j_1 \cdots j_6$. As a result we obtain

$$J^\alpha_\beta = \begin{pmatrix} j_3 + \frac{j_8}{\sqrt{3}} & j_1 + ij_2 & j_4 + ij_5 \\ j_1 - ij_2 & -j_3 + \frac{j_8}{\sqrt{3}} & j_0 + ij_7 \\ j_4 - ij_5 & j_6 - ij_7 & -\frac{2j_8}{\sqrt{3}} \end{pmatrix}$$

In our new notation the weak hadronic current has the form

$$\cos\vartheta(j_1 + ij_2) + \sin\vartheta(j_4 + ij_5).$$

(We assume that one and the same angle ϑ enters the expression for the axial and vector hadronic current.)

The electromagnetic current in the Sakata model has the form

$$j_{el} = \bar{p}p = j_3^v + \frac{1}{\sqrt{3}} j_8^v + \sqrt{\frac{2}{3}}\, j_0^v .$$

Here the index v shows that the current is a vector current (γ_μ). The unitary-singlet current j_0 in the Sakata model has the form

$$j_0 = \frac{1}{\sqrt{6}} (\bar{p}p + \bar{n}n + \bar{\Lambda}\Lambda).$$

The presence of the term j_0 in the electromagnetic current is associated with the fact that in the Sakata model there are supercharged particles. Since the mean electric charge of the supercharged multiplet differs from zero ($Z = 3 \ll Q \gg \neq 0$), the current j_0 for supercharged particles interacts with photons. For superneutral particles the current j_0 is electroneutral, and does not enter the expression for the electromagnetic current. Thus, for example, in the quark model

$$j_{el} = \frac{2}{3} \bar{q}_1 q_1 - \frac{1}{3} \bar{q}_2 q_2 - \frac{1}{3} \bar{q}_3 q_3 = j_3^v + \frac{1}{\sqrt{3}} j_8^v .$$

An analogous relation is valid also in the eightfold way. The matrix J_β^α can be written in the form

$$J_\beta^\alpha = j_i \lambda_{i\beta}^\alpha ,$$

where $i = 1, \dots 8$, while the matrices λ_i, introduced by Gell-Mann, have the form

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} 1/\sqrt{3} & 0 & 0 \\ 0 & 1/\sqrt{3} & 0 \\ 0 & 0 & -2/\sqrt{3} \end{pmatrix}.$$

It is easy to verify that the matrices λ_i satisfy the relations

$$\begin{aligned} Sp\lambda_i\lambda_j &= 2\delta_{ij}, \\ [\lambda_i\lambda_j] &= 2if_{ijk}\lambda_k, \\ \{\lambda_i\lambda_j\} &= \tfrac{4}{3}\delta_{ij}\,.\,1 + 2d_{ijk}\lambda_k. \end{aligned}$$

The symbols $d_{ijk}(f_{ijk})$ are real and completely symmetric (antisymmetric). This is easy to verify if one multiplies by λ_k the last two relations and calculates the trace:

$$\begin{aligned} Sp\lambda_k[\lambda_i\lambda_j] &= 4if_{ijk}, \\ Sp\lambda_k\{\lambda_i\lambda_j\} &= 4d_{ijk}. \end{aligned}$$

Let us write the elements f_{ijk} and d_{ijk} differing from zero:

ijk	f_{ijk}	ijk	d_{ijk}	ijk	d_{ijk}
123	1	146	$\frac{1}{2}$		
147	$\frac{1}{2}$	157	$\frac{1}{2}$	448	$-\frac{1}{2\sqrt{3}}$
156	$-\frac{1}{2}$	228	$1/\sqrt{3}$		
246	$\frac{1}{2}$	247	$-\frac{1}{2}$	558	$-\frac{1}{2\sqrt{3}}$
257	$\frac{1}{2}$	256	$\frac{1}{2}$		
345	$\frac{1}{2}$	338	$1/\sqrt{3}$	668	$-\frac{1}{2\sqrt{3}}$
367	$-\frac{1}{2}$	344	$\frac{1}{2}$		
458	$\sqrt{(3)}/2$	355	$\frac{1}{2}$	778	$-\frac{1}{2\sqrt{3}}$
678	$\sqrt{(3)}/2$	366	$-\frac{1}{2}$		
118	$1/\sqrt{3}$	377	$-\frac{1}{2}$	888	$-1/\sqrt{3}$

It is easy to see that an element $d_{ijk}(f_{ijk})$ is equal to zero if among its indices there is an odd (even) number of indices 2, 5, 7. The peculiarity of these indices is associated with the fact that the matrices λ_2, λ_5, λ_7 are antisymmetric, in contrast to the other λ matrices. We have given the properties of the matrices λ and symbols f and d for reference. In the meanwhile we return to the currents j_i.

We assume that the weak vector hadronic current is in an octet with the electromagnetic hadronic current. Furthermore, we assume that the weak axial hadronic current is also a member of an octet. These assumptions are a unitary generalization of the isotopic properties of the hadronic current with $\Delta Y = 0$. (Note that the vector component of this current enters an isotopic triplet with the isovector component of the electromagnetic current, while the axial component of this current is a member of an isotroplet.) But now both the isoscalar component of the electromagnetic current and the strange hadronic current (with $|\Delta Y| = 1$) are involved. Such a generalization possesses a number of merits. In particular, it has as a consequence, for the current with $|\Delta Y| = 1$, that it should be an isospinor and satisfy the $\Delta Q = \Delta Y$ rule. Before, these properties of the strange current could only be related to those of the strangeness conserving current within the framework of a minimal model. Now they turn out to be among the manifestations of a more general symmetry property.

Besides the prediction of the isotopic properties of the strange current we obtained as a bonus one more important consequence—in the approximation in which the moderately strong interaction is switched off, the strange vector current must be conserved. This property follows from the fact that the strange vector current enters an octet with the conserved electromagnetic current, and that all members of the octet must possess identical properties.

Let us now consider consequences resulting from these properties of the weak hadronic current. We begin with the decay of mesons.

LEPTONIC DECAYS OF MESONS

An octet of pseudoscalar mesons, like a current octet, is described by a rank-two tensor P^α_β with trace zero:

$$P^\alpha_\beta = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\dfrac{2\eta^0}{\sqrt{6}} \end{pmatrix}$$

(the upper index is the column number, while the lower one is the row number). We consider first decays due to the axial current. Proceeding from the form of the weak hadronic current it is easy to conclude that the amplitudes of the decays $K \to \mu\nu$ and $\pi \to \mu\nu$ must be in the ratio $\sin\vartheta : \cos\vartheta$. Comparing with experiment the theoretical relation

$$\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)} = \tan^2\vartheta \frac{m_K}{m_\pi} \frac{(1 - m_\mu^2/m_K^2)^2}{(1 - m_\mu^2/m_\pi^2)^2},$$

we find $\vartheta \approx 0{\cdot}26$.

We shall construct the amplitudes of the decays π_{e3}, K_{e3} and $K_{\mu 3}$, due to vector currents, in the following way. Since the weak current is an 8-vector, also the amplitudes must be 8-vectors. From two 8-vectors (of the initial and the final meson) an amplitude which transforms like an 8-vector can be obtained, in general, in two different ways:

$$\bar{P}^{\gamma}_{\beta} P^{\alpha}_{\gamma} + \bar{P}^{\alpha}_{\gamma} P^{\gamma}_{\beta} \quad \text{and} \quad \bar{P}^{\gamma}_{\beta} P^{\alpha}_{\gamma} - \bar{P}^{\alpha}_{\gamma} P^{\gamma}_{\beta}.$$

The first of these tensors is symmetric with respect to the exchange $\bar{P} \leftrightarrow P$, whereas the second is antisymmetric. The interaction of the first tensor with the leptonic current is called the D-coupling, while that of the second tensor is called the F-coupling.

In the case of K_{e3}- and π_{e3}-decays, only the F-coupling gives a contribution. This can be seen in the following way:

As is known, in general, the amplitude of decays of the type "meson → meson + leptons" (for instance, K_{l3}) can contain two terms:

$$[(P_K + P_\pi)\alpha + \xi(P_K - P_\pi)\alpha]\, l\alpha.$$

However, the requirement of current conservation in the approximation of a unitary symmetric strong interaction and equal masses for K- and π-mesons leads to the condition $\xi = 0$. This means that in the annihilation channel in a system of two mesons there remains only a P-wave, and, consequently, the space part of their wave function is antisymmetric. But, the total wave function of two mesons belonging to the same supermultiplet must be symmetric. Consequently, it must be antisymmetric in unitary spin variables.

If we take into account that to the strangeness conserving interaction there corresponds a term $\bar{P}^{\gamma}_{2} P^{1}_{\gamma} - \bar{P}^{1}_{\gamma} P^{\gamma}_{2}$, while to the strangeness violating interaction there corresponds a term $\bar{P}^{\gamma}_{3} P^{1}_{\gamma} - \bar{P}^{1}_{\gamma} P^{\gamma}_{3}$, then it is easy to obtain the expression for the amplitude of the vector interaction of mesons with leptons:

$$- \Big\{ [\sqrt{2}(\bar{\pi}^0 \pi^+ - \bar{\pi}^- \pi^0) + \bar{K}^- \tilde{K}^0 - \tilde{K}^0 K^+] \cos\vartheta$$
$$+ \left[\frac{1}{\sqrt{2}} (\bar{\pi}^0 K^+ - \bar{K}^- \pi^0) + \sqrt{\frac{3}{2}}\, (\bar{\eta}^0 K^+ - \bar{K}^- \eta^0) + \bar{\pi}^- K^0 + \bar{\tilde{K}}^0 \pi^+ \right]$$
$$\times \sin\vartheta \Big\}\, l.$$

Here l stands for the leptonic current; spin-space variables are omitted. We see that the ratio of the amplitudes of K_{e3}- and π_{e3}-decays is equal to $\frac{1}{2}\tan\vartheta$. This ratio is also easily obtained in an elementary way. In the Sakata model the amplitudes for the decays $K^+ \to \pi^0 e\nu$ and $\pi^+ \to \pi^0 e\nu$ are, respectively, equal to

$$\sin\vartheta \frac{1}{\sqrt{2}} \langle \bar{p}p - \bar{n}n | \bar{p}\Lambda \rangle = + \frac{\sin\vartheta}{\sqrt{2}} \langle \bar{p} | \Lambda \rangle,$$

$$\cos\vartheta \frac{1}{\sqrt{2}} \langle \bar{p}p - \bar{n}n | \bar{p}n \rangle = \frac{\cos\vartheta}{\sqrt{2}} \{ \langle \bar{p} | n \rangle - \langle \bar{n} | p \rangle \}$$

$$= \frac{2}{\sqrt{2}} \cos\vartheta \langle \bar{p} | n \rangle .$$

It follows that their ratio, in the limit of unitary symmetry, is equal to $\frac{1}{2}\tan\vartheta$. When comparing with experiment the ratio $\Gamma_{K_{e3}} : \Gamma_{\pi_{e3}}$, calculated by means of these amplitudes, we obtain for the angle ϑ a value of $\sim 0{\cdot}22$, which is in good agreement with that obtained from $K_{\mu 2}$ and $\pi_{\mu 2}$-decays.

Data on the $K_{\mu 3}$-decay also yield an argument in favour of the validity of the picture described above. The ratio of the $K_{\mu 3}$ to K_{e3} decay rate is expressed as follows in terms of the parameter ξ quoted above:

$$R_{\mu e} = \frac{\Gamma(K_{\mu 3})}{\Gamma(K_{e3})} = 0{\cdot}65 + 0{\cdot}13\xi + 0{\cdot}019\xi^2 .$$

As we already know, in the limit of unitary symmetry ξ must be equal to zero. From experimental data on $R_{\mu e}$ and on the polarization of muons from $K_{\mu 3}$-decay it follows that $-1 \lesssim \xi \lesssim +1$. Now this result is in a reasonable agreement with theory, especially when one takes into account that another possible value of ξ (for given $R_{\mu e}$) was $\xi \sim -9$. (The latter turned out to be in contradiction with data on the muon polarization.)

LEPTONIC DECAYS OF BARYONS

The baryon octet $J^P = \frac{1}{2}^+$ is described by a matrix B^α_β, which can be written in analogy with the meson matrix:

$$B^\alpha_\beta = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

To D-coupling there corresponds a combination $\bar{B}^{\gamma}_{\beta} B^{\alpha}_{\gamma} + \bar{B}^{\alpha}_{\gamma} B^{\gamma}_{\beta}$, whereas to the F-coupling there corresponds a combination $\bar{B}^{\gamma}_{\beta} B^{\alpha}_{\gamma} - \bar{B}^{\alpha}_{\gamma} B^{\gamma}_{\beta}$. Let us write the corresponding amplitudes of the interaction of baryons with leptons.

D-Coupling:

$$\Big\{\Big[\sqrt{\frac{2}{3}}\,(\bar{\Lambda}^0\Sigma^+ + \bar{\Sigma}^-\Lambda^0) + \bar{\Xi}^-\Xi^0 + \bar{n}p\Big]\cos\vartheta$$

$$+\Big[\frac{1}{\sqrt{2}}\,(\bar{\Sigma}^0 p + \bar{\Xi}^-\Sigma^0) - \frac{1}{\sqrt{6}}\,(\bar{\Lambda}^0 p + \bar{\Xi}^-\Lambda^0) + \bar{\Sigma}^- n + \bar{\Xi}^0\Sigma^+\Big] \times$$

$$\times \sin\vartheta\Big\}\, l.$$

F-Coupling:

$$-\Big\{[\sqrt{2}\,(\bar{\Sigma}^0\Sigma^+ - \bar{\Sigma}^-\Sigma^0) + \bar{\Xi}^-\Xi^0 - \bar{n}p]\cos\vartheta$$

$$+\Big[\frac{1}{\sqrt{2}}\,(\bar{\Sigma}^0 p - \bar{\Xi}^-\Sigma^0) + \sqrt{\frac{3}{2}}\,(\bar{\Lambda}^0 p - \bar{\Xi}^-\Lambda^0) + \bar{\Sigma}^- n - \Xi^0\bar{\Sigma}^+ \times$$

$$\times \sin\vartheta\Big\}\, l.$$

For the vector and axial baryon amplitudes one can, in general, write

$$V = \alpha_V[(1 - d_V)\, V_F + d_V V_D]\,\gamma_\alpha,$$

$$A = \alpha_A[(1 - d_A)\, A_F + d_A A_D]\,\gamma_\alpha\gamma_5,$$

where the indices F and D correspond respectively to the F-coupling and D-coupling. From the fact that the vector weak current is in the same octet as the electromagnetic current it follows immediately that $d_V = 0$, $\alpha_V = 1$. (One can easily convince oneself of the validity of the first of these statements, for example, as follows: if $d_V \neq 0$ there would exist an electromagnetic amplitude $\bar{\Lambda}\gamma_\alpha\Sigma$ with a coefficient which does not become zero as $q^2 \to 0$, which would contradict gauge invariance.) As to d_A and α_A, their values can be found by comparing the amplitudes written above with corresponding experimental data. The neutron decay enables one to determine $\alpha_A \cos\vartheta$:

$$A_{n\to p} = \alpha_A[(1 - d_A) + d_A]\cos\vartheta\,\gamma_\alpha\gamma_5 = \alpha_A\cos\vartheta\,\gamma_\alpha\gamma_5.$$

From comparison with experiment it follows that $\alpha_A = 1{\cdot}15 \pm 0{\cdot}04$. The value of d_A can be determined proceeding, for instance, from the experimental rate for the $\Sigma^- \to \Lambda^0 e^- \bar{\nu}$ decay. This decay is forbidden when $d_A = 0$. Indeed, as follows from the expression for D-coupling and F-coupling, the $\Sigma \to \Lambda$ transition is contained only in the first of these:

$$A_{\Sigma\to\Lambda} = \sqrt{\tfrac{2}{3}}\, d_A \alpha_A \cos\vartheta\, \gamma_\alpha \gamma_5 .$$

Comparison of the theoretical decay rate, calculated by means of this amplitude, with the experimental one gives

$$\tfrac{2}{3} \lesssim d_A \lesssim \tfrac{3}{4} .$$

The value of ϑ can be found by comparing corresponding theoretical expressions with the experimental probabilities of the decays $\Sigma \to ne\nu$, $\Lambda \to pe\nu$ and so on. The amplitudes of these decays have the form

$$\Lambda \to pe^-\bar{\nu}: \quad -\sqrt{\frac{3}{2}}\left\{\gamma_\alpha + \alpha_A\left(1 - \frac{2}{3}d_A\right)\gamma_\alpha\gamma_5\right\}\sin\vartheta,$$

$$\Sigma^- \to ne^-\bar{\nu}: \quad -\{\gamma_\alpha + \alpha_A(1 - 2d_A)\gamma_\alpha\gamma_5\}\sin\vartheta,$$

$$\Xi^- \to \Lambda e^-\bar{\nu}: \quad \sqrt{\frac{3}{2}}\left\{\gamma_\alpha + \alpha_A\left(1 - \frac{4}{3}d_A\right)\gamma_\alpha\gamma_5\right\}\sin\vartheta,$$

$$\Xi^- \to \Sigma^0 e^-\bar{\nu}: \quad \frac{1}{\sqrt{2}}\{\gamma_\alpha + \alpha_A\gamma_\alpha\gamma_5\}\sin\vartheta .$$

The value of ϑ, as determined from the first two decays, amounts to about 0·24, which is in good agreement with the values of ϑ found above from meson decays. The experimental probability of the $\Xi^- \to \Lambda^0 e^- \bar{\nu}$ decay is several times larger than the theoretical value calculated on the basis of the amplitude written above. However, experimental data are still very scarce—only a few cases of the $\Xi^- \to \Lambda^0 e^- \bar{\nu}$ decay have been observed. As to the $\Xi^- \to \Sigma^0 e^- \bar{\nu}$ decay, it has not been observed at all as yet (see Table 8).

The knowledge of the amplitude allows one, of course, to predict not only the decay rates but also the angular and spin correlations in these decays. The experimentally observed angular distribution of electrons from the decay of polarized Λ-hyperons is in good qualitative agreement with the amplitude of the $\Lambda \to pe\nu$ decay and the value of d_A quoted above. The test of other correlations is of great interest.

The quantitative comparison of the leptonic decays of mesons and baryons within the framework of octet symmetry, described above, was first carried out by Cabibbo, who, in analysing the experimental data, obtained $\vartheta \sim 1/4$, $d_A \sim 3/4$. Since the experimental data have large errors, they are not in contradiction with another possible value of the parameter d_A ($d_A \sim 1/3$) for about the same Cabibbo angle ($\vartheta \sim 1/4$). As is seen from the amplitudes quoted above, the latter possibility corresponds to a higher theoretical probability for the $\Xi^- \to \Lambda^0 e^- \tilde{\nu}$ decay, a lower theoretical probability for the $\Sigma^- \to \Lambda^0 e^- \tilde{\nu}$ decay, and a negative sign for the A/V ratio in the $\Sigma^- \to n e^- \tilde{\nu}$ decay. (Note that, according to the generally adopted definition, $V = +\gamma_\alpha$, $A = +\gamma_5\gamma_\alpha$.)

We have considered only the dominant, "allowed" terms of the amplitudes of the leptonic decays of hyperons. Analogous relations exist also between terms of the "weak magnetism" and "effective pseudoscalar" type. Taking into account these terms will give only small corrections and will slightly change the value of the total probabilities of the corresponding decays.

All of what we said above refers to the limit in which the strong interaction is unitary symmetric. It is natural to raise the question as to how the relations obtained above will change when the moderately strong interaction is taken into account. An answer to this question was given by Ademollo and Gatto who showed that in first order in the moderately strong interaction, having the transformation properties of a component of an octet, the above expressions for vector amplitudes remain unchanged. No unitary relations remain between axial amplitudes.

HADRONIC DECAYS

We have considered the consequences of the hypothesis that weak hadronic currents are components of an 8-vector. We shall now assume that the Lagrangian of the weak interaction between hadrons also represents a component of an 8-vector. From this assumption it follows immediately that the $\Delta T = \frac{1}{2}$ rule for hadronic decays holds: indeed, the components of an octet with $|Y| = 1$ have $T = \frac{1}{2}$.

The non-leptonic Lagrangian, which transforms like a component of an 8-vector and obeys the $\Delta T = \frac{1}{2}$ rule, is not a consequence of the current × current hypothesis. This is connected with the fact that the product of two hadronic currents, each of which is an 8-vector, contains more than the representation 8. In particular, it contains the representation 27 ($p = 2$, $q = 2$), having components with $|Y| = 1$ and $T = 3/2$. There are two possible explanations of the hadronic rule $\Delta T = \frac{1}{2}$.

First, in the Lagrangian the terms with $\Delta T = 3/2$ belonging to the representation 27 can be compensated for by additional terms in the Lagrangian, for instance, by those due to the product of additional neutral hadronic currents.

Second, the representation 8, containing terms with $\Delta T = \frac{1}{2}$, can be selectively enhanced by some dynamical mechanism, whereas the representation 27 is not enhanced, and may even be suppressed.

In what follows, without trying to predetermine which of the hypotheses quoted is correct, we shall make use only of the fact that the amplitudes of weak hydronic decays are dominated by the octet component of the Lagrangian, and that the latter conserves CP-parity. From these assumptions it follows that the dominant component of the Lagrangian of the strangeness changing weak hadronic interaction is the 6th component of the octet. In order to demonstrate this, we recall that an octet of mesons, currents or interactions that goes over into itself under CP-transformation contains 8 real components (see above the paragraph on the eight-current matrix). Five of the eight components (3, 8, 1, 4, 6) have the same CP-parity, whereas remaining three components (2, 5, 7) have opposite parity.

The Lagrangian of the weak hadronic interaction might, in general, contain four charge-conserving components of an octet: H_3, H_8, H_6, H_7. Two of these, H_3 and H_8, conserve strangeness, and their CP-parity is even. The other two components, H_6 and H_7, do not conserve strangeness but do conserve electric charge. However, a Lagrangian conserving CP-parity can contain only one of these, i.e. H_6, having the same CP-parity as H_3 and H_8.

θ-DECAY

We shall now show that the Lagrangian H_6 forbids $K_1^0 \to 2\pi$ decays which are allowed by the $\Delta T = \frac{1}{2}$ rule. For this purpose we shall first show that the K_1^0-meson is the 7th component of a pseudoscalar meson octet. Indeed, the CP-parity of the K_1^0-meson is even, whereas that of the π^0-meson (the 3rd component) and the η^0-meson (the 8th component) is odd. Consequently, the K_1^0-meson is the seventh component of the octet.

From this it follows immediately, for instance, that SU_3-invariance forbids diagrams of the tadpole type (Fig. 78), which describe the transition of the virtual K_1^0-meson into the vacuum under the action of the weak interaction. (In a number of studies it was assumed that it is these diagrams that are responsible for the dynamical enhancement of transitions with $\Delta T = \frac{1}{2}$.)

Let us now consider the $K_1^0 \to 2\pi$ decay. The discussion is more conveniently given if a spurion h, representing the 6th component of an octet, is introduced. Then the $K_1^0 \to 2\pi$ decay can be written in the form of a unitary-invariant reaction in which four 8-vectors take part:

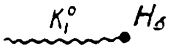


FIG. 78

$$h + K_1^0 \to 2\pi .$$

The most general amplitude of a reaction $A + B \to C + D$, where A, B, C, D are 8-vectors, has the form

$$\begin{aligned} M = {} & a_1(AB\bar{C}\bar{D}) + a_2(AB\bar{D}\bar{C}) + a_3(A\bar{C}\bar{D}B) + a_4(A\bar{C}B\bar{D}) \\ & + a_5(A\bar{D}B\bar{C}) + a_6(A\bar{D}\bar{C}B) + a_7(AB)(\bar{C}\bar{D}) \\ & + a_8(A\bar{C})(\bar{D}B) + a_9(A\bar{D})(B\bar{C}). \end{aligned}$$

Here parentheses denote a trace, for example,

$$ABCD = A_\alpha^\beta B_\beta^\gamma C_\gamma^\delta D_\delta^\alpha .$$

It should be noted that nine unitary scalars satisfy one linear relation:

$$\begin{aligned} & (AB\bar{C}\bar{D}) + (AB\bar{D}\bar{C}) + (A\bar{C}\bar{D}B) + (A\bar{C}B\bar{D}) + (A\bar{D}B\bar{C}) + (A\bar{D}\bar{C}B) \\ & \qquad = (AB)(\bar{C}\bar{D}) + (A\bar{C})(\bar{D}B) + (A\bar{D})(B\bar{C}) \end{aligned}$$

However, it is sometimes more convenient not to employ this relation and to write the amplitude in the symmetric form shown above.

In order to prove the forbiddenness of the $K_1^0 \to 2\pi$ decay, we consider the charge conjugation operation. Under charge conjugation

$$A = A_\beta^\alpha \to A_C = \eta \bar{A}_\alpha^\beta = \eta \tilde{\bar{A}} .$$

In the case that $A \neq \bar{A}$ we shall assume $\eta = 1$. When $A = \bar{A}$ we shall take $\eta = +1$ if the 8th (and, consequently, 1st, 3rd, 4th, 6th) component of the octet A has even charge parity, whereas, if the charge parity of the 8th component is odd, we shall take $\eta = -1$.

The $K_1^0 \to 2\pi$ decay goes with violation of both P- and C-parity. The amplitude of this decay satisfies the conditions $CP = +1$, $P = -1$, $C = -1$. The same conditions must be satisfied by the amplitude of the reaction $h + K_1^0 \to 2\pi$. In particular, from the last condition it follows that the relation $M_C = -M$ must hold. We shall now convince ourselves of the fact that this relation contradicts the unitary symmetry requirement. Performing the substitution $A = h$, $B = C = D = \varphi$ and taking into account that the three mesons enter symmetrically into the space part of the matrix element of the $K_1^0 \to 2\pi$ decay, we obtain:

$$\begin{aligned} M = a[&(h\varphi_1\varphi_2\varphi_3) + (h\varphi_1\varphi_3\varphi_2) + (h\varphi_2\varphi_3\varphi_1) \\ &+ (h\varphi_2\varphi_1\varphi_3) + (h\varphi_3\varphi_1\varphi_2) + (h\varphi_3\varphi_2\varphi_1)] \\ &+ b[(h\varphi_1)(\varphi_2\varphi_3) + (h\varphi_2)(\varphi_3\varphi_1) + (h\varphi_3)(\varphi_1\varphi_2)]. \end{aligned}$$

Taking into account that under charge conjugation $\varphi \to \tilde{\varphi}$ and that by definition $h \to h$, we get

$$(h\varphi_1\varphi_2\varphi_3) \to (\varphi_3\varphi_2\varphi_1 h) = (h\varphi_3\varphi_2\varphi_1)$$

$$(h\varphi_1)(\varphi_2\varphi_3) \to (h\varphi_1)(\varphi_2\varphi_3) \quad \text{and so on.}$$

From this it follows that $M_C = M$, and, consequently, $M = 0$.

Thus, in the limit of strict unitary symmetry the $K_1^0 \to 2\pi$ decay is forbidden, while under the conditions of approximate unitary symmetry it is suppressed. In experiment this decay is found to be about 500 times more probable than the $K^+ \sim 2\pi$ decay which takes place on account of the violation of the $\Delta T = \frac{1}{2}$ rule. It can be assumed that, if there were no unitary forbiddenness for $K_1^0 \to 2\pi$, this ratio would be of the order of 10^4.

This conclusion is of great importance. It means that the accuracy to within which the $\Delta T = \frac{1}{2}$ rule holds is about the same as that to within which the isotopic invariance ("$\Delta T = 0$ rule") holds in strong interactions.

VIOLATION OF THE $\Delta T = \frac{1}{2}$ RULE

The problem of the mechanism of violation of the selection rule $\Delta T = \frac{1}{2}$ is closely connected with that of the origin of this rule. Therefore it can be expected that an investigation of the first problem will help us to find an answer to the second.

If the accuracy of the $\Delta T = \frac{1}{2}$ rule is of the same order as that of the conservation of isotopic spin in strong interactions, then it would be natural to assume that departures from the $\Delta T = \frac{1}{2}$ rule, like the violation of isotopic invariance, is due to virtual photons. However, the situation here is perhaps not so simple. Our suspicion is inspired by the following numerical agreement, pointed out by Schwinger. The product of two phenomenological currents —a vector current $K^+\pi^0$ with $T = \frac{1}{2}$ (from K_{l3}-decays) and an axial current π^+ with $T = 1$ (from the $\pi_{\mu 2}$-decay)—gives for the matrix element of the $K^+ \to 2\pi$ decay, forbidden by the $\Delta T = \frac{1}{2}$ rule, a value very close to the observed one (the calculated value is twice as large as the experimental one).

If this estimate is taken seriously, then we should conclude that the transitions with $\Delta T = \frac{1}{2}$, contained in the product of charged hadronic currents, are enhanced, and that it is those with $\Delta T = 3/2$, although slightly suppressed, that are responsible for departures from the $\Delta T = \frac{1}{2}$ rule, which have been observed in the $K^+ \to 2\pi$ decay and are to be observed in other decays when the experimental accuracy becomes higher. The role of virtual photons is in this case unessential. On the other hand, the following possibilities are not excluded: the component of the initial Lagrangian with $\Delta T = 3/2$ is suppressed (for example, owing to neutral currents), the observed departures from the $\Delta T = \frac{1}{2}$ rule are due to virtual photons, and the numerical agreement in the $K^+ \to 2\pi$ decay is accidental.

Finally, it may be that the order of magnitude of the contribution of virtual photons is the same as that of the contribution of original current × current transitions with $\Delta T = 3/2$. In this case the interaction with $\Delta T = 3/2$ would be in the same ratio to the interaction with $\Delta T = \frac{1}{2}$ as the virtual electromagnetic interaction is to the strong isotopically invariant interaction.

In finding out which one of these possibilities is realized in nature, valuable information can be yielded by data on the relative intensity of transitions with $\Delta T = 5/2$ and $\Delta T = 3/2$. This is connected to the fact that the ratio between these transitions is different for the different theories described above.

Thus, if virtual photons are unessential, then we expect that transitions with $\Delta T = 3/2$ will be considerably more intense than those with $\Delta T = 5/2$ (the latter exist only owing to virtual photons). If virtual photons play an essential role in violating the $\Delta T = \frac{1}{2}$

rule, then the intensities of the transitions with $\Delta T = 3/2$ and with $\Delta T = 5/2$ should be of the same order.

Data on transitions with $\Delta T = 5/2$ can be obtained from an analysis of the τ-decay of K^+- and K_2^0-mesons.

HADRONIC DECAYS OF HYPERONS

It is known that the amplitude of the non-leptonic decay of hyperons contains two components—S and P—the first of which corresponds to a decay with $l = 0$, and the second to a decay with $l = 1$. The $\Delta T = \frac{1}{2}$ rule interrelates the $S(P)$ amplitudes of various decays within a given isotopic multiplet:

$$S(\Lambda^0 \to p\pi^-) = -\sqrt{2}S(\Lambda^0 \to n\pi^0),$$

$$S(\Xi^- \to \Lambda^0\pi^-) = -\sqrt{2}S(\Xi^0 \to \Lambda^0\pi^0),$$

$$\sqrt{2}S(\Sigma^+ \to p\pi^0) = S(\Sigma^- \to n\pi^-) - S(\Sigma^+ \to n\pi^+).$$

The same relations hold between corresponding P-amplitudes. As a result, the seven observed decays of hyperons are described by four independent $S(P)$ amplitudes.

The hypothesis that the Lagrangian of the non-leptonic weak interaction represents the 6th component of an octet imposes an additional condition upon the four independent S-amplitudes. To obtain this condition, we again make use of the spurion. By means of the spurion, the hyperon decay $B \to B + \pi$ can be written as a unitary-invariant reaction:

$$h + B \to B + \pi.$$

The most general form of the S-amplitude of this reaction is

$$S = \sum_{i=1}^{9} S_i I_i,$$

where

$$\begin{array}{lll} I_1 = (h\bar{B}\varphi B) & I_4 = (hB\bar{B}\varphi) & I_7 = (h\varphi\bar{B}B) \\ I_2 = (hB\varphi\bar{B}) & I_5 = (h\bar{B})(B\varphi) & I_8 = (h\varphi B\bar{B}) \\ I_3 = (h\bar{B}B\varphi) & I_6 = (hB)(\bar{B}\varphi) & I_9 = (h\varphi)(\bar{B}B). \end{array}$$

Making use of the explicit matrix form of the wave functions of 8-vectors, it is easy to obtain the expression for the S-amplitudes of various decays (we recall that $h = \lambda_6$):

$$S(\Lambda^0 \to p\pi^-) = \frac{1}{\sqrt{6}}(S_1 + S_3 - 2S_4),$$

$$S(\Xi^- \to \Lambda^0\pi^-) = \frac{1}{\sqrt{6}}(S_2 - 2S_3 + S_4),$$

$$S(\Sigma^+ \to p\pi^0) = \frac{1}{\sqrt{2}}(S_1 - S_3),$$

$$S(\Sigma^+ \to n\pi^+) = S_1 + S_5,$$

$$S(\Sigma^- \to n\pi^-) = S_3 + S_5.$$

Let us see what restrictions are imposed on the S-amplitudes by the requirement of combined parity conservation. It should be recalled that the amplitude for hyperon decay has the form $\bar{\psi}(S + P\gamma_5)\psi\pi$, where ψ and π are the space parts of the wave functions of the baryon and the meson, respectively. Both components in the amplitude must have even CP-parity. The space part of the amplitude has $P = +1$, $C = +1$ for the P-wave, and $P = -1$, $C = +1$ for the S-wave.

Consequently, by virtue of CP-parity conservation the unitary-isotopic factors P and S must possess, respectively, even and odd charge parity. (If the factor S did not change sign under charge conjugation, the amplitude $S\bar{\psi}\psi\pi$ would be forbidden by virtue of CP-parity conservation, as is the case, for instance, for the vertex $\bar{p}p\pi$.) In what follows we shall make use of this.

Under charge conjugation

$$B = B^\alpha_\beta \to B_C = \bar{B}^\beta_\alpha = \tilde{\bar{B}}, \quad \varphi \to \tilde{\varphi}, \quad h \to h,$$

and, consequently,

$$\begin{array}{ll} I_1 \to I_1 & I_3 \leftrightarrow I_7 \\ I_2 \to I_2 & I_4 \leftrightarrow I_8 \\ I_9 \to I_9 & I_5 \leftrightarrow I_6. \end{array}$$

Since the S-amplitude must have odd charge parity ($S = -S_C$), it follows that the relations

$$S_1 = S_2 = S_9 = 0, \quad S_3 = -S_7, \quad S_4 = -S_8, \quad S_5 = -S_6$$

must hold. Making use of the fact that S_1 and S_2 are equal to zero, the relation

$$S(\Lambda^0 \to p\pi^-) + 2S(\Xi^- \to \Lambda^0\pi^-) = \sqrt{3}S(\Sigma^+ \to p\pi^0)$$

can easily be derived. This relation is in rather a good agreement with experiment.

Writting the amplitude of the P-wave in the form

$$P = \sum_{i=1}^{9} P_i I_i$$

and making use of the condition $P_C = P$, it is easy to obtain the relations $P_3 = P_7$, $P_4 = P_8$, $P_5 = P_6$. However, these equalities do not lead to any relations among the observed P-amplitudes of hyperon decays.

PARITY NON-CONSERVING NUCLEAR FORCES

In elucidating the nature of the hadronic rule $\Delta T = \frac{1}{2}$ the knowledge of the isotopic structure of parity non-conserving nuclear forces may turn out to be of fundamental importance.

According to the "current × current" hypothesis a strangeness conserving weak non-leptonic interaction arises (in multiplying the total weak current by the Hermitian conjugate current) along with the strangeness changing weak non-leptonic interaction.

A weak hadronic interaction that does not conserve parity but does conserve strangeness has recently been discovered experimentally (see the beginning of Chapter 20) in the form of a small admixture in nuclear forces. A strangeness conserving weak interaction between two nucleons might, in general, contain terms with $\Delta T = 0$, $\Delta T = 1$ and $\Delta T = 2$. However, if this interaction enters an octet with the non-leptonic interaction H_6 considered above (with $|\Delta Y| = 1$ and $\Delta T = \frac{1}{2}$) then $\Delta T \neq 2$, since in the octet there are no components with $T = 2$ (they enter the representation 27). The strangeness conserving interaction is described in this case by two components: H_8 ($\Delta Y = 0, \Delta T = 0$) and H_3 ($\Delta Y = 0$, $\Delta T = 1$). The ratio between the components H_3 and H_8 turns out to be sensitive to the nature of the mechanism giving rise to the octet properties of the weak hadronic Lagrangian (and, consequently, to the $\Delta T = \frac{1}{2}$ rule).

If octet properties arise without additional currents, and are due to a dynamical enhancement of the octet part of the product of the total charged current, then a ratio $H_8 \gg H_3$ must hold. This is associated with the fact that the strangeness conserving current $(j_1 + ij_2) \cos\vartheta$ gives a contribution only to H_8, whereas a contribution to H_3 is given only by the strange current $(j_4 = ij_5) \sin\vartheta$.

Hence $H_3 \sim \tan^2\vartheta H_8$, where, from the data on leptonic decays, $\tan^2\vartheta \sim 1/15$, and we may conclude that parity non-conserving nuclear forces must be approximately isotopically invariant ($\Delta T \sim 0$). But, if octet properties arise from a compensation due to additional currents, then there are no special reasons for assuming H_8 to be considerably larger than H_3. Therefore, in this case presumably $H_3 \sim H_8$ and parity non-conserving nuclear forces are not isotopically invariant.

To elucidate the isotopic structure of parity non-conserving nuclear forces, experiments with light nuclei are necessary. Thus, for instance, the intensity of the interaction with $\Delta T = 0$ might be estimated on the basis of the rate of the α-decay from the excited 8·8 MeV level of the O^{16} nucleus:

$$O^{16*}(2^-, T = 0) \rightarrow C^{12}(0^+, T = 0) + He^4(0^+, T = 0).$$

The presently established upper limit for the intensity of this decay amounts to $\sim 10^{-12}$ of the normal intensities for allowed α-decays.

The magnitude of the admixture with $\Delta T = 1$ might be estimated from the degree of circular polarization of photons produced in transitions from an excited level of the B^{10} nucleus, having an energy of 5·11 MeV, $J^P = 2^-$, $T = 0$, to lower levels with $J^P = 1^+$, 2^+, 3^+ and $T = 0$. The circular polarization will arise from the interference of the parity conserving electric transition EI and the parity violating magnetic transition MI. Beside the 5·11 MeV level with $J^P = 2^-$ and $T = 0$ there is a 5·16 MeV level with $J^P = 2^+$ and $T = 1$. The closeness of this level having different parity may enhance the MI-transition, if parity non-conserving nuclear forces have a considerable component with $\Delta T = 1$. Otherwise this enhancement will be small.

It should be noted that the experiment under discussion is very difficult, and that its theoretical interpretation may conceal many a "sunken reef". Nevertheless, information which this and similar experiments can yield is so fundamental that they deserve the most serious attention.

OTHER GROUPS OF RANK TWO

The group SU_3 is a rank-two group, since two additive quantum numbers, T_3 and Y, correspond to it. It is one of the four so-called semi-simple Lie groups of rank two. The remaining three are denoted by O_4, O_5 and G_2. The groups O_4 and O_5 are rotation groups in four-dimensional and five-dimensional space, respectively. The group G_2 is a special Cartan group.

A disadvantage of all these groups in comparison with SU_3 is the fact that the T_3Y-diagrams of their representations are symmetric with respect to the line $Y = 0$, whereas in nature, apparently, there are supermultiplets which do not possess such a symmetry. The group SU_3, as is known, contains not only symmetric representations (e.g. 8) but also unsymmetric ones (e.g. 10). However, it is not excluded that, with discovery and systematization of new resonances, we shall have to recur to other groups.

GROUPS OF A HIGHER RANK

If supercharged particles are discovered, then in the strong interaction we shall have to deal no longer with two but with three quantum numbers: T_3, Y, Z. In essence, we shall pass from the group SU_3 to the product of groups $U_1 . SU_3$. Naturally, the question arises as to whether in this case the particles will be separated into super-supermultiplets, grouping particles having different T, Y, Z but the same spin and parity, closely spaced masses and similar strong interactions.

Such super-supermultiplets have been considered in a number of studies for the unitary rank-three group SU_4 and symplectic rank-three group SP_6. Referring the reader to the survey of literature, we find it useful to make a remark concerning the notation. In some studies, instead of the relation

$$Q = T_3 + \frac{Y}{2}$$

that we are using, a relation

$$Q = T_3 + \frac{Y'}{2} + \frac{Z}{3}$$

is used. In such a definition of charge, T_3, Y' and Z are the eigenvalues of the group generators, and this, in a number of cases, may present certain advantages. But, if the conservation of Z turns out to be violated by the moderately strong or electromagnetic interaction, then the conservation of Y' will also be violated in these interactions, and so it is more reasonable to make use of the hypercharge Y, which is conserved in these interactions.

SPACE-UNITARY SYMMETRIES

Besides supermultiplets grouping particles with given values of spin and parity, the possibility is also discussed in the literature of the existence of super-supermultiplets grouping particles with different spatial properties, for instance, particles of different parity. Such theoretical schemes (e.g. those for the $SU_3 \times SU_3$ symmetry) have as an underlying hypothesis that the fundamental Lagrangian of the SU_3-invariant strong interaction possesses also γ_5-invariance. Degeneracy in parity, associated with γ_5-invariance, must also be manifested in the case of physical particles, provided the violation of this invariance is not too strong. One of the main consequences of the scheme of $SU_3 \times SU_3$ symmetry is the prediction of the existence of nine scalar mesons with masses of the same order as the nine pseudoscalar mesons.

CHAPTER 21

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The literature covered by the survey will help the reader to get a more detailed knowledge of the present state of the problems considered in the book and also, in a number of cases, of the history of these problems. In accordance with the general design of the book, this survey of the literature is divided into the following sections:

Particles. Models. Interactions
Weak interaction Lagrangian
Interactions of leptons
Strangeness conserving leptonic decays
Strangeness changing leptonic decays
Non-leptonic decays
Neutral K-mesons
Parity non-conservation in nuclear forces
Weak interactions at small distances
Weak interaction and unitary symmetry

PARTICLES. MODELS. INTERACTIONS

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WEAK INTERACTION AND UNITARY SYMMETRY

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WEAK INTERACTIONS OF ELEMENTARY PARTICLES

L. B. Okun'

ERRATA

p. 34, line 6: for $\varphi = \chi_1 + \chi_2$ read $\varphi = \chi_1 - \chi_2$
p. 35, line 15: for $e\bar{\psi}\gamma_\alpha\psi_e A_\alpha$ read $e\bar{\psi}_e\gamma_\alpha\psi_e A_\alpha$
p. 52, line 10: for $C = (-1)^{1+s}$ read $C = (-1)^{l+s}$
p. 73, line 12: for $\hat{k}_1\gamma_\alpha k_2$ read $\hat{k}_1\gamma_\alpha\hat{k}_2$
p. 85, line 10: for $\bar{\psi}\gamma_{\alpha\tau^+\psi}$ read $\bar{\psi}\gamma_\alpha\tau^+\psi$
p. 90, line -4: for $\bar{p}n + \sqrt{2}\varphi^+ + \varphi_0 + \sqrt{2}\varphi_0^+\varphi_-$
read $\bar{p}n + \sqrt{2}\varphi^\pm\varphi_0 + \sqrt{2}\varphi_0^+\varphi_-$
p. 90, line -1: for $T^- = T_1 = iT_2$ read $T^- = T_1 - iT_2$